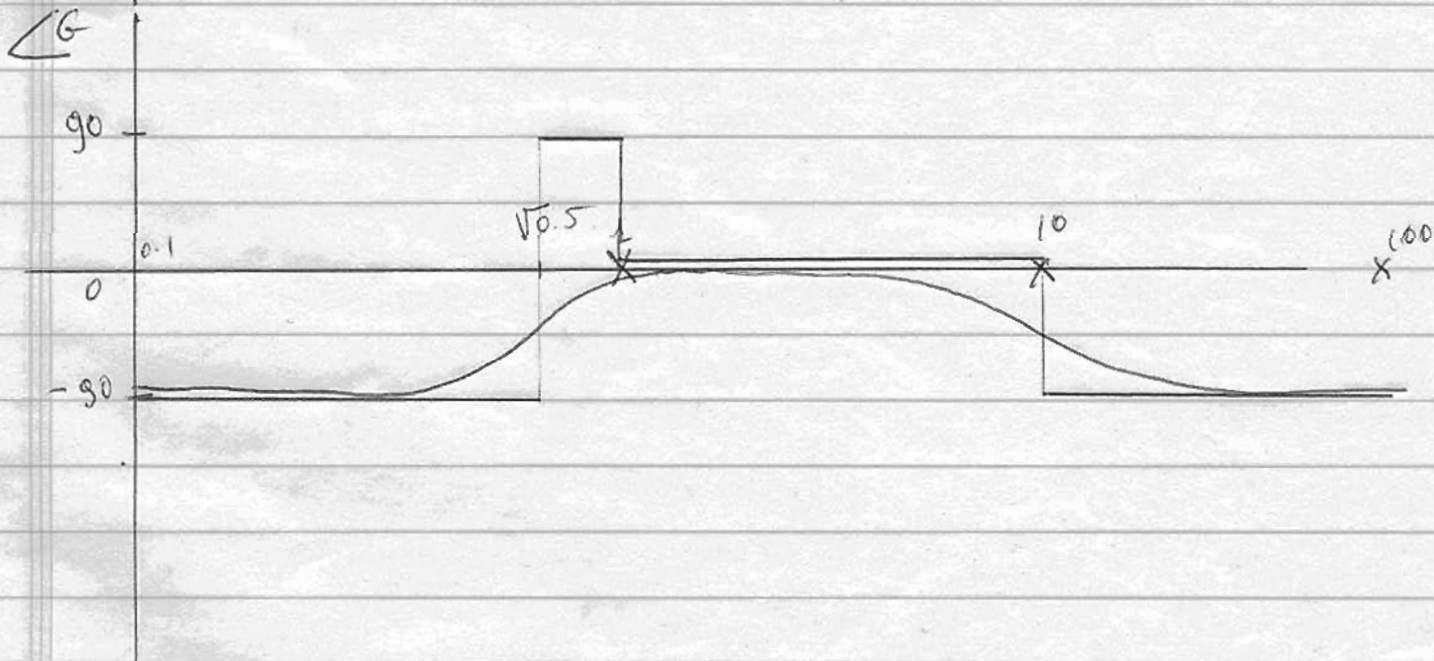
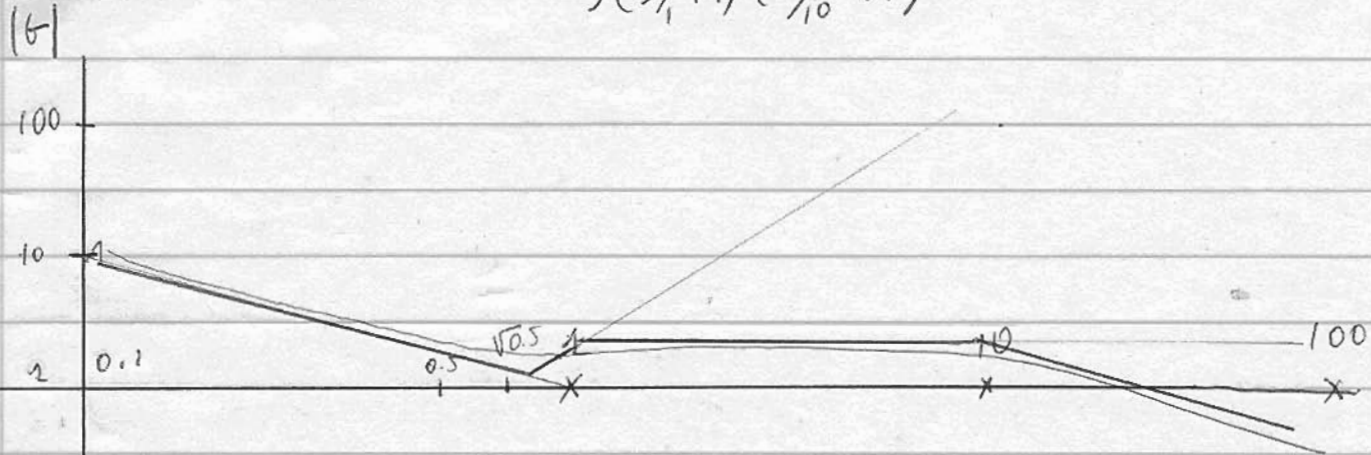


