Dynamic System Analysis & Why do we Care?

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SBE - 2006 Dynamics Lectures

(03/21/06 & 03/23/06)

Learning Objectives

Introduce the

Have a little Fun!

What is Dynamics Useful For?

Equilibrium Point Hypothesis

Dynamics vs. Kinematics

- **Dynamics**:
  Propagation of motion due to forces / torques

- **Kinematics**:
  Movement from one pose to another

Greek 101

dynamis:
power

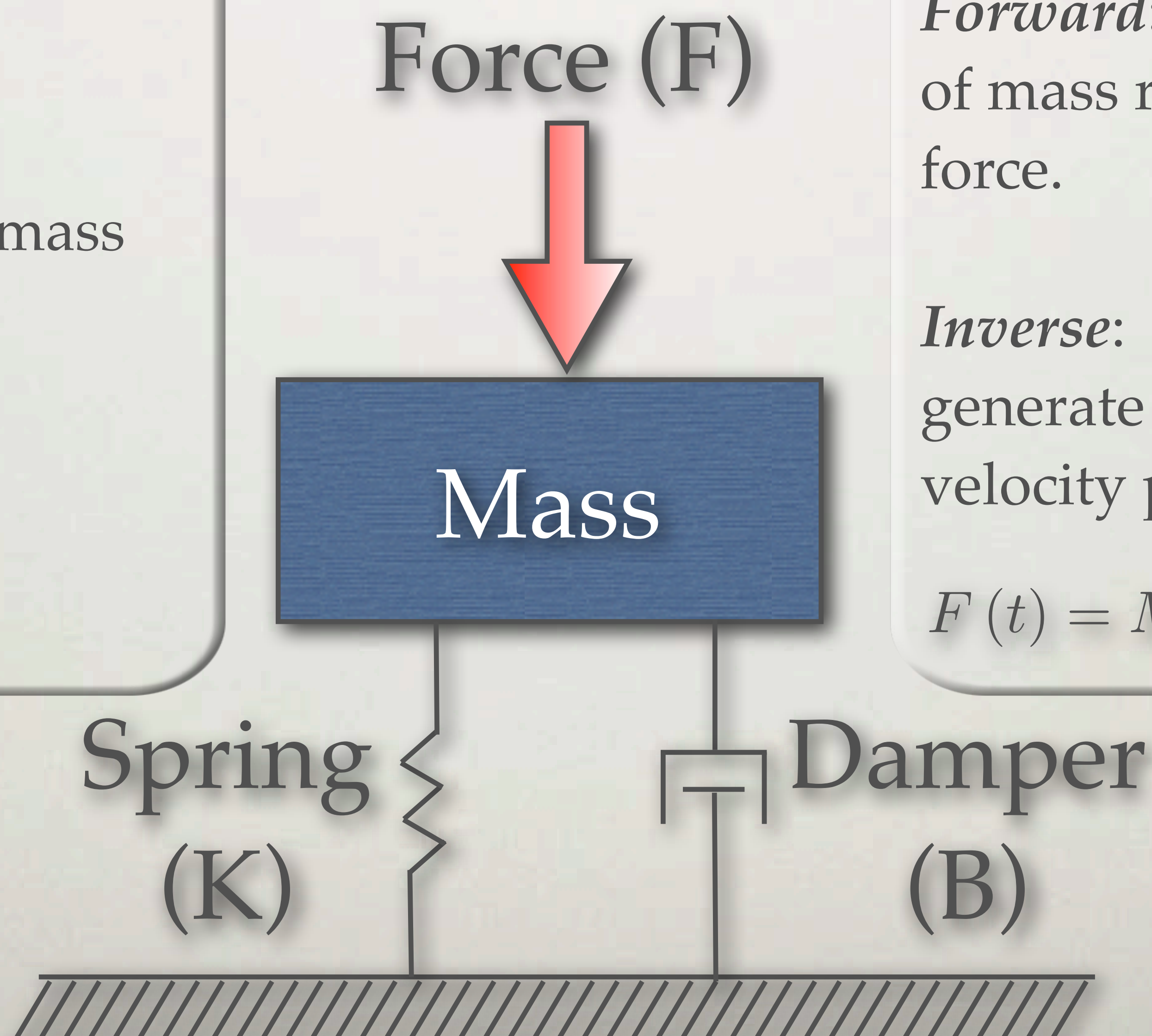
kinein:
to move

Example

Kinematics

Position and velocity of mass over time.

$$\dot{x}(t),\ \ x(t)$$



Dynamics

Forward: Position and velocity of mass resulting from applied force.

Inverse: Force required to generate specific position and velocity profile.

$$F(t) = M\ddot{x}(t) + B\dot{x}(t) + Kx(t)$$

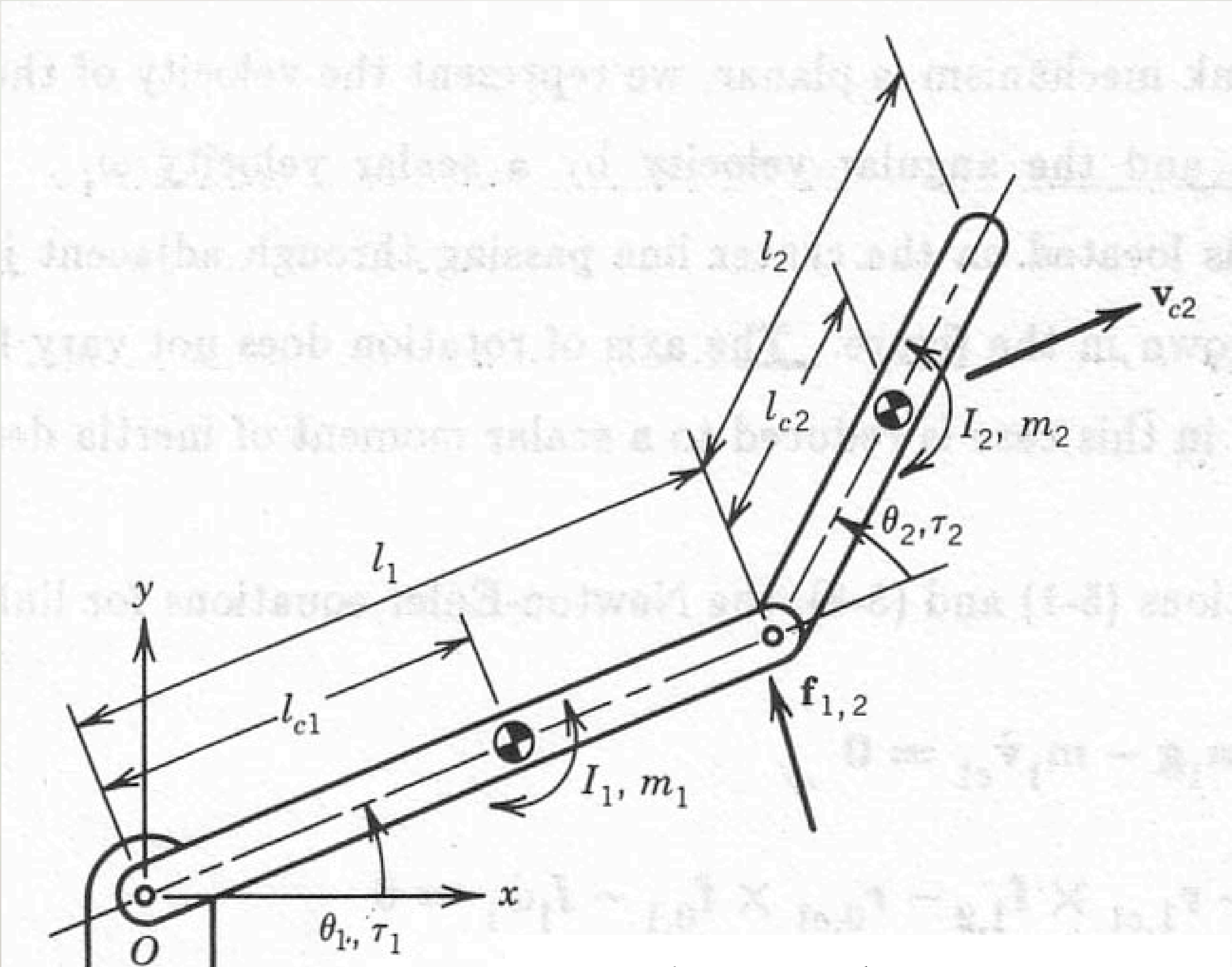
What's the Big Deal?

- Force input can look very different from desired trajectory
- Simple trajectories can require complicated driving forces and vice versa
- Example: Multi-limb dynamics

$$\tau = H(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + D(\theta, \dot{\theta})\dot{\theta} + G(\theta)$$

Control Torques = Inertia Torques + Coriolis Torques + Friction Torques + Gravity Torques

