Examples of Estimation Filters from Recent Aircraft Projects at MIT

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Vehicles & Navigation Sensors

OHS (Outboard Horizontal Stabilizer)

<u>Navigation Sensors</u> (Piccolo from Cloudcap Tech)

- GPS Motorola M12
- Inertial
  - 3 Tokin CG-16D rate gyros
  - 3 ADXL202 accelerometers
- Air Data
  - Dynamic & absolute pressure sensor
  - Air temperature sensor
- MHX 910/2400 radio modem
- MPC555 CPU

- **Crista Inertial Measurement Unit**
  - 3 Analog Devices ADXL accelerometers
  - 3 ADXRS MEMs rate sensors

<u>Navigation Sensors</u>

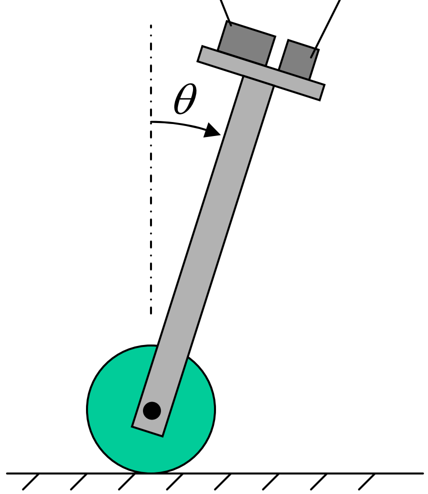
- GPS Receiver (Marconi, Allstar)
- Inertial Sensors

  - Crossbow 3-axis Accelerometer, Tokin Ceramic Gyro (MINI) or Crossbow IMU (OHS)
- Pitot Static Probe: measures airspeed
- Altitude Pressure Sensor

Complementary Filter (CF)

Often, there are cases where you have *two* different measurement sources for estimating *one* variable and the noise properties of the two measurements are such that one source gives good information only in low frequency region while the other is good only in high frequency region.
→ You can use a complementary filter !

Example : Tilt angle estimation using accelerometer and rate gyro

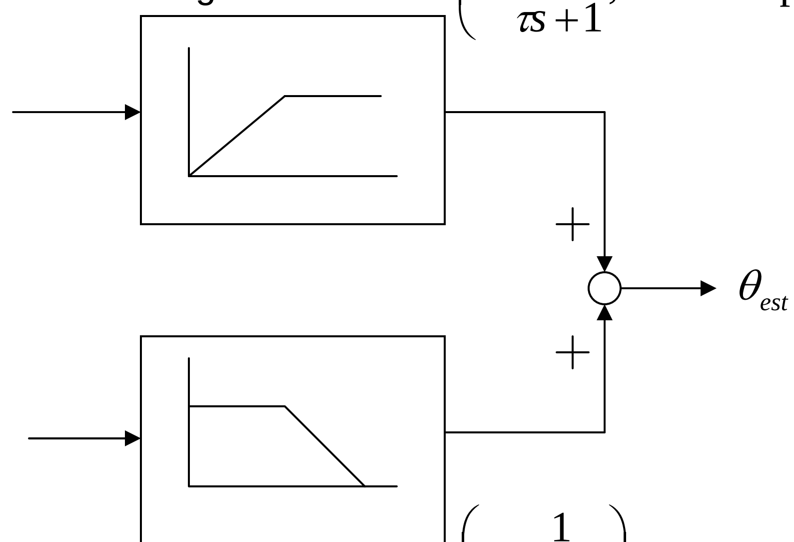


$$\theta \approx \int (\text{angular rate})\, dt$$

- not good in long term due to integration

$$\theta \approx \sin^{-1}\left(\frac{\text{accel. output}}{g}\right)$$

- only good in long term
- not proper during fast motion



High Pass Filter $\left(= \dfrac{\tau s}{\tau s + 1}, \text{ for example}\right)$

Low Pass Filter $\left(= \dfrac{1}{\tau s + 1}\right)$

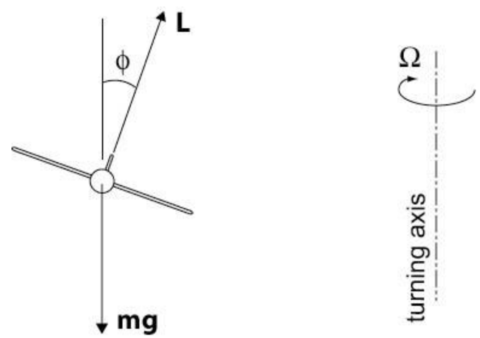
Complementary Filter(CF) Examples

- **CF1. Roll Angle Estimation**
- **CF2. Pitch Angle Estimation**
- **CF3. Altitude Estimation**
- **CF4. Altitude Rate Estimation**

