Muscle Spindle Response: Review, Model, and Simulations

1. Length, velocity and bursts:

Everybody agrees that spindles give information about not only its length, but also about its rate of change of length. (T. McMahon pg. 153-154)

"Primary endings of muscle spindle are highly sensitive to the rate of change of muscle length, a property referred to as *velocity sensitivity*."

"Because of their high degree of dynamic sensitivity, primary endings respond with bursts of firing to transient stimuli." (C. Ghez in Principles of Neural Science pg. 569)

Bottom Line: • Primary endings encode information about:

$$\begin{cases} -\text{ length of muscle} \\ -\text{ speed of movement} \end{cases}$$

• Primary endings are most sensitive to small changes in muscle length. "This sensitivity is reflected in transient increase in firing rate at the beginning of a stretch." (C. Ghez)

2. Dynamic/Static γ motoneurons (mn)

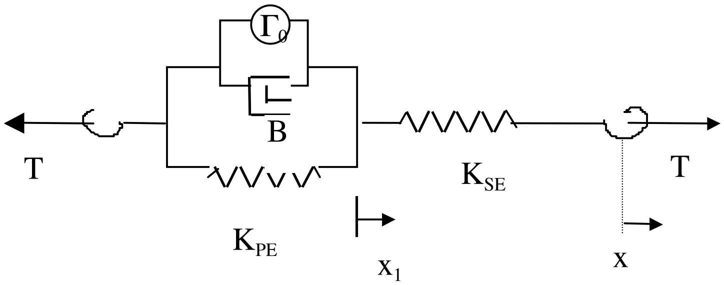
The extent that Ia (primary afferents) contain information about "x" and "v" is determined by the amount of activation of static and dynamic γ's (S-γ and D-γ).

$$\begin{cases} \text{High activation of S-}\gamma \rightarrow \text{ high sensitivity to "x"} \\ \text{High activation of D-}\gamma \rightarrow \text{ high sensitivity to "V"} \end{cases}$$

Utility: as the speed and/or difficulty of the movement increases, both static and dynamic γ motor neurons are set at high levels. Their activation is dependent on task or context.

For example, think of a clown walking on an elevated tight rope, she will have high activation levels of both γ ⇒ higher resolution in the measurement of x and v ⇒ better stabilization.

3. Conceptual mechanical model of spindle (McMahon, 1984)



Γ_0 is the fusimotor drive (≈ isometric tension developed by spindle)

given in Mc-Mahon

$$T = \Gamma_o + B\dot{x}_1 + K_{PE}x_1 \quad (1)$$ for the parallel elements

a

$$T = K_{SE}(x - x_1) \quad (2)$$

Then he proposes that the frequency of discharge of the Ia, ν(t), is proportional to the tension T (pg. 152).

$$\left\{ v(t) = \frac{A}{K_{SE}} \cdot T \right. \quad (3)$$ (A constant)

$$\Rightarrow \frac{d}{dt}(3) \Leftrightarrow \dot{v} = \frac{A}{K_{SE}}\dot{T} = \frac{A}{K_{SE}}(\dot{x} - \dot{x}_1) \quad (4)$$

(3) and (4) $\Rightarrow \begin{cases} x_1 = x - \dfrac{K_{SE}}{A} v & (5) \\ \dot{x}_1 = \dot{x} - \dfrac{1}{\alpha}\dot{v} & (6) \end{cases}$

derived in Mc-Mahon but not in this form (pg. 64-165)

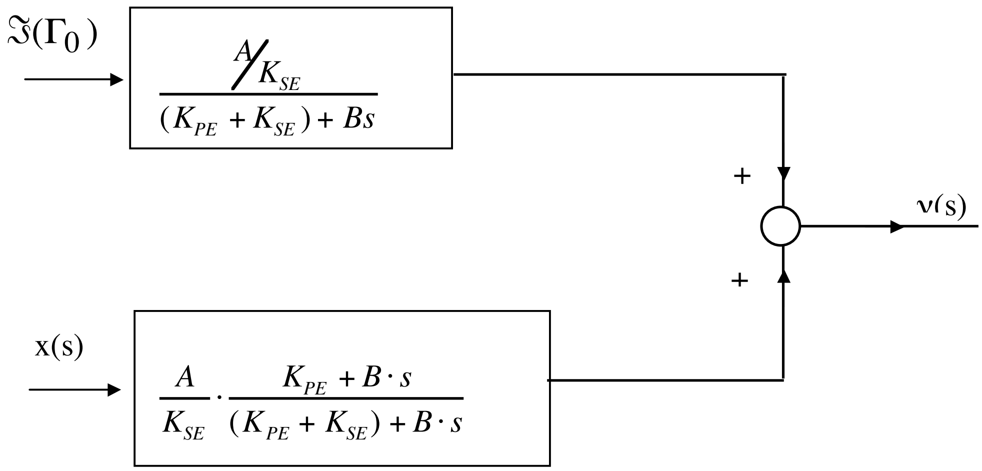
Now, taking all the previous equations together, we get:

$$\boxed{\frac{K_{SE}}{A}\left[(K_{PE} + K_{SE})v + B\dot{v}\right] = \Gamma_o + K_{PE}x + B\dot{x} \quad (7)}$$

Or we can write it as a transfer function (not in McMahon):

$$v(s) = \frac{A}{K_{SE}}\left[\frac{K_{PE} + B\cdot s}{\left(K_{PE} + K_{SE}\right) + B\cdot s}\cdot x(s) + \frac{\Im(\Gamma_0)}{\left(K_{PE} + K_{SE}\right) + B\cdot s}\right] \qquad (8)$$

where $\Im(\Gamma_o)$ is the Laplace transform of Γ_o.