Two-Link Arm Dynamics



Asada & Slotine, 1986

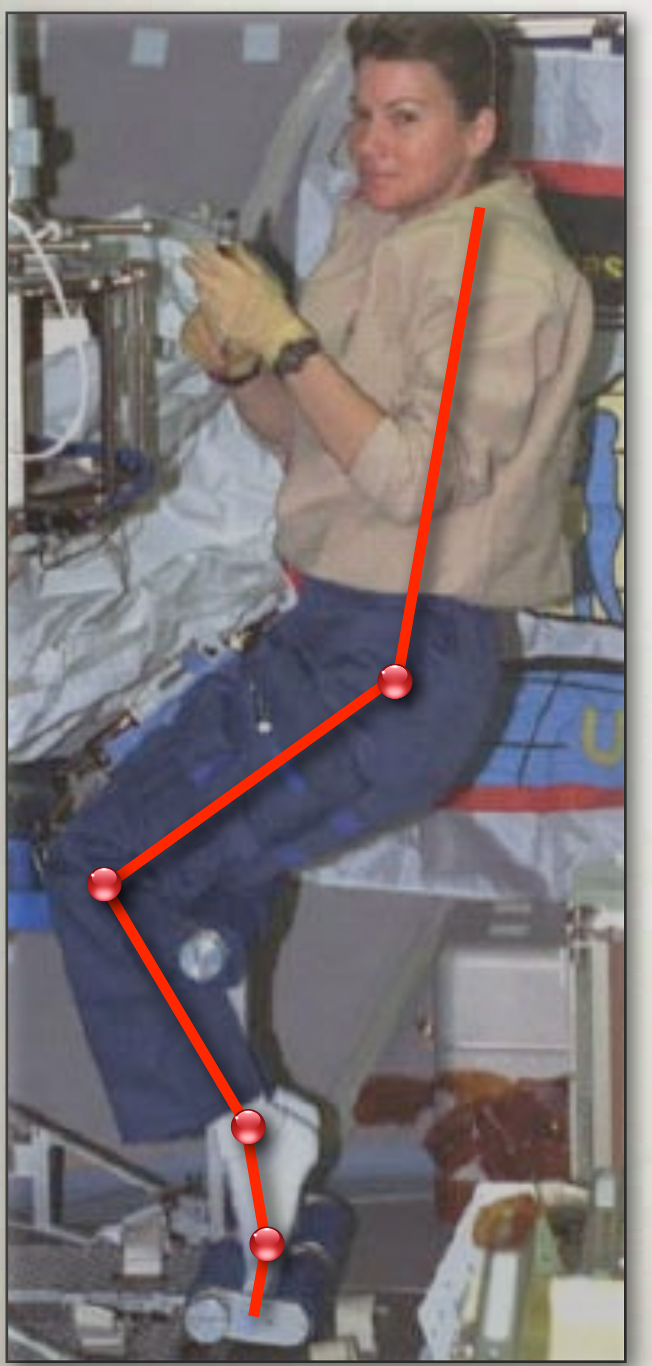
Two-Link Arm Dynamics

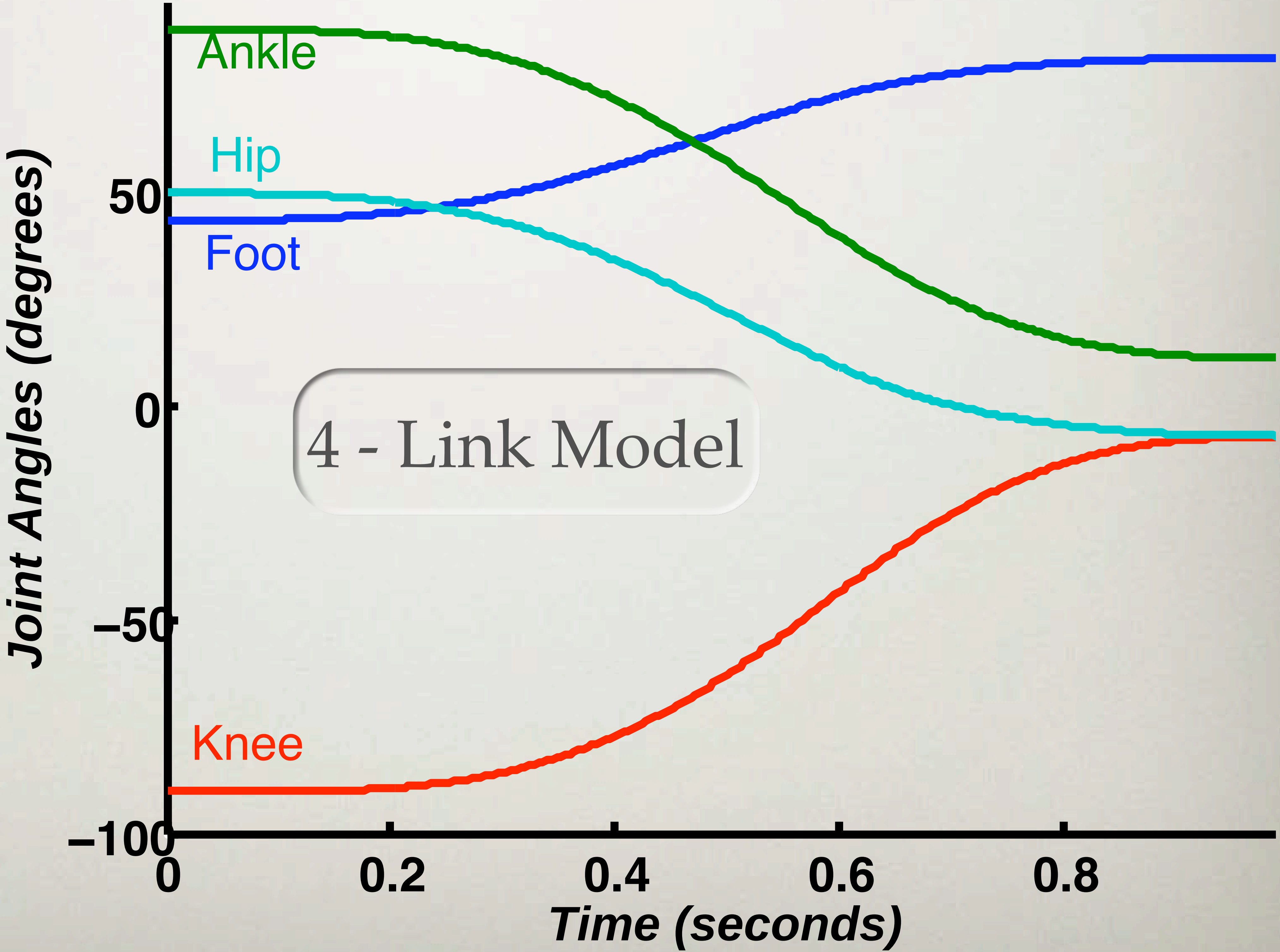
Imagine the complexity for more than two links!

$$C(\theta,\dot{\theta}) = \begin{bmatrix} -2h\dot{\theta}_2 & -h\dot{\theta}_2 \\ h\dot{\theta}_1 & 0 \end{bmatrix}$$

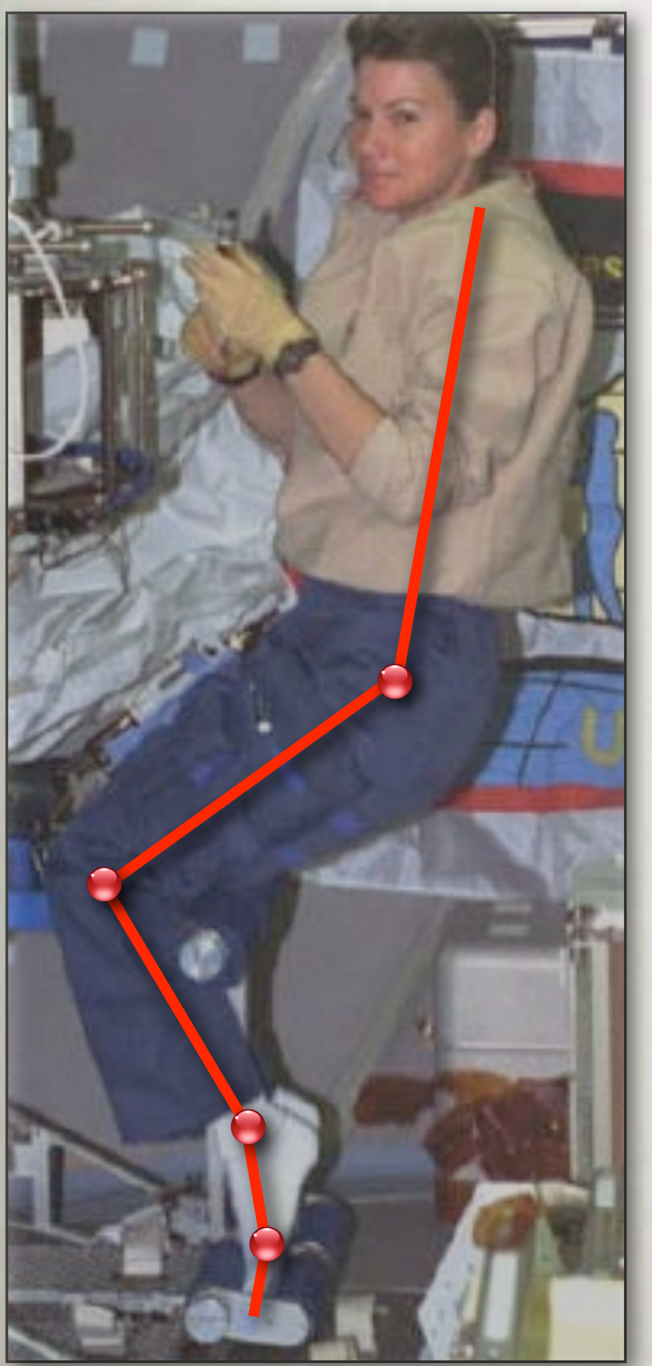
$$G(\theta) = \begin{bmatrix} g_1 & g_2 \end{bmatrix}^T$$

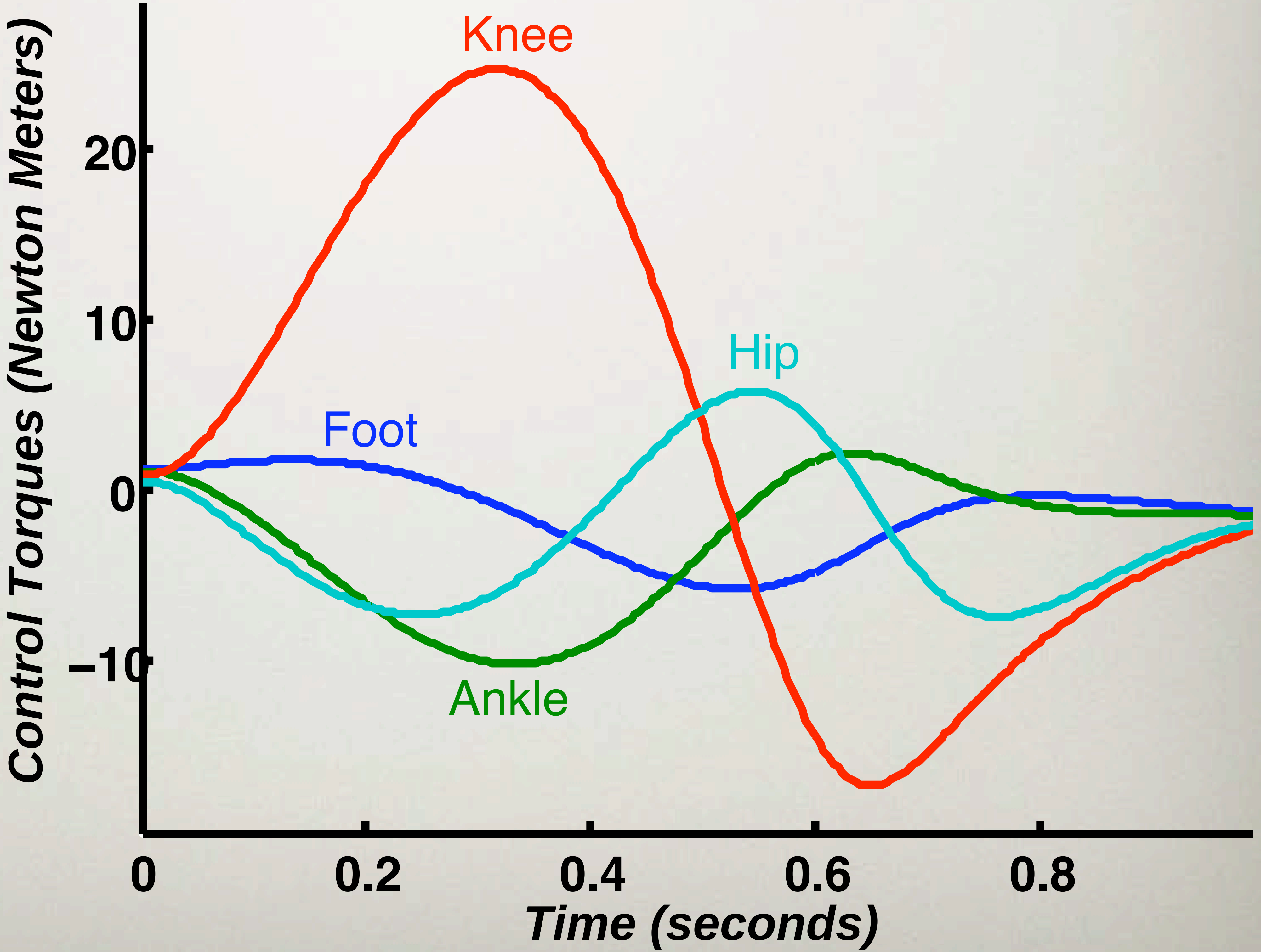
Joint Angles





Joint Torques





Impact on Control Strategy

- Complicated control strategies may be difficult or undesirable to realize
  - Dynamic Cost Functions:
    - Minimum Joint Torque
    - Minimum Torque Change
    - Minimum Jerk (Snap, Crackle, Pop?)
  - Cannot be described by kinematics alone

- But, are dynamics really a part of human movement control?