CF1. Roll Angle Estimation

- High freq. : integrating roll rate (p) gyro output
- Low freq. : using aircraft kinematics

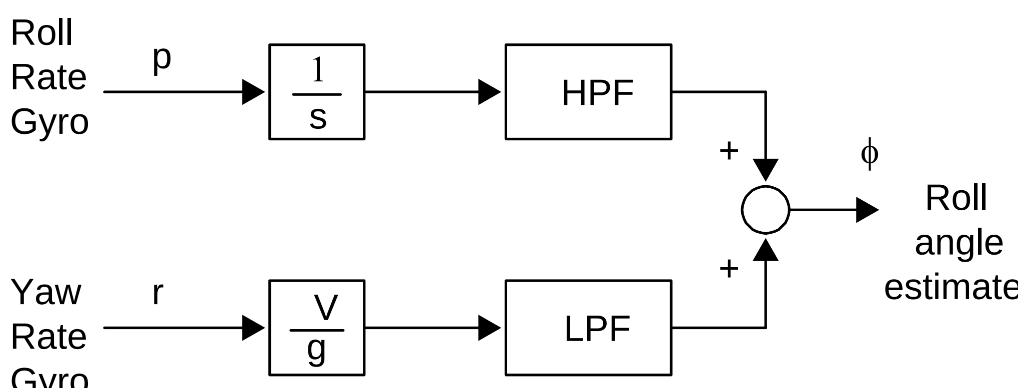
  - Assuming steady state turn dynamics,
    roll angle is related with turning rate, which is close to yaw rate (r)



$$L \sin\phi = mV\Omega$$
$$L \approx mg$$
$$\Omega \approx r$$
$$\sin\phi \approx \phi$$

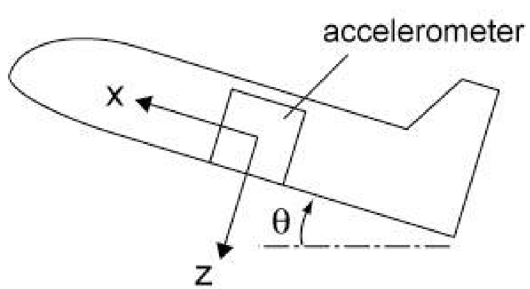
$$\Rightarrow \quad \phi \approx \frac{V}{g} r$$

CF setup



CF2. Pitch Angle Estimation

- High freq. : integrating pitch rate (q) gyro output
- Low freq. : using the sensitivity of accelerometers to gravity direction
  - *"gravity aiding"*



In steady state

$$A_X = g\sin\theta \qquad A_Z = -g\cos\theta \quad \Rightarrow \quad \theta = \tan^{-1}\left(-\frac{A_x}{A_z}\right)$$

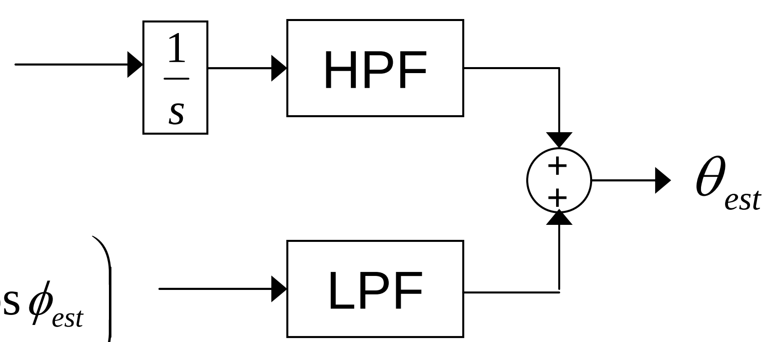
A_X , A_Z – accelerometer outputs

- Roll angle compensation is needed

<u>CF setup</u>

$$q_{meas},\ \phi_{est} \Rightarrow \dot{\theta} \approx q_{meas}\cos\phi_{est}$$

