Second Homework

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1. **Singularities on $j\omega$- axis and Nyquist plots.**

   Draw a Nyquist diagram for each of the following systems, choosing the contour to be to the right of any singularities on the $j\omega$-axis.

   (a) $KG(s) = \dfrac{K(s+10)}{s^2(s+100)}$.

   (b) $KG(s) = \dfrac{K(s+0.1)}{s(s+0.2)}$.

   (c) $KG(s) = \dfrac{K}{(s+2)(s^2+20)}$.

   (d) Redo the Nyquist plots in parts (b) and (c), this time choosing the contour to be to the left of all singularities on the imaginary axis.

   (e) Use the Nyquist stability criterion to determine the range of K for which each system is stable, and check your answer using a root-locus plot, generated by hand of a computer.

2. **Bode and Nyquist plots for unstable systems.**

   Consider the transfer function

$$G(s) = \frac{100(s/100+1)}{s(s/10-1)(s/1000+1)}$$

   (a) Using Matlab, draw the Bode plot for this system

   (b) Sketch the Nyquist plot for $G(s)$.

   (c) Is the corresponding closed-loop system stable?

3. **Closed-loop formulas for Phase Margin.**

(a) If a system has the open-loop transfer function

$$G(s) = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$

with unity feedback, then what is the closed-loop transfer function?

Compute the values of the phase margin for $\zeta = 0.1$, 0.4, and 0.7.

(b) Plot the step response of the closed-loop system for $\zeta = 0.1$, 0.4, 0.7 and 1. Plot the overshoot vs. ζ (use six or seven values of ζ).

(c) Show that the phase margin PM can be computed as follows:

$$\mathrm{PM} = \tan^{-1} \frac{2\zeta}{\sqrt{\sqrt{1 + 4\zeta^4} - 2\zeta^2}}.$$

4. Consider the unity feedback system with the open-loop transfer function

$$G(s) = \frac{K}{s(s+1)(s^2/16 + 0.4s/4 + 1)}.$$

(a) Draw the Bode plot (gain and phase) for $G(j\omega)$ assuming $K = 1$.

(b) What gain K is required for a phase margin of 45 degrees? What is the gain margin for that value of K?

(c) Sketch a root locus with respect to K and indicate the roots for a PM of 45 degrees.