Equilibrium Point Hypothesis

- Predicts multi-joint motion based on programmable springs and dampers at each joint
- Stiffness and damping adjusted such that equilibrium point of arm matches desired position
- Trajectory formed out of time varying equilibrium points

$$\tau(t) = K_p[\theta_{ep}(t) - \theta(t)] + K_d[\dot{\theta}_{ep}(t) - \dot{\theta}(t)]$$

Advantages

- Very simple form
  - Does not require computation of full nonlinear dynamics
- Excellent performance shown in deafferented monkeys as well as humans [Flash, Hogan, Bizzi]
- Simple joint PD was very common in early “pick-and-place” robotics [Slotine and Li]
- Architecture of spinal cord lends itself to “natural” PD control
  - Avoids large transport delays to / from cortex

Drawbacks

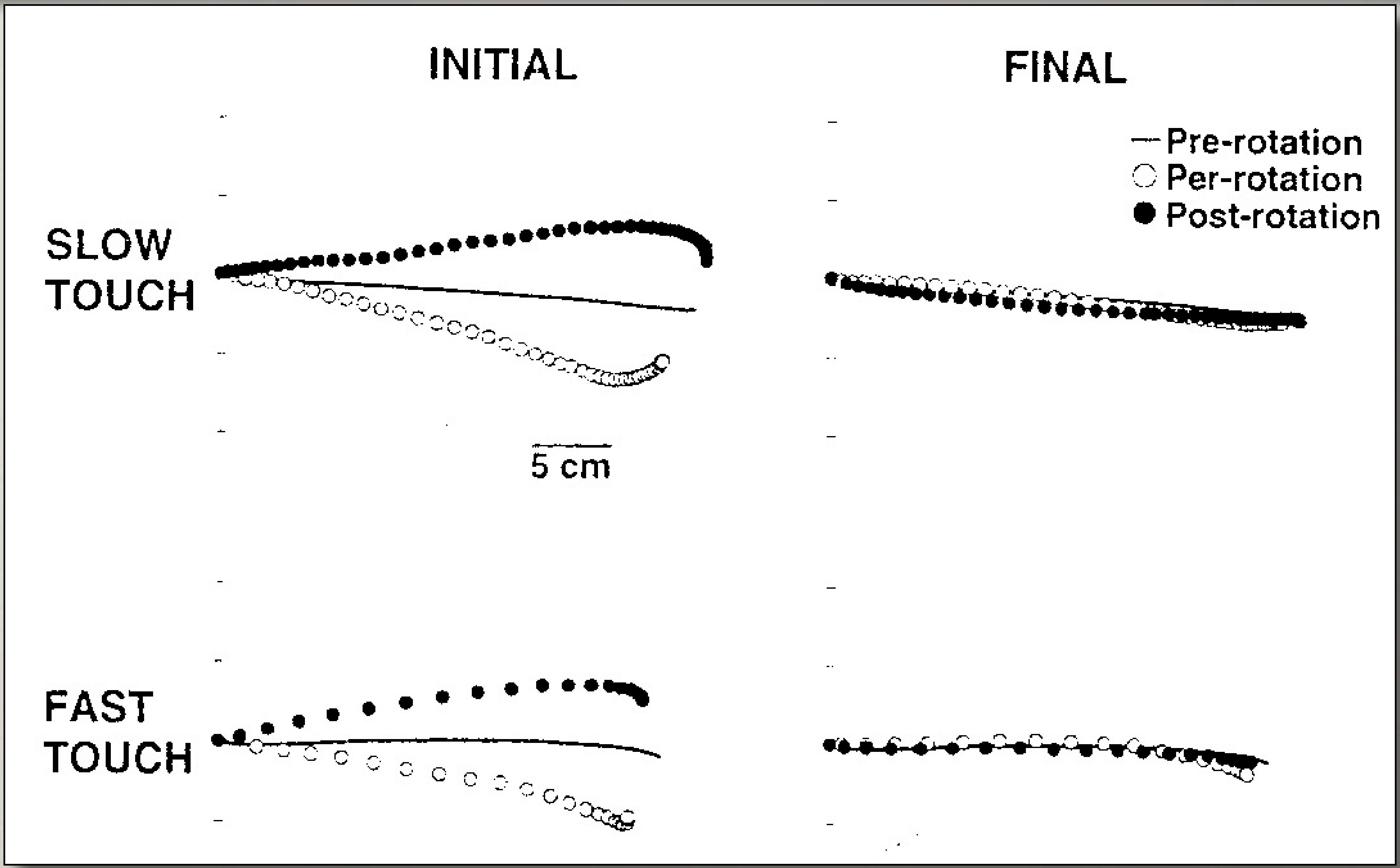
- Joint PD controllers NOT good at complicated tracking problems [Slotine & Li]
  - Most experiments supporting EPH use very simple trajectories
- Subjects reaching for objects in rotating rooms have end-point errors [DiZio & Lackner]
  - Also adapt to trajectory
- Astronauts on-orbit show remarkable adaptation to micro-gravity [Newman]
  - Not predicted by the equilibrium point hypothesis
- Counter-intuitive that humans with large mental capacity would opt for sub-optimal (in the performance sense) tracking controllers

Brandeis Rotating Room

Coriolis Forces

- Reaching in rotating room
  - Coriolis forces applied without mechanical arm contact
  - Velocity dependent so no force at end of trajectory
- Equilibrium point hypothesis would predict no end-point error
  - Also would not predict any adaptation

DiZio & Lackner's Results



Credit DiZio & Lackner, Brandeis University 1994.

Simulation Results

- Computed torques due to rotation using Lagrangian Dynamics:

$$\tau_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i}$$

- 3-link manipulator assumed to compute coriolis and centripetal torques
    - Link lengths, angular displacement of first link set to zero
    - Terms depending on angular velocity of first joint were thus related to coriolis and centripetal accelerations

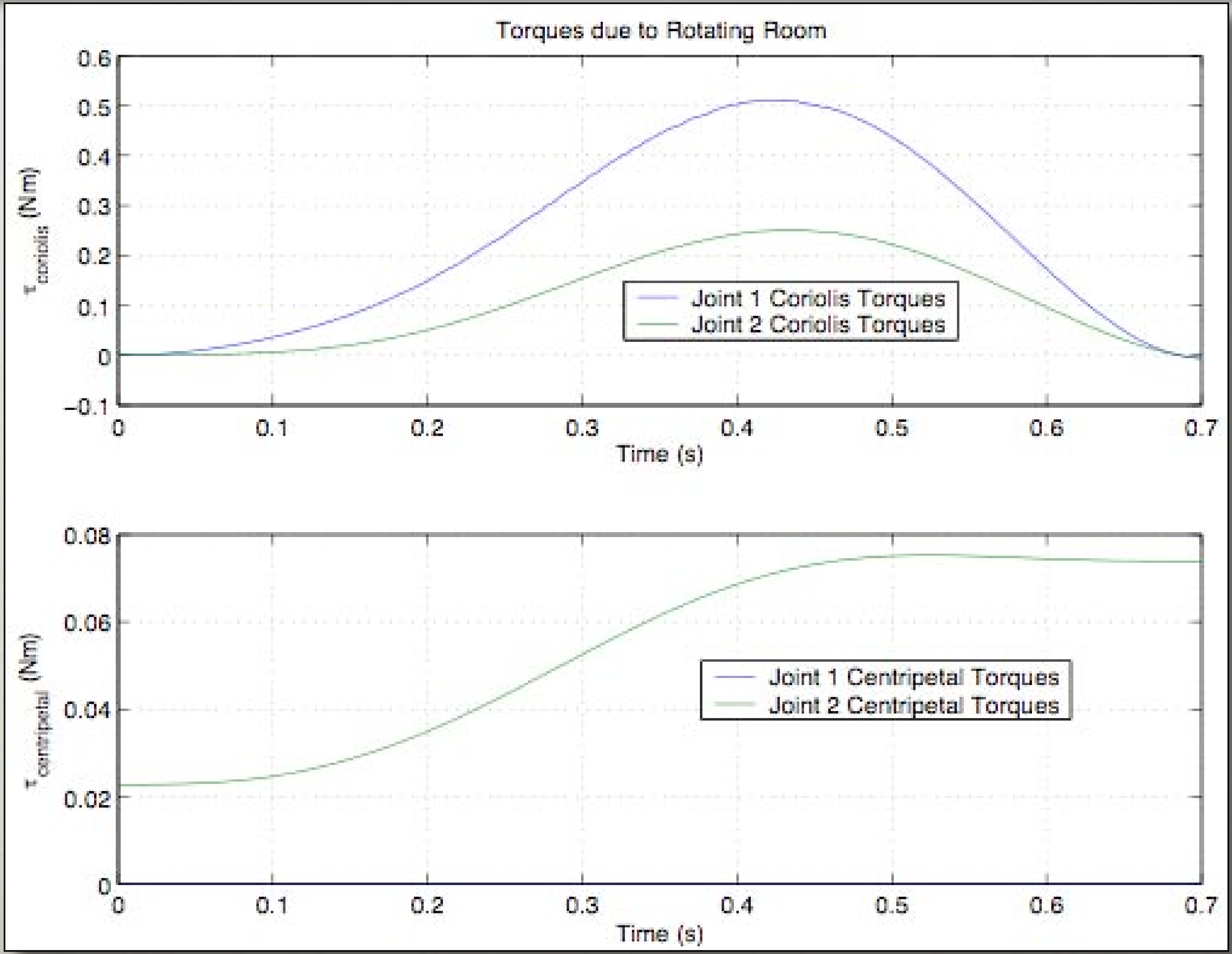
Rotational Torques / Forces

$$\boldsymbol{\tau}_{coriolis} = \begin{bmatrix} -2\omega_0 m_2 l_1 l_{c2} \sin(q_2)\dot{q}_2 \\ 2\omega_0 m_2 l_1 l_{c2} \sin(q_2)\dot{q}_1 \end{bmatrix}$$