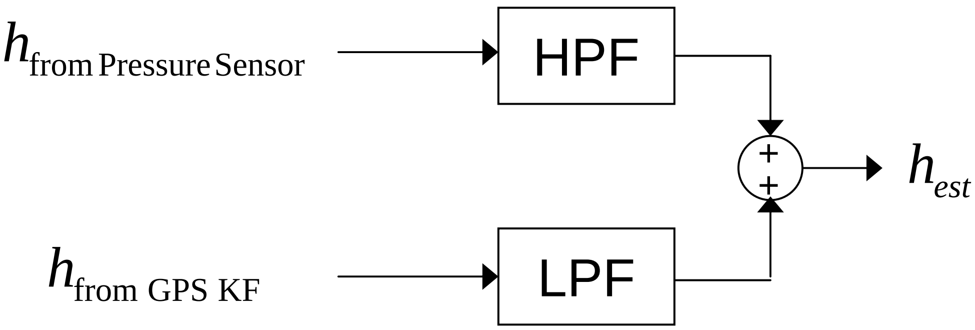
$$A_x,\ A_z,\ \phi_{est} \Rightarrow \theta_{ss} = \tan^{-1}\left(-\frac{A_x}{A_z}\cos\phi_{est}\right)$$

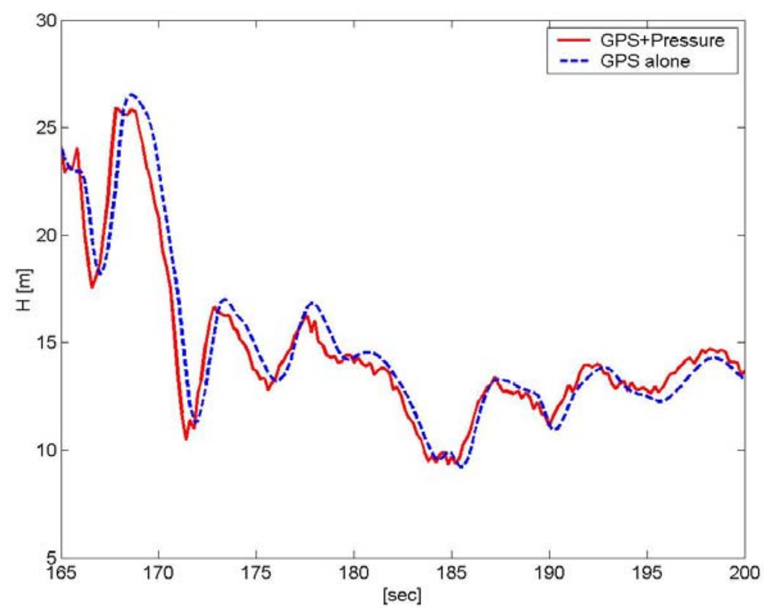


CF3. Altitude Estimation

- Motivation : GPS receiver gives altitude output, but it has ~0.4 seconds of delay. In order of overcome this, pressure sensor was added.
- Low freq. : from GPS receiver
- High freq. : from pressure sensor