Quiz solutions

$$G(s) = \frac{((s/\sqrt{0.5})^2 + \frac{s}{0.5} + 1)}{s(s_1+1)(s_{10}+1)}$$



2. $G(s) = \frac{sT}{1+sT}$

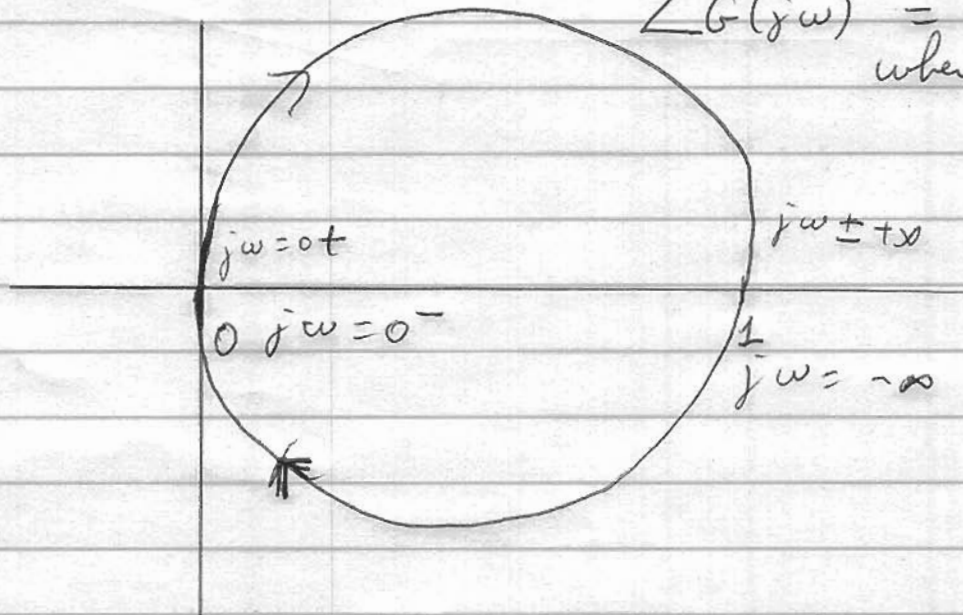
a) We use the reduced variable:

$s_1 = sT$

Then $G(s_1) = \frac{s}{1+s}$

$j\omega = 0 \quad G(j\omega) = 0$
 $j\omega = \infty \quad G(j\omega) = 1$

$\angle G(j\omega) = 90^\circ$
 when $j\omega \ll 0$
 $\omega > 0$



b) If it's a circle, the center is most likely $+1/2$.