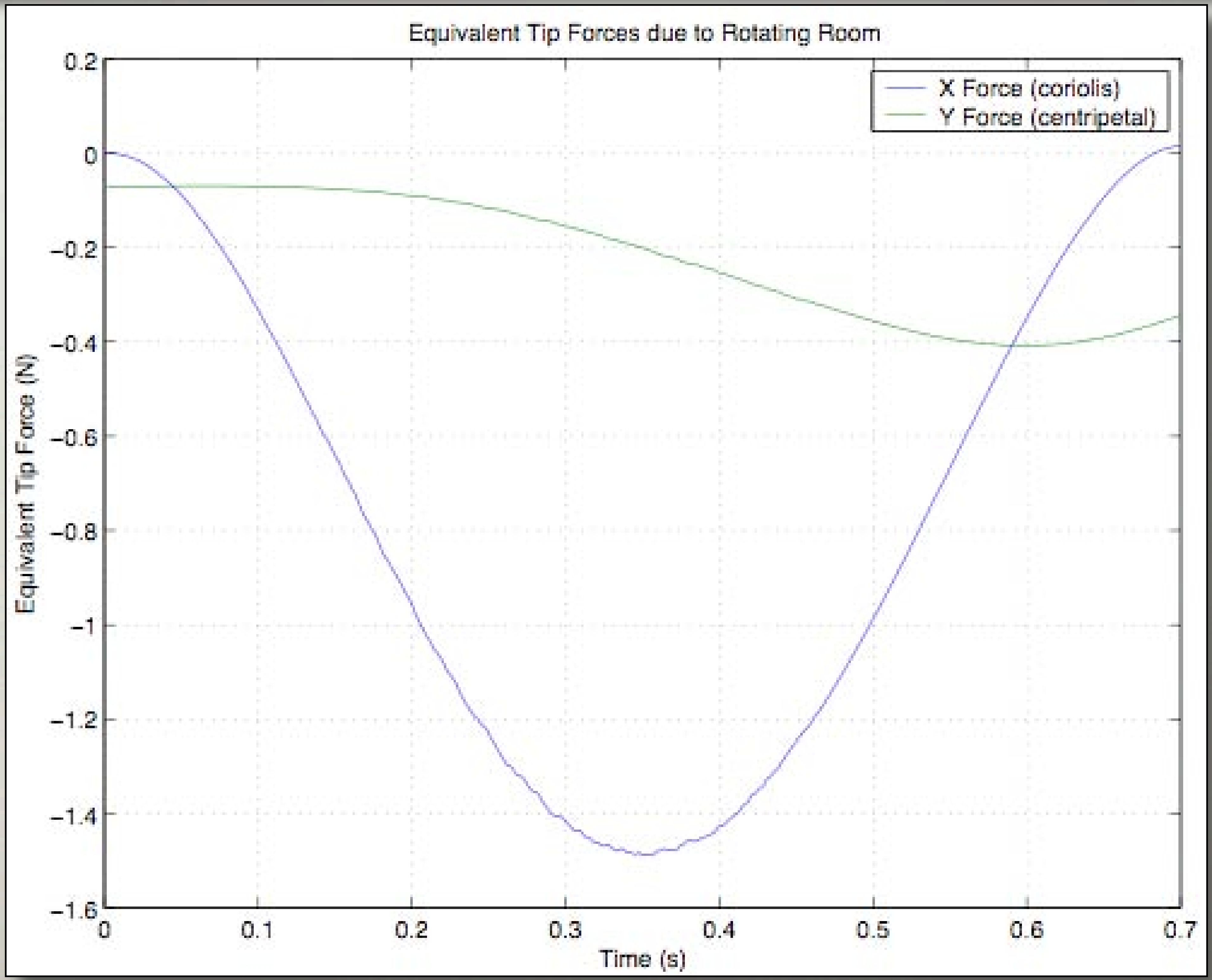
$$\boldsymbol{\tau}_{centripetal} = \begin{bmatrix} 0 \\ \omega_0^2 m_2 l_1 l_{c2} \sin(q_2) \end{bmatrix}$$

$$F_{tip} = \left(J^T\right)^{-1}\left(\boldsymbol{\tau}_{coriolis} + \boldsymbol{\tau}_{centripetal}\right)$$