CF setup & flight data

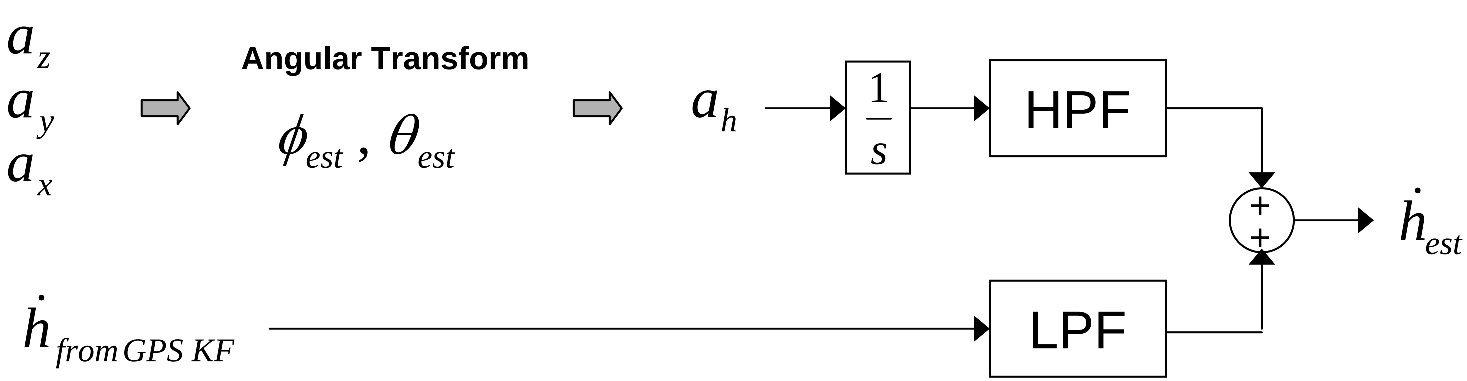




CF4. Altitude Rate Estimation

- Motivation : GPS receiver gives altitude rate, but it has ~0.4 seconds of delay. In order of overcome this, inertial sensor outputs were added.
- Low freq. : from GPS receiver
- High freq. : integrating acceleration estimate in altitude direction from inertial sensors

<u>CF setup</u>



$$\text{note :}\begin{Bmatrix} a_x \\ a_y \\ a_z \end{Bmatrix} = \begin{Bmatrix} A_x \\ A_y \\ A_z \end{Bmatrix} - [\phi_{est}][\theta_{est}]\begin{Bmatrix} 0 \\ 0 \\ -g \end{Bmatrix}$$

A_x , A_z – accelerometer outputs

$[\phi_{est}]$, $[\theta_{est}]$: angular transformation matrices

Kalman Filter(KF) Examples

- KF1. Manipulation of GPS Outputs
- KF2. Removing Rate Gyro Bias Effect

KF 1. Manipulation of GPS Outputs

<u>Background & Motivation</u>

- Stand-alone GPS receiver gives position and velocity

- These are obtained by independent methods :
  - position ← pseudo-ranges
  - velocity ← Doppler effect

  and are certainly related $(\dot{x} = v)$

  → *Kalman filter can be used to combine them !*

- Motivation :