This is just a representation of (8)

4. Problems with the model

- The model does not explicitly include information about "v"

$$\Gamma_o = 0 \Rightarrow \frac{v(s)}{x(s)} = \frac{1 + a\tau s}{1 + \tau s} \qquad \begin{array}{l} a > 1 \\ \tau > 0 \end{array}$$

which is just a "phase-advance" filter or an approximation of a derivative.

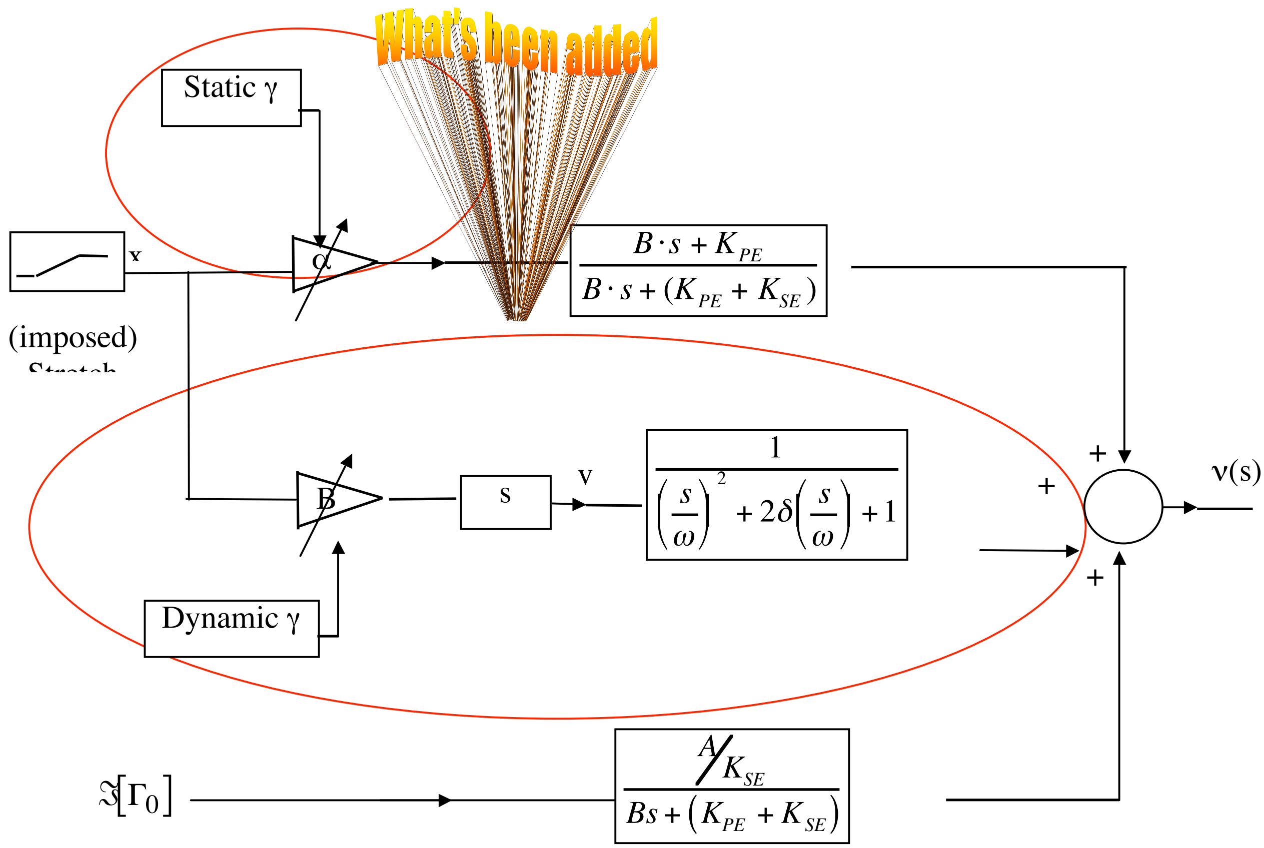
⇒ Changing its parameter (of the above transfer function), we cannot separately include information about x and v or decouple the sensitivity of ν to x and $\frac{dx}{dt} = \text{v}$.

However, this *is* what is observed and what the literature talks about.

- This model cannot reproduce the "bursts" mentioned on page 1, due to its structure

$\frac{N(s)}{D(s)}$ where N(s) is first order and D(s) as well.

5. Proposed enhanced model



- α and β are just the "symbolic" sensitivity (which could be in terms of a simple gain or changing the parameters in the transfer functions) to γ levels of activation.
- In the second branch: The 's' has been separated out from its transfer function to highlight that this is the information about the derivative of x ⇒ v.