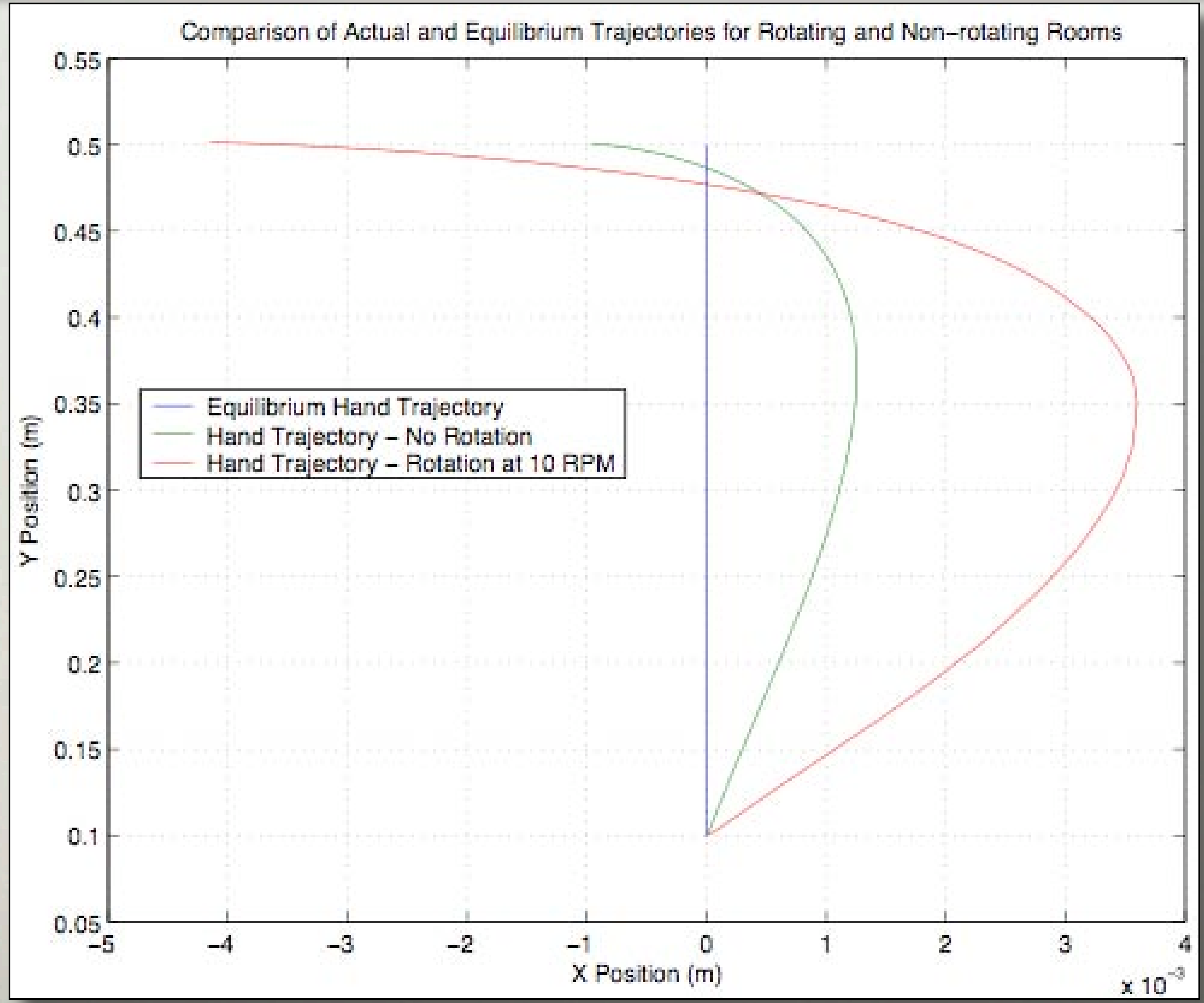
Rotational Torques



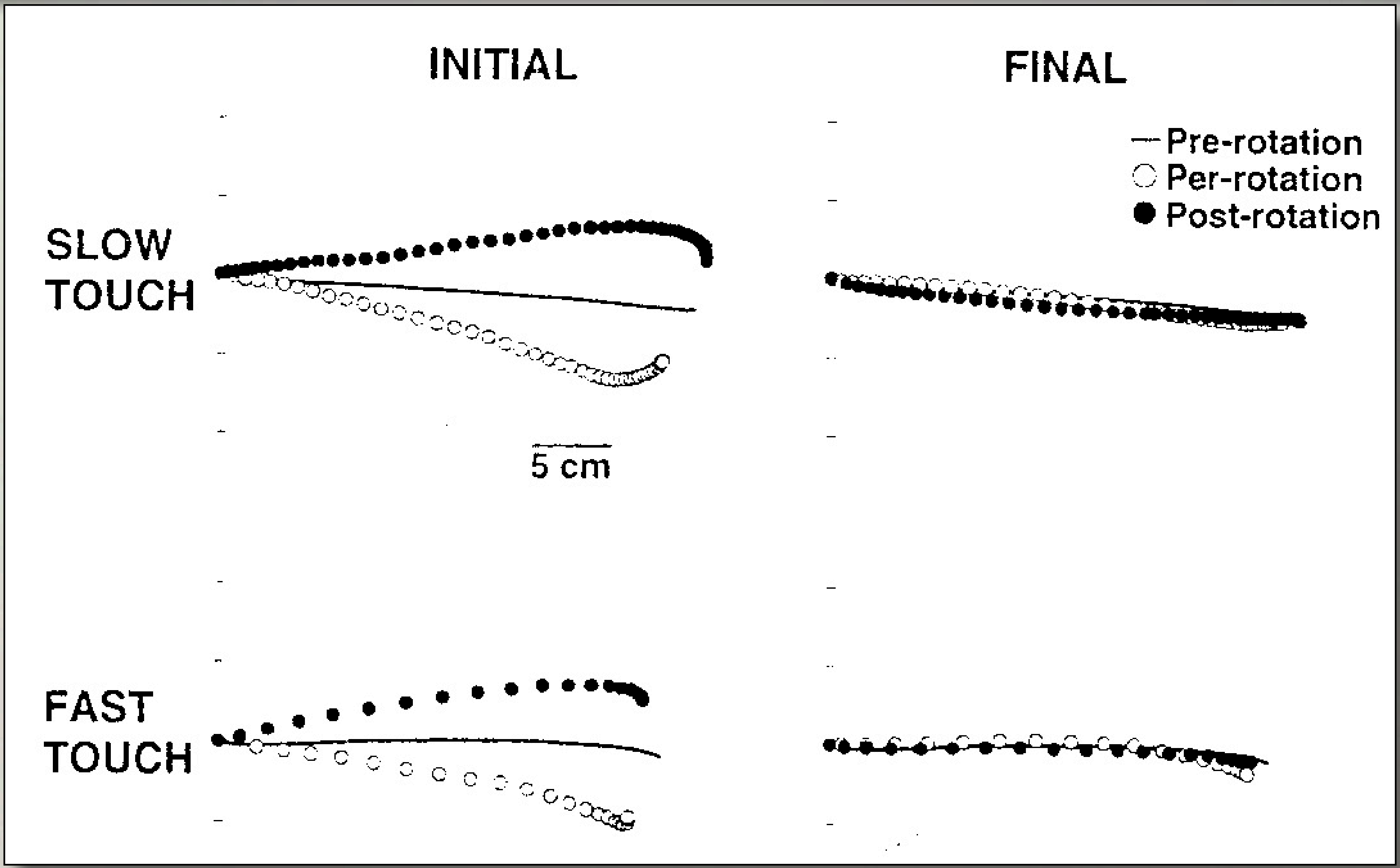
Rotational Forces



Tip Trajectory



Experimental Results



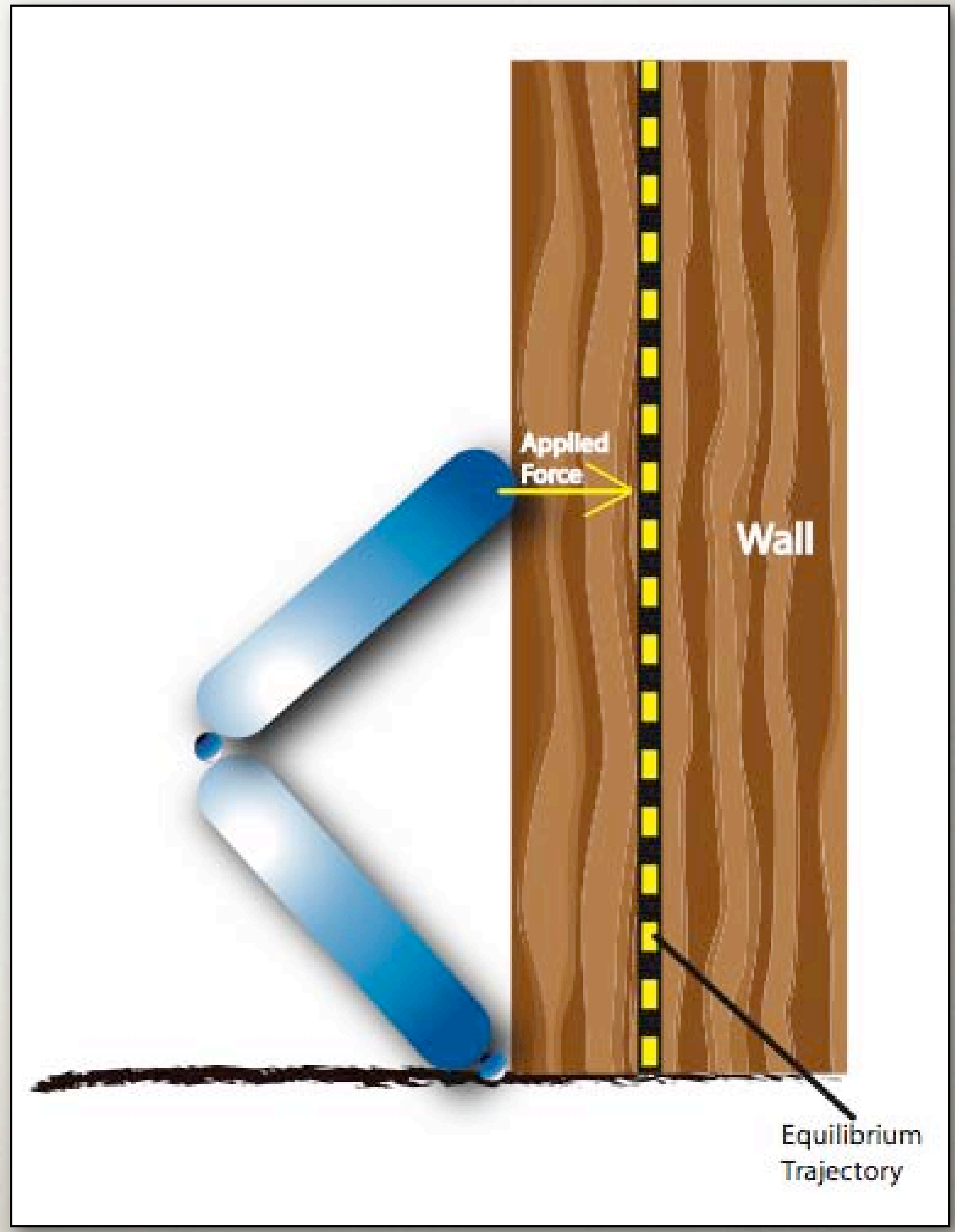
Credit DiZio & Lackner, Brandeis University 1994.

Virtual Rotation

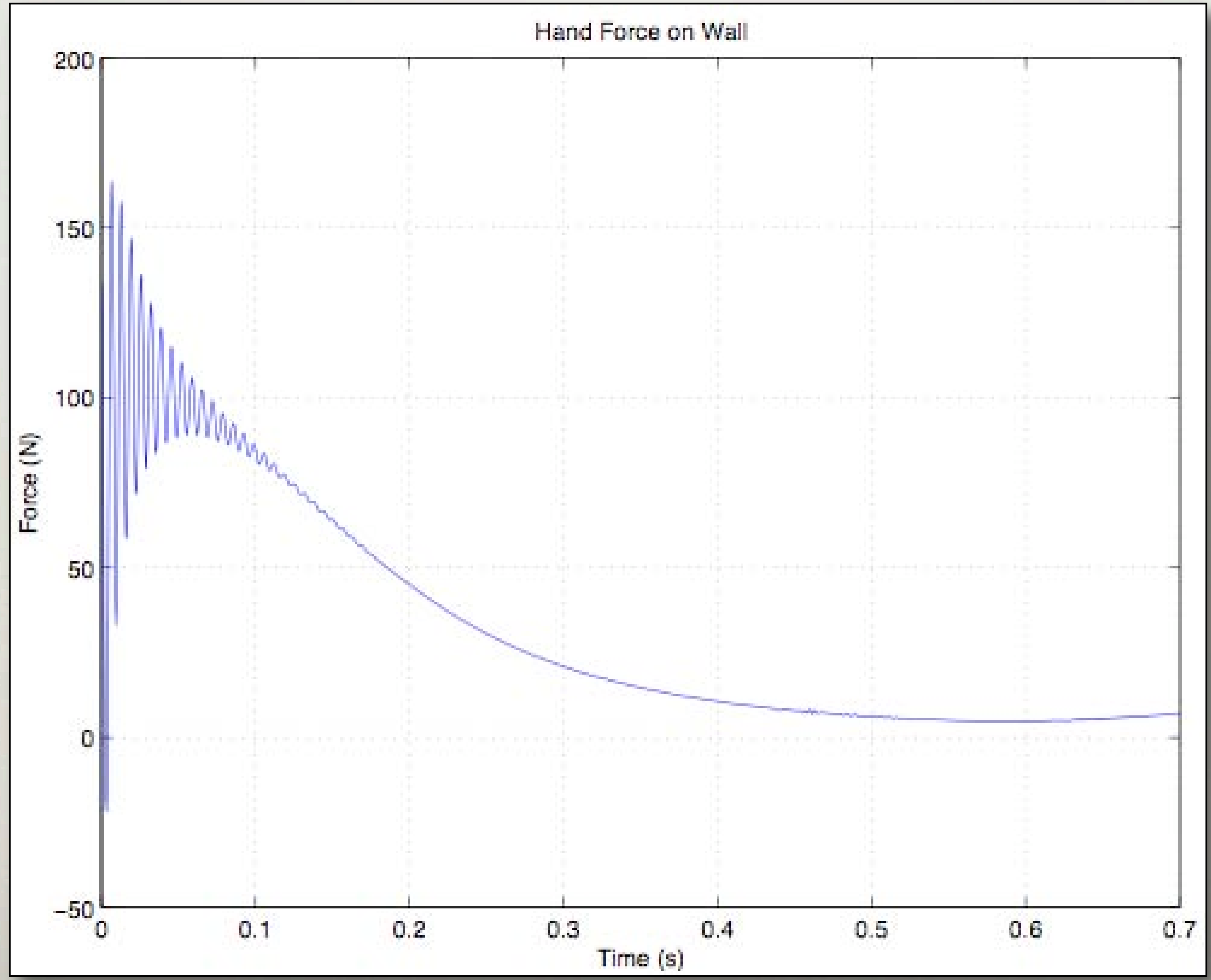
- Similar experiments carried out with virtual reality helmets
  - Subjects felt like they were rotating
- Arm trajectories deviated in the direction opposite to where the coriolis force would have been
- Suggests knowledge of arm dynamics in control strategy
- Interesting addition to experiment
  - Subjects did NOT exhibit errors immediately after rotation stoppage (unlike previous rotating room expts.)

Force Application

- Equilibrium trajectory placed within a stiff wall
- Force computed by measuring insertion depth into wall
- Requires STIFF ODE solvers.