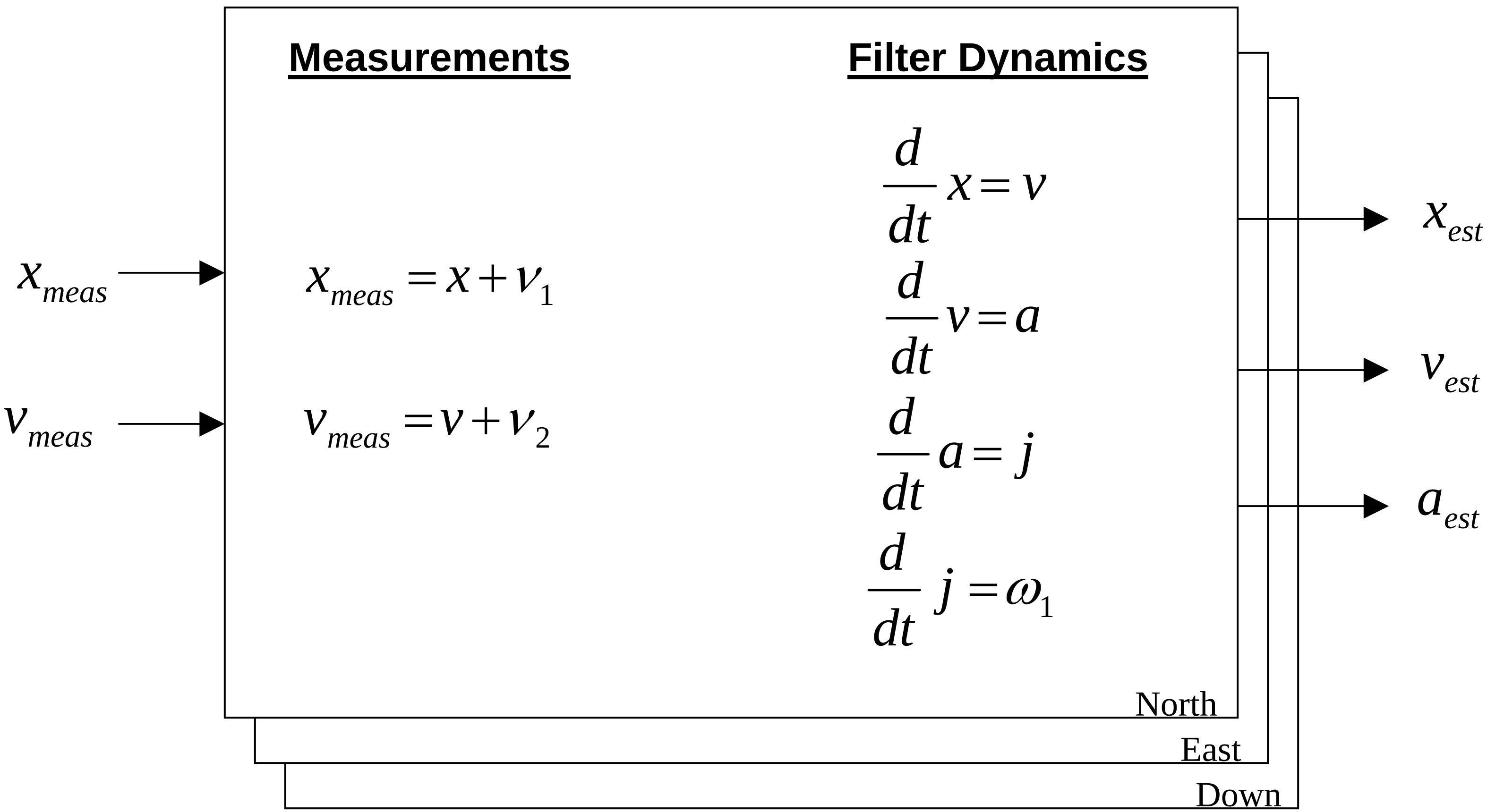
Typical Accuracies

Position	~ 30 m
Velocity	~ 0.15 m/s

Many GPS receivers provide high quality velocity information

→ *Use high quality velocity measurement to improve position estimate*

KF 1. Kalman Filter Setup



x : position $\quad$ v : velocity

a : acceleration $\quad$ j : jerk

ν_i , ω_i : white noises

a_{est} :

- noisy, but not biased
- combined with rate gyros in removing the gyro biases (KF2)

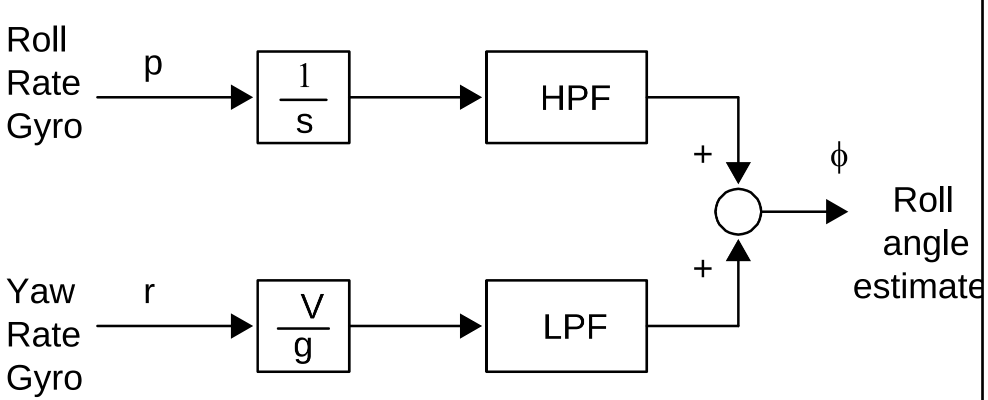
KF 2. Removing Rate Gyro Bias Effect

Background & Motivation

• In aircraft control, *roll angle* control is commonly used in inner-loop to create required *lateral acceleration* which is commanded from guidance outer-loop

• Biased roll angle estimate can cause steady-state error in cross-track

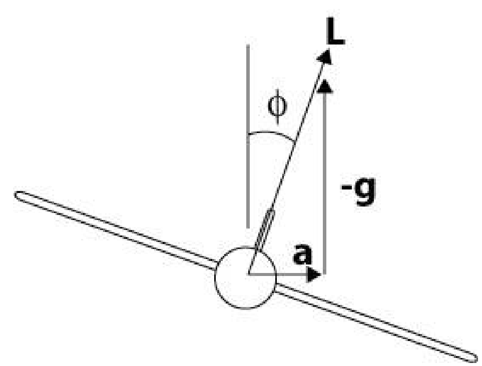
Complementary filter with roll & raw gyros (CF1)