Compute: $\left| G(j\omega) - \frac{1}{2} \right| = \left| \frac{s}{1+s} - \frac{1}{2} \right|$

$s = j\omega \quad = \left| \frac{s - 1/2 - s/2}{1+s} \right|$

$= \left| \frac{s/2 - 1/2}{1+s} \right|$

3

$$= \left| \frac{1}{2} \left(\frac{j\omega - 1/2}{j\omega + 1/2} \right) \right| = \frac{1}{2}$$

~~radius is const~~

distance between $G(j\omega)$ and $1/2$ is constant, so

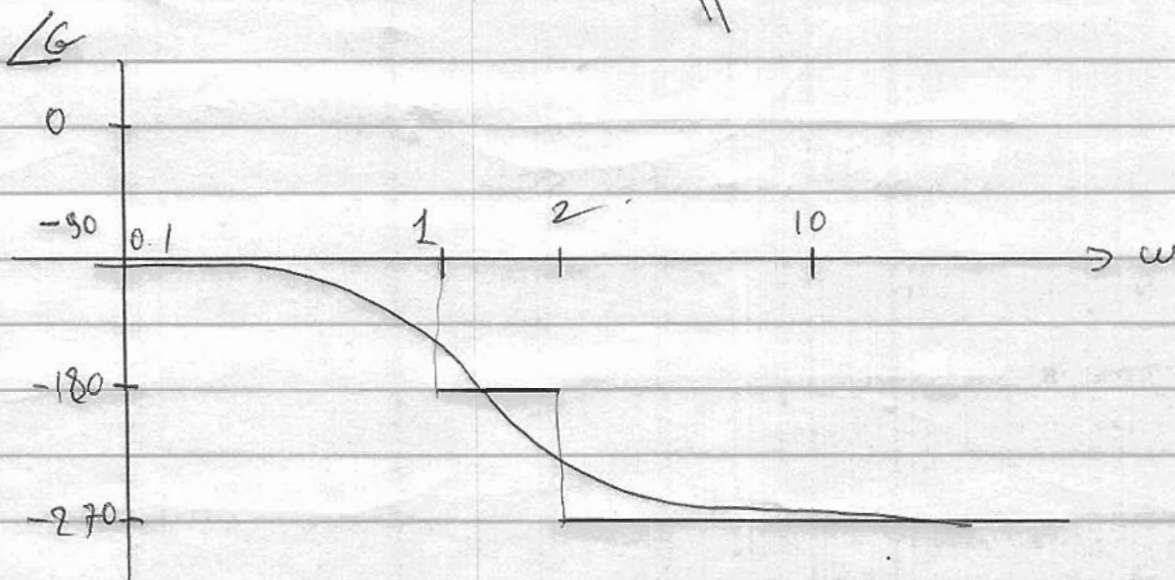
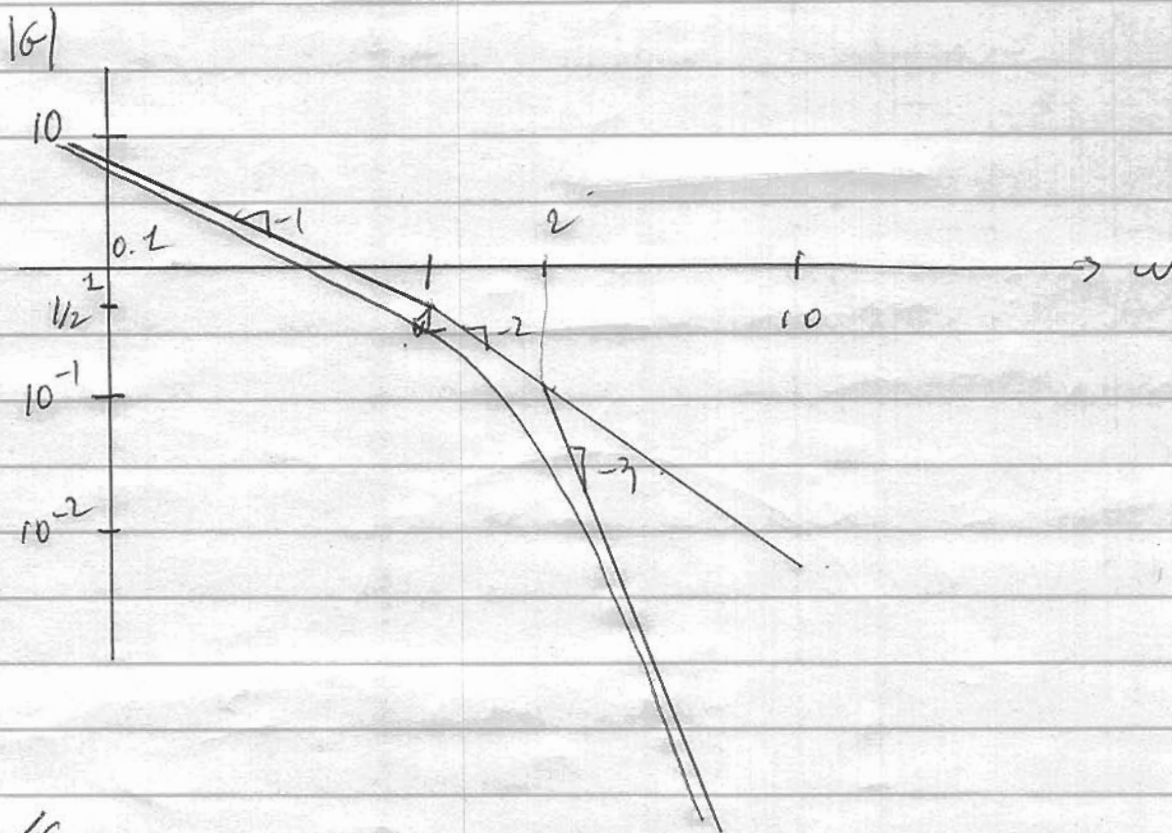
it's a circle of radius $1/2$.