EPH Force Application



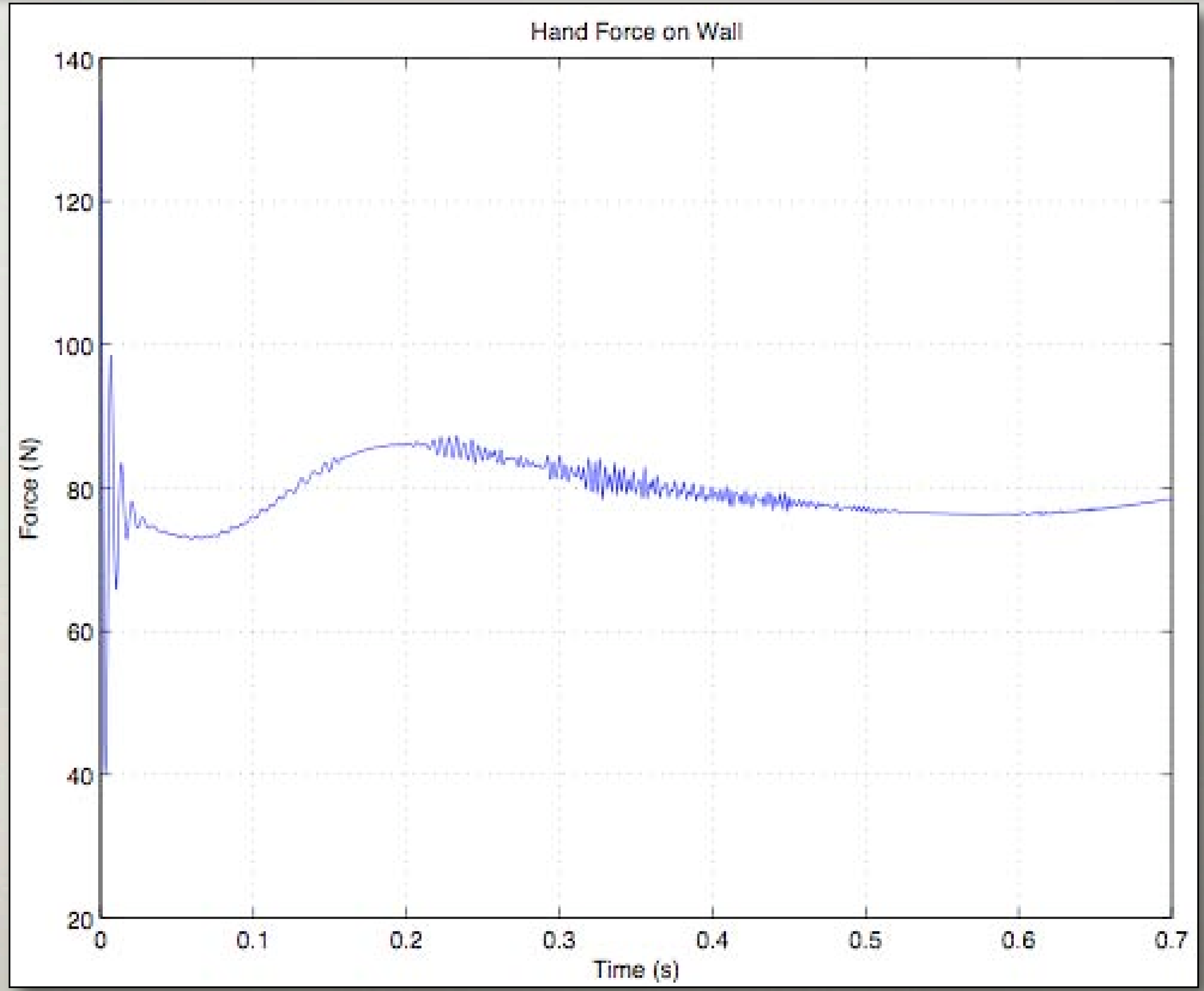
Tip Controller

- Another control possibility is to use the manipulator Jacobian to track the tip position … not the joint position

$$\tau = J^T\left(K\tilde{x} + B\dot{\tilde{x}}\right)$$

- More complicated controller (requires some kinematic knowledge)

With Tip Control



Jumping Experiments

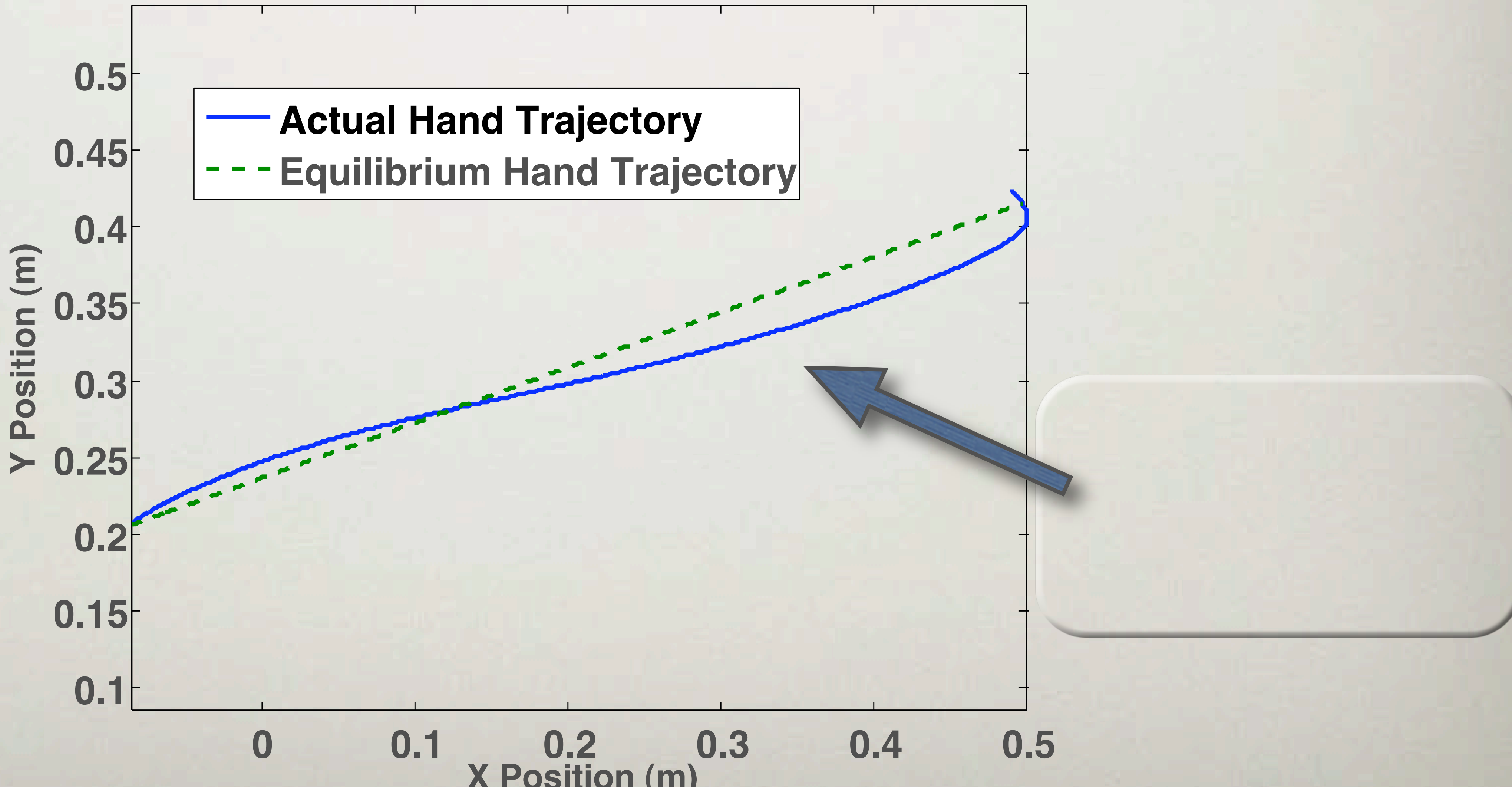
- False platform jumps
  - Sometimes floor is present, other times, substantially lower
- Astronaut jumps post-flight

- **Observation:** Subjects develop internal model predicting landing times
  - Astronauts not "ready" for floor when it arrives
- **Why?**

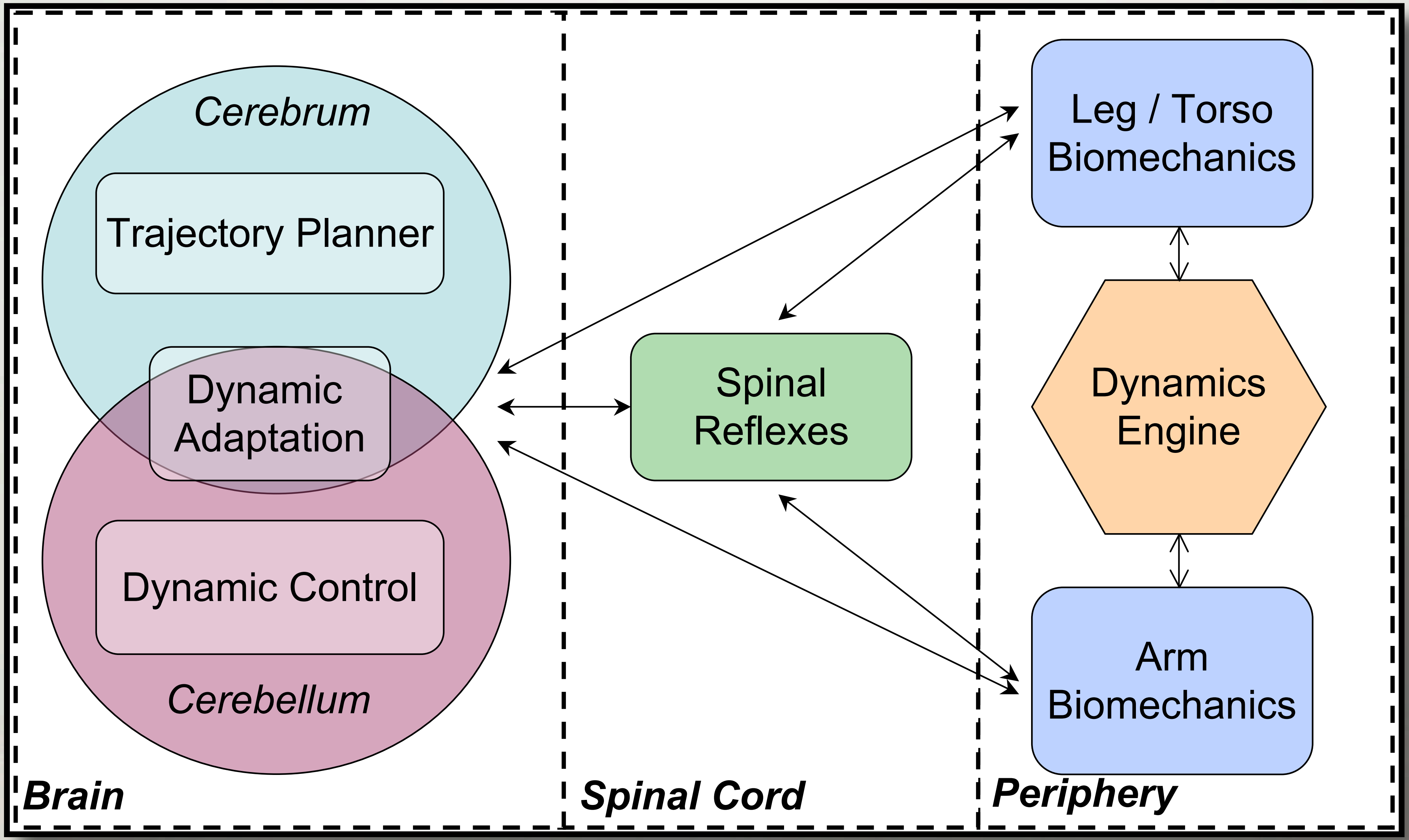
$$g_{internal} < g_{true} \Rightarrow \hat{T}_{impact} > T_{true}$$

What about fast motion?

