Handout 2: Gain and Phase margins

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Nyquist plots and Cauchy's principle

Let $H(s)$ be a transfer function. eg $H(s) = \dfrac{s^2 + s + 1}{(s+1)(s+3)}$

Evaluate H on a contour in the s-plane. (your plots here)

$$H = \frac{s^2 + s + 1}{(s+3)(s-3)}$$

Evaluate H on another contour of the s-plane (your plots here)

Cauchy's Principle:

Control application: Given $KG(s)$, we encircle the entire
to get the contour evaluation of

Closed-loop roots are poles of

They are zeros of

If there are no RHPs, then $1 + KG$ encirclement of 0 means

With no RHP poles, KG encirclement of -1 means

With right half plane open-loop poles

A clockwise contour enclosing a zero of $1 + KG(s)$ will result in

A clockwise contour enclosing a pole of $1 + KG(s)$ will result in

Nyquist plot rules

1. Plot $KG(s)$ for $s = -j\infty$ to $+j\infty$

2. Count number of

3. Determine number of

4. Nunber of unstable closed-loop roots is

Example: $G(s) = \dfrac{1}{s^2 + 3s + 1}$

Bode plot

Nyquist plot

Example: $G(s) = \dfrac{1}{s(s+1)^2}$

Bode plot

Nyquist plot

Gain and Phase margins

Nyquist plot for $G(s)$.

Gain Margin is

Phase Margin is

$$G(s) = \frac{1}{s^2 + 3s + 1}$$

$$G(s) = \frac{1}{s(s+1)^2}$$