So c is done too -

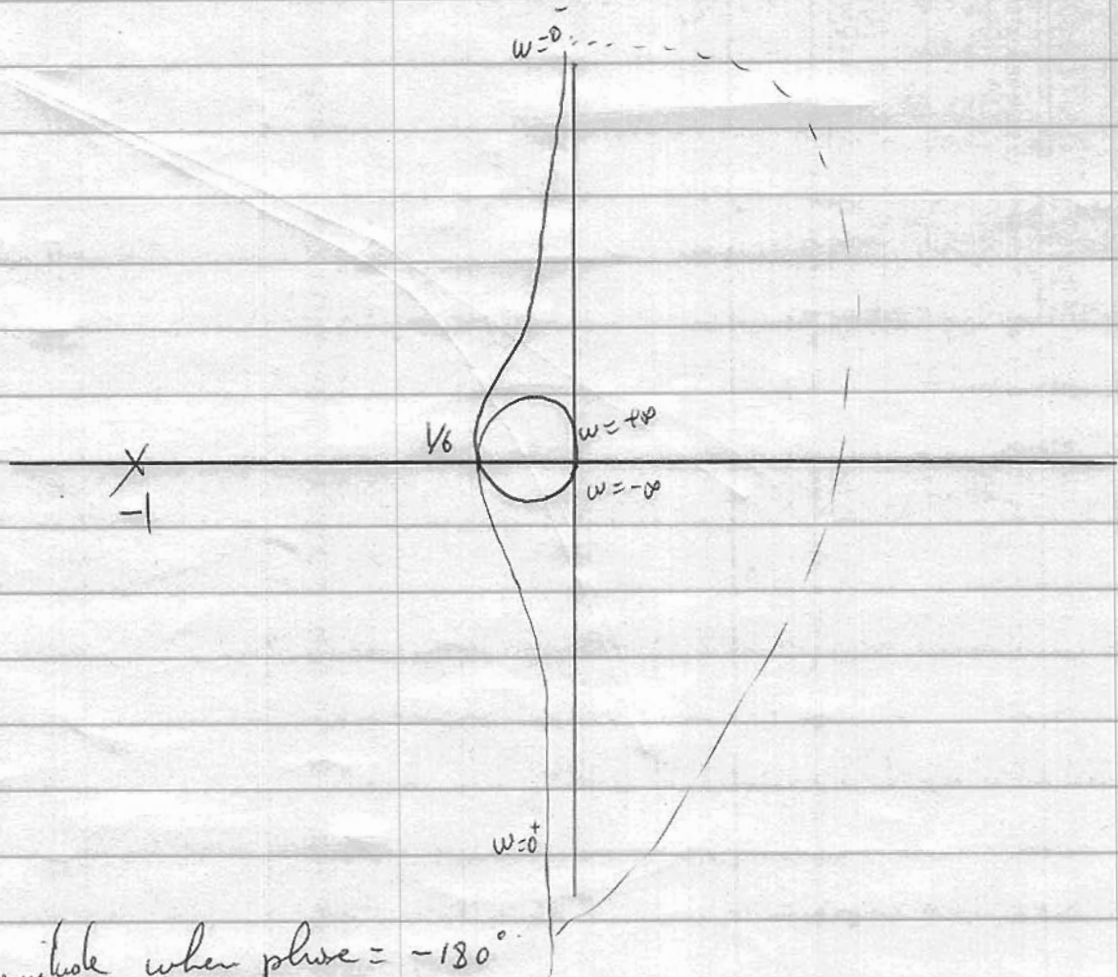
3) $G(s) = \frac{K}{z(s)(s_1+1)(s_2+1)}$