Drawback : biased estimate

Single-Antenna GPS Based Aircraft Attitude Determination

\- *Richard Kornfeld, Ph.D.* (1999)



Drawback : sampling rate limit (GPS), typical filter time constant ~ 0.5 sec.

$$\boxed{a_s \approx g \cdot \phi \approx V \cdot r \qquad \dot{\phi} \approx p}$$

KF 2. Kalman Filter Setup

from Rate Gyros

Inputs: p_{meas}, r_{meas}, $(a_s)_{est}$

from GPS Kalman Filter

Measurement Equations

$$p_{meas} = p + bias_p + \nu_2$$

$$r_{meas} = \frac{g}{V}\phi + bias_r + \nu_3$$

$$a_s = g\phi + \nu_1$$

Filter Dynamics

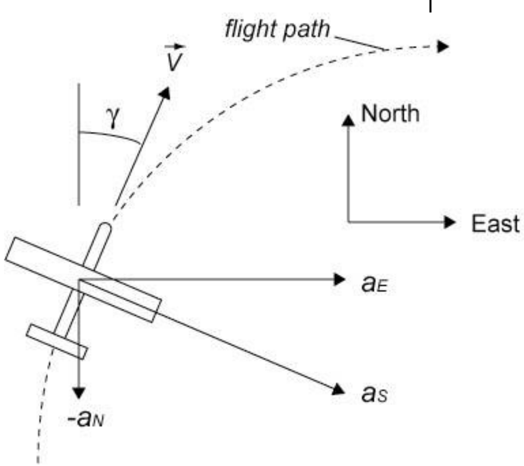
$$\frac{d}{dt}\phi = p + \omega_1$$

$$\frac{d}{dt}p = \omega_2$$

$$\frac{d}{dt}bias_p = \omega_3$$

$$\frac{d}{dt}bias_r = \omega_4$$

Outputs: ϕ_{est}, p_{est}, $(bias_p)_{est}$, $(bias_r)_{est}$



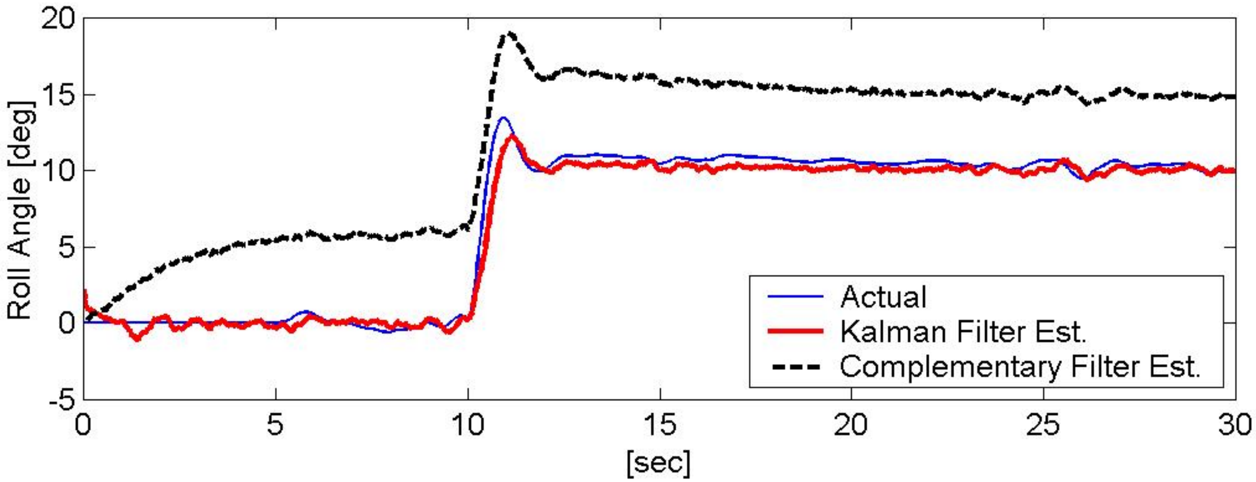
ϕ : bank angle $\quad V$: velocity

a_s : acceleration in sideways direction

p : roll rate $\quad r$: yaw rate

ν_i, ω_i : white noises

KF 2. Simulation Result



- Simulation for 10 degree bank angle hold
- Roll rate gyro bias=0.03 rad/s, yaw rate gyro bias = 0.02 rad/s were used in simulation

References

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