- Common argument against EPH is that it requires high arm stiffnesses
  - But human arm stiffnesses measured by Gomi and Kawato are quite low
- So, how can we make accurate, fast motions if the EPH is true?



General Consensus



Trajectory generation with feed-forward internal model

Equilibrium-point trajectory tracking

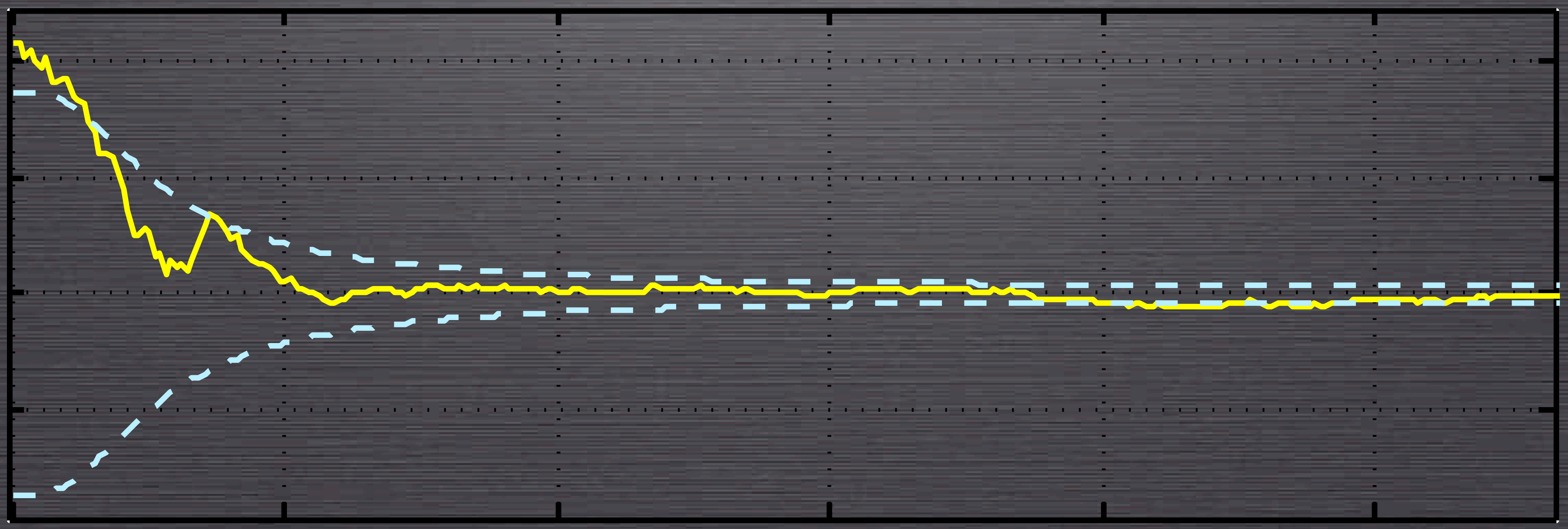
Spring-like muscle properties

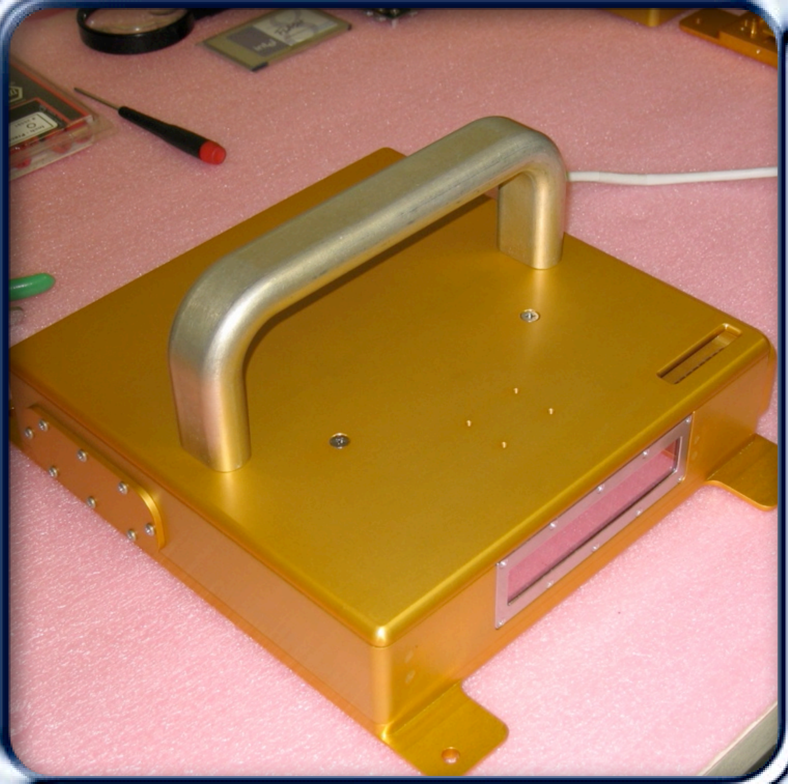
Story So Far ...

- Shown role of dynamics in human motion
  - Current debate over whether or not dynamics play
- What else is dynamics useful for?

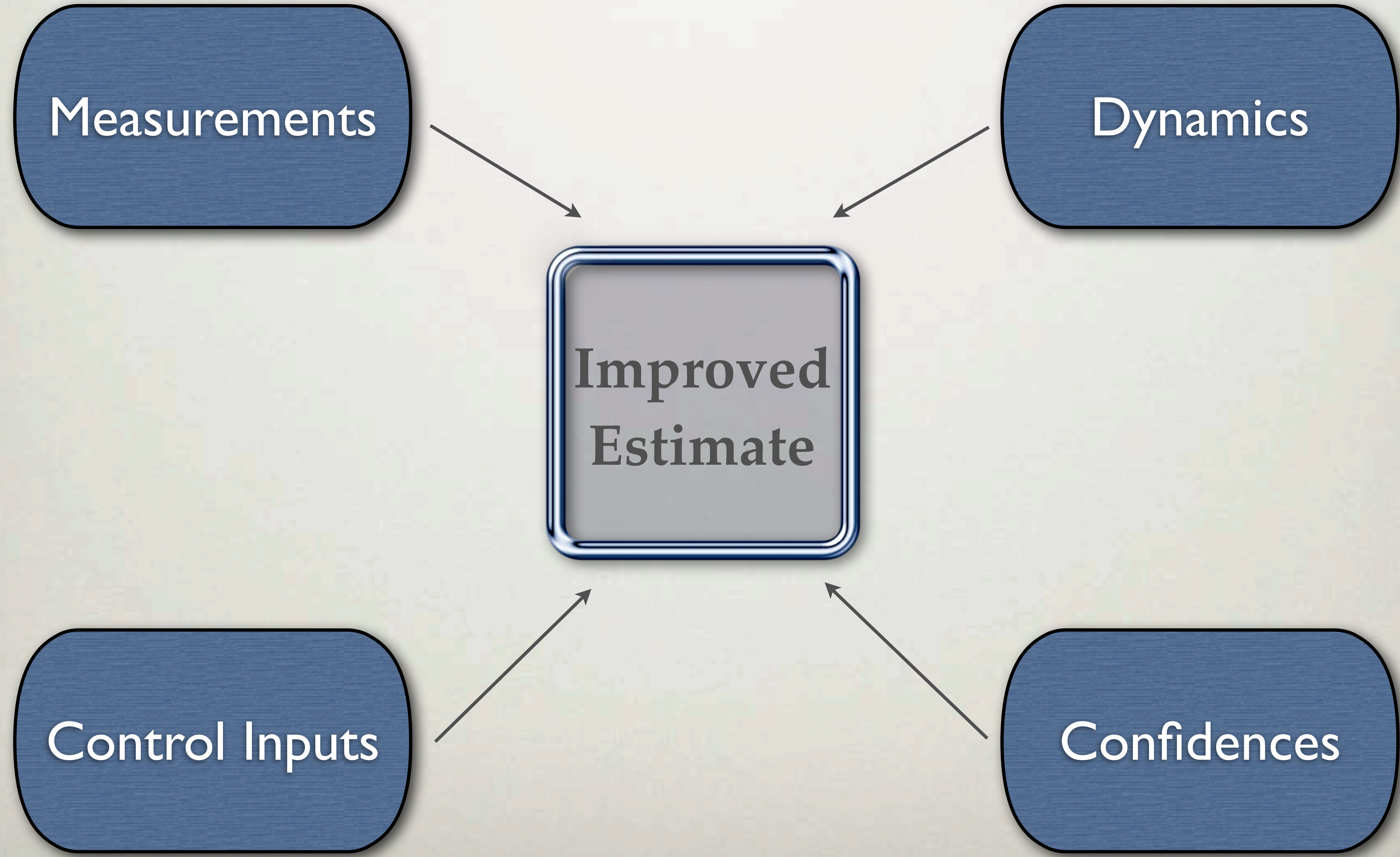
ANALYSIS

Dynamic Estimation





- Lots of (noisy) data
- Indirect measures
- Complicated dynamics
- Must understand control strategy


Measurements
Dynamics
Improved Estimate
Control Inputs
Confidences

Thought Example

Clear View

Out of focus

Distorted

Bad Angle

Colour blind

Thought Example

Clear View

Out of focus

Distorted

Known Dynamics

Bad Angle

Colour blind

$$\bar{X} = \left(\sum_{i=1}^{N} \mathbf{y}_i c_i \right) / \left(\sum_{i=1}^{N} c_i \right)$$

Weighted Average

Measurements

Weights

- Weight estimates based on confidence of measures

- State Vector X
  - Quantity to be estimated
- Covariance P
  - "Sigma Squared" variance of estimate
- Measurement Variance R
  - "Sigma Squared" noise on measurements
- Measurement Equation $\mathbf{y} = HX$
  - Relates measurements to state vector