4

a) Assume $K=1$.



b) Nyquist plot:



Magnitude when phase = -180°
~~1~~ $\frac{1}{j\omega(j\omega+1)(j\omega+2)} = X_{\text{real}}$

$$1 = X(j\omega)(j\omega+1)(j\omega+2)$$

$$= X(-\omega^2 + j\omega)(j\omega+2)$$

$$= X(-j\omega^3 - 2\omega^2 - \omega^2 + 2j\omega)$$

$$= X(-j\omega^3 - 3\omega^2 + 2j\omega)$$

So: $-\omega^3 + 2\omega = 0$ or: $\omega^2 = 2$ or $\omega = \sqrt{2}$

and: $X(-3 \times 2) = 1$ or: $X = 1/6$

c) The system goes unstable for $K=6$. ⑥

d) From the bode plot, the phase of the transfer function is 135° when ω is about 0.5. pick $\omega = 0.2$ for safety.

$$G(0.2) = \frac{1}{0.2j(0.2j+1)(0.2j+2)}$$

$$\approx \frac{5}{j(0.2j+1)(0.2j+2)}$$

$$\text{phase} \approx -\frac{\pi}{2} - 0.2 - 0.1 \approx$$

$$-1.8 \text{ still } > \frac{3\pi}{4}$$

$$G = \frac{5}{1.1 \cdot 2.1} \approx 2$$

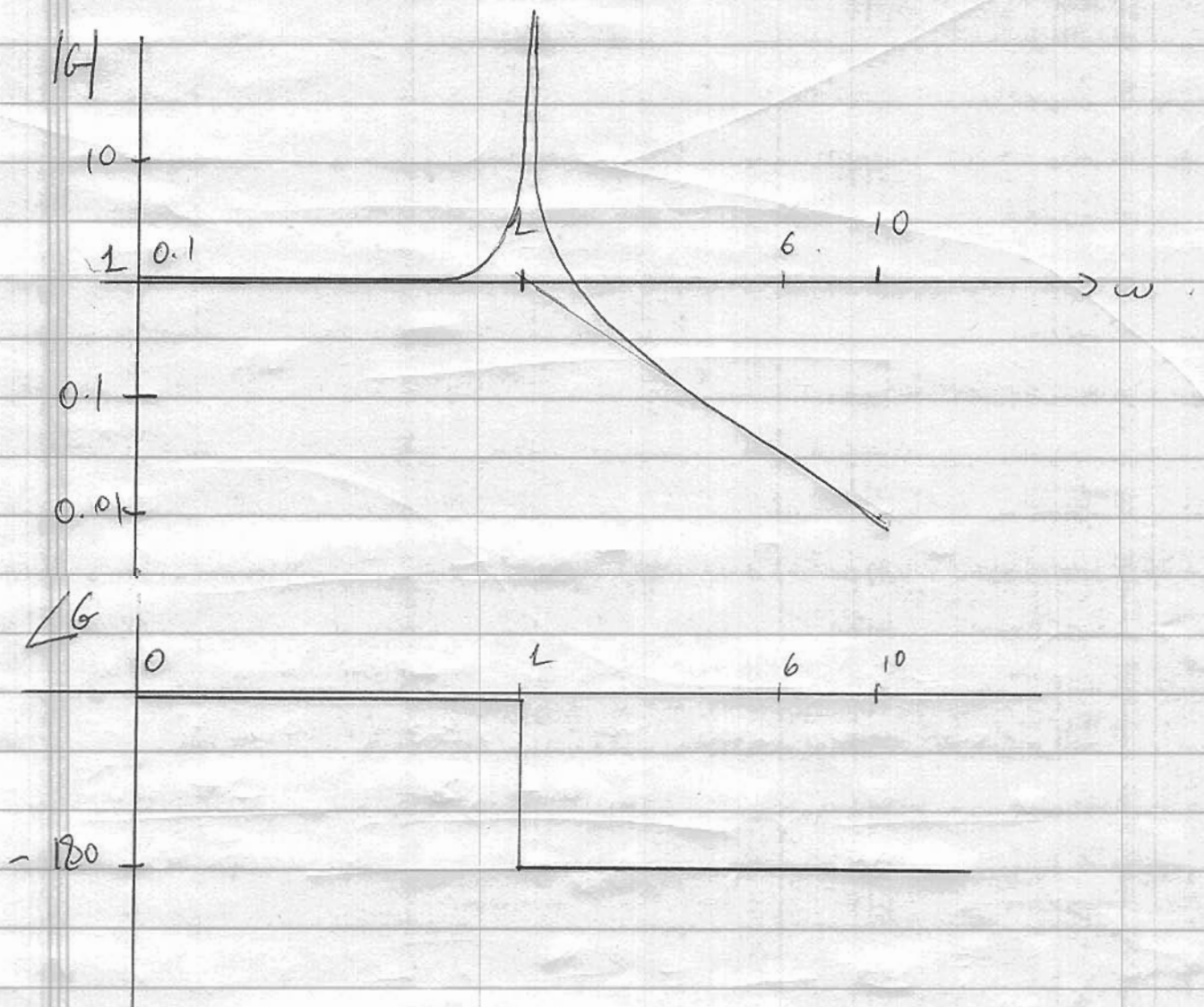
choose $K \approx 1/2$

not the only answer!

e) the resulting $\dot{\omega}$ is 0.2 rad/sec .
It's a shy controller!

4)
$$G(s) = \frac{1}{s^2 + 1}$$

Let's do a Bode plot.



8) To get perfect tracking ^{at DC}, let's add an integrator (log compensation) (2)

eg:
$$K_{lag}(s) = \left(\frac{(s/0.1 + 1)}{s} \right)$$

Now our desired BW is above 6 rad/sec.

Let's pick a lead compensator that gives us 60° lead at least ~~over the~~ at that frequency (6 rad/sec)

