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### 16.346 Astrodynamics

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## Lecture 10 Transformation of the Boundary-Value Problem \#6.7

According to Lambert's Theorem

$$
\sqrt{\mu}\left(t_{2}-t_{1}\right)=F\left(a, r_{1}+r_{2}, c\right)
$$

the orbit of the boundary-value problem can be transformed to a rectilinear orbit ( $e=1$ ), keeping the sum of the radii $r_{1}+r_{2}$, the length of the chord $c$ and the semimajor axis $a$ all fixed in value, and the time-of-flight will be unchanged. The transformation is illustrated in the following figure:


Figure by MIT OpenCourseWare.

The flight time for the rectilinear orbit is

$$
\begin{aligned}
\sqrt{\frac{\mu}{a^{3}}}\left(t_{2}-t_{1}\right) & =(\alpha-\sin \alpha)-(\beta-\sin \beta) \\
& =\left(E_{2}-\sin E_{2}\right)-\left(E_{1}-\sin E_{1}\right)
\end{aligned}
$$

in terms of the Lagrange parameters and the eccentric anomalies.

Transformation of the Four Basic Ellipses


Fig. 6.20 from An Introduction to the Mathematics and Methods of Astrodynamics. Courtesy of AIAA. Used with permission.

We adopt the convention for assigning quadrants to the Lagrange parameters $\alpha$ and $\beta$

$$
\begin{array}{|lrlr|}
\hline 0 \leq \alpha \leq 2 \pi & 0 \leq \beta \leq \pi & \text { for } & \theta \leq \pi \\
0 \leq \alpha \leq 2 \pi & -\pi \leq \beta \leq 0 & \text { for } & \theta \geq \pi \\
\hline
\end{array}
$$

which will include all elliptic orbits.