"Cost"

$$J = (\mathbf{y} - HX)^T R^{-1} (\mathbf{y} - HX)$$

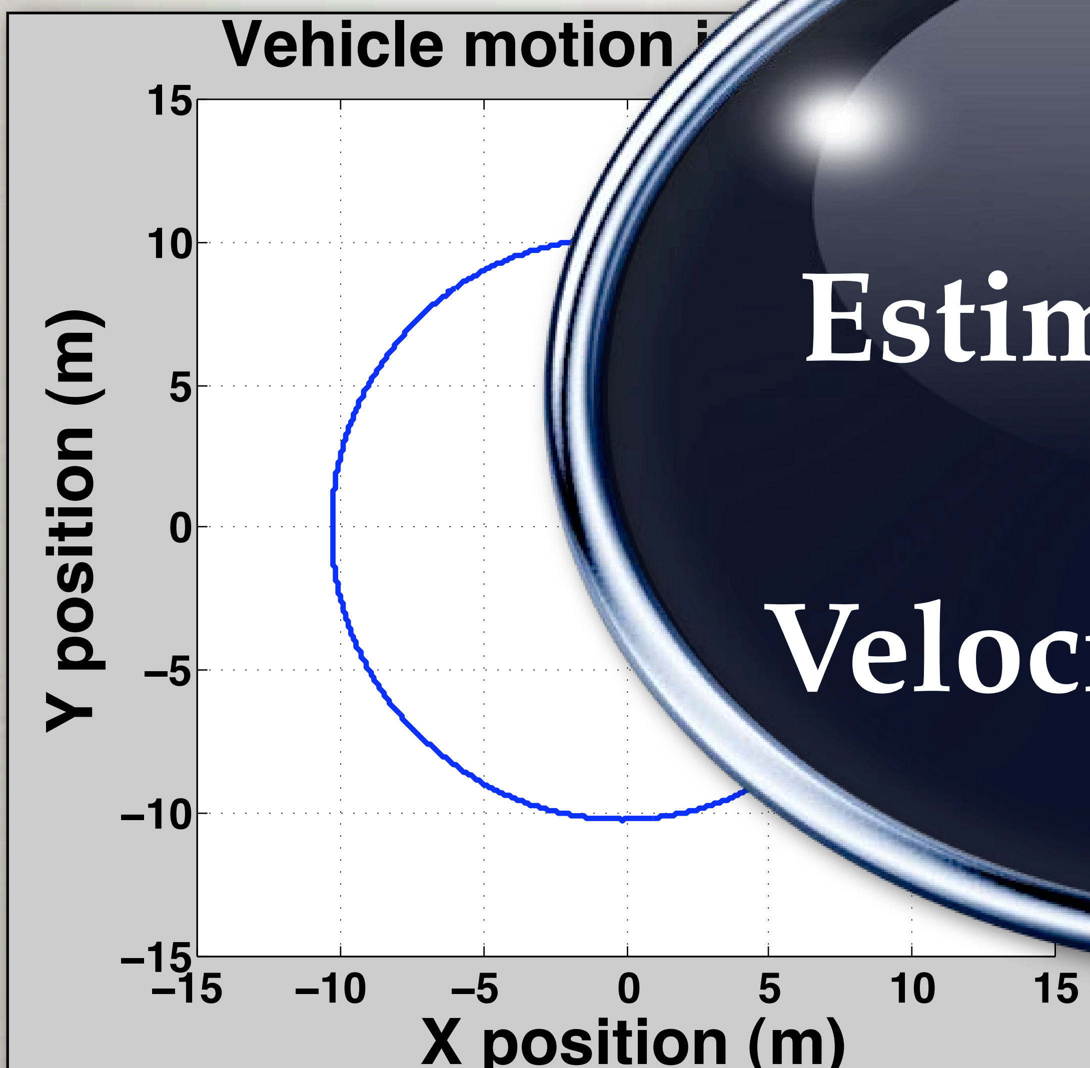
First Variation

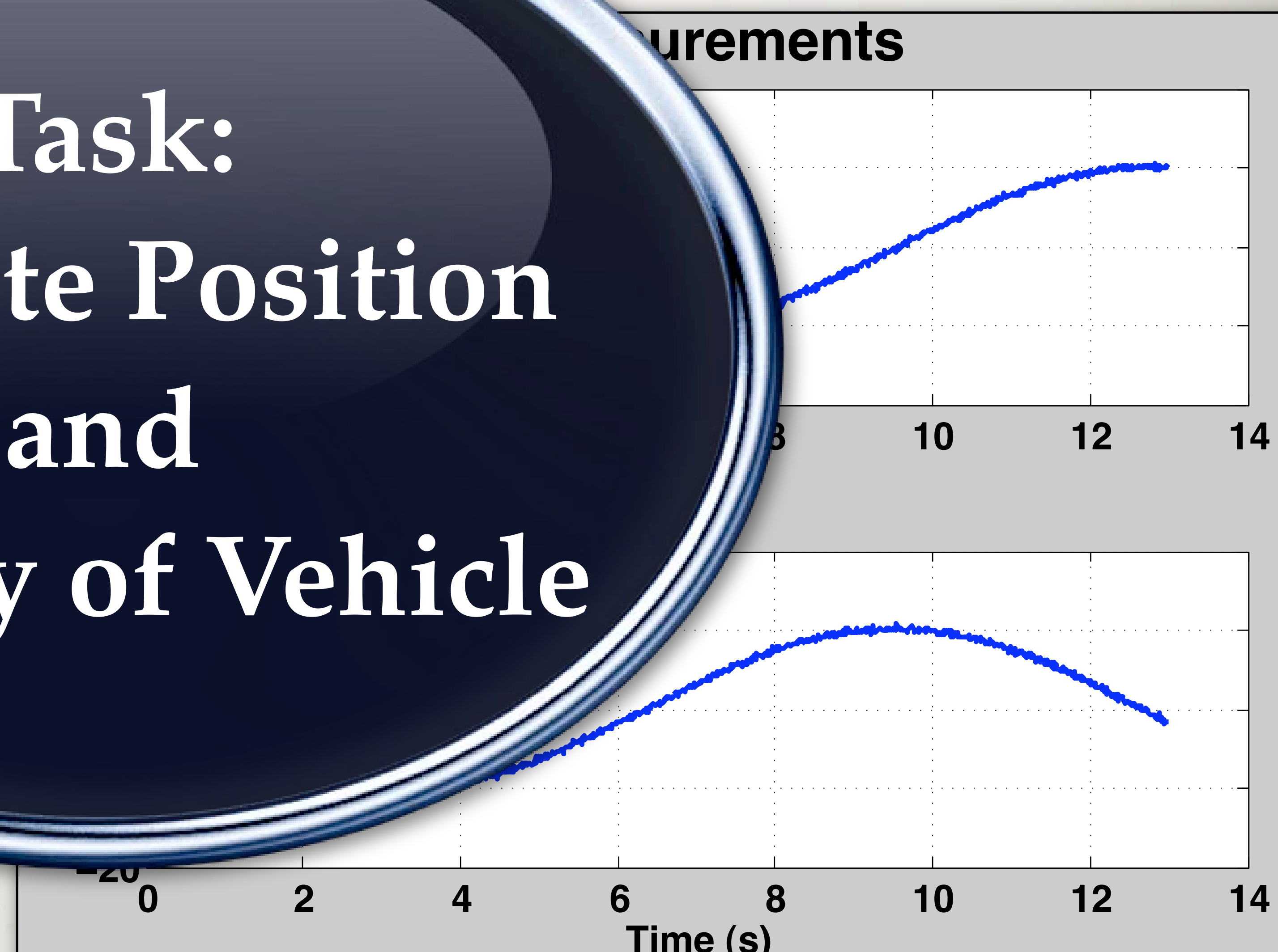
$$\frac{\partial J}{\partial X} = (\mathbf{y} - HX)^T R^{-1} (0 - H) = 0$$

Stationary Point

$$X = \left(H^T R^{-1} H\right)^{-1} H^T R^{-1} \mathbf{y}$$

Numeric Example

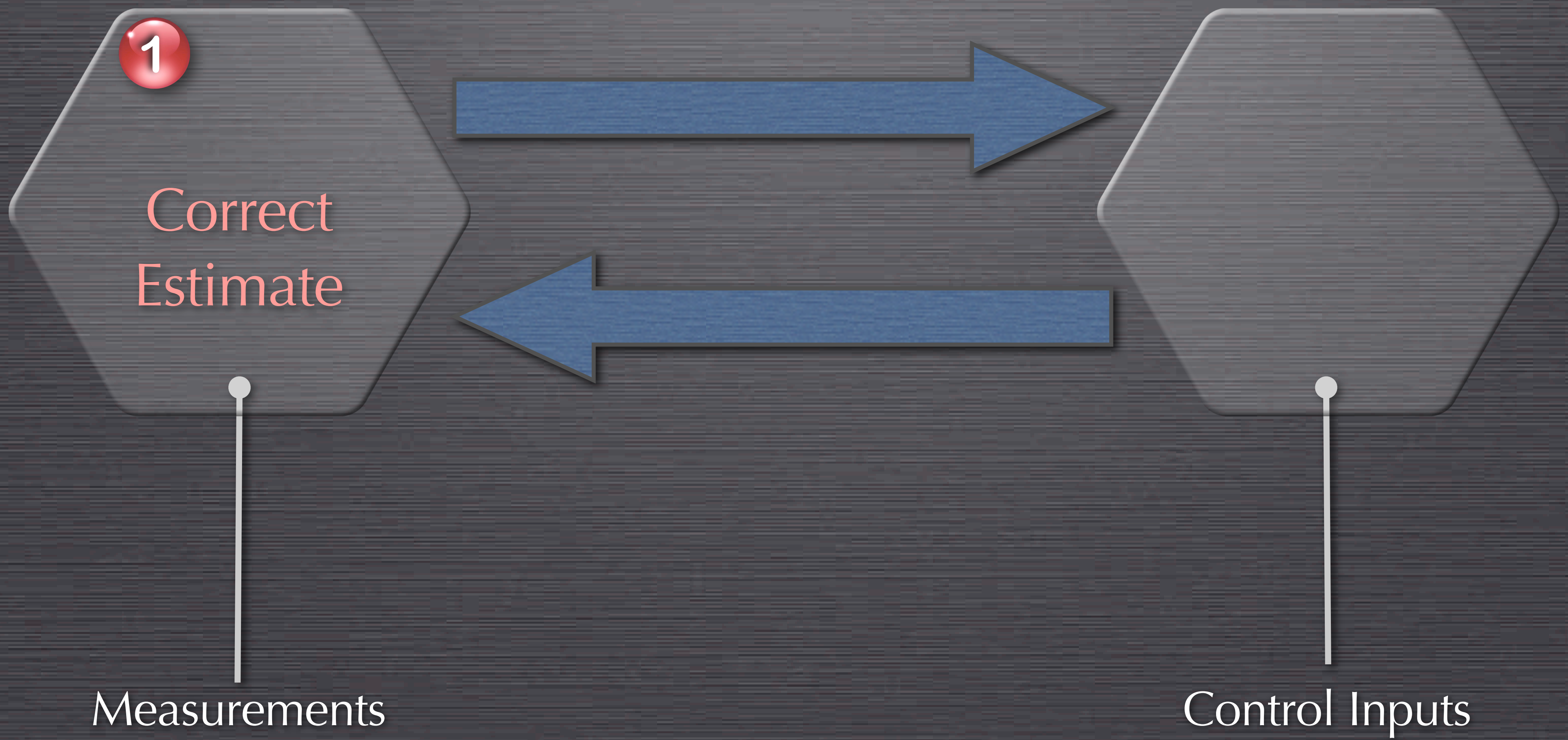




Task:
Estimate Position
and
Velocity of Vehicle

Circular Vehicle Motion

Kalman Filter