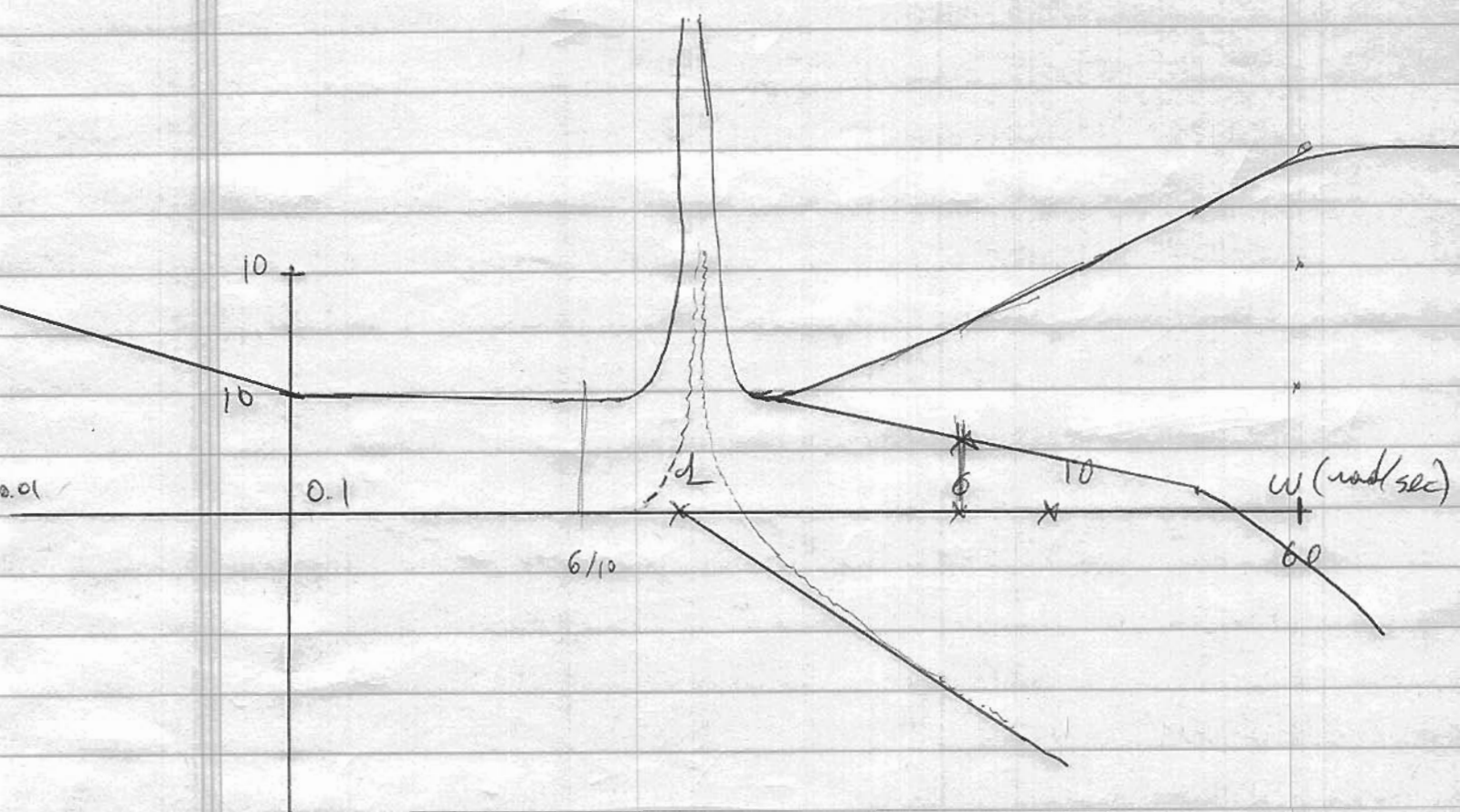
choose:

$$K_{lead} = \frac{s + 6/10}{s + 6 \times 10} \quad (\text{gives } 78^\circ \text{ lead})$$

our compensation so far looks like:

$$K = K_0 \left(\frac{s/0.1 + 1}{s} \right) \left(\frac{s + 6/10}{s + 60} \right) \times 100$$

Bode plot:



We must now adjust the gain - K_0
 at 6 rad/sec, the loop gain is:

$$\left(\frac{50j + 1}{6j} \right) \left(\frac{6j + 6/10}{6j + 60} \right) \times \frac{1}{-36 + 1} \times 100$$

$$\approx 100 \times 10 \times \frac{1}{10} \times \frac{1}{36} = \frac{100}{36} \approx 3$$

So pick $K_0 \approx 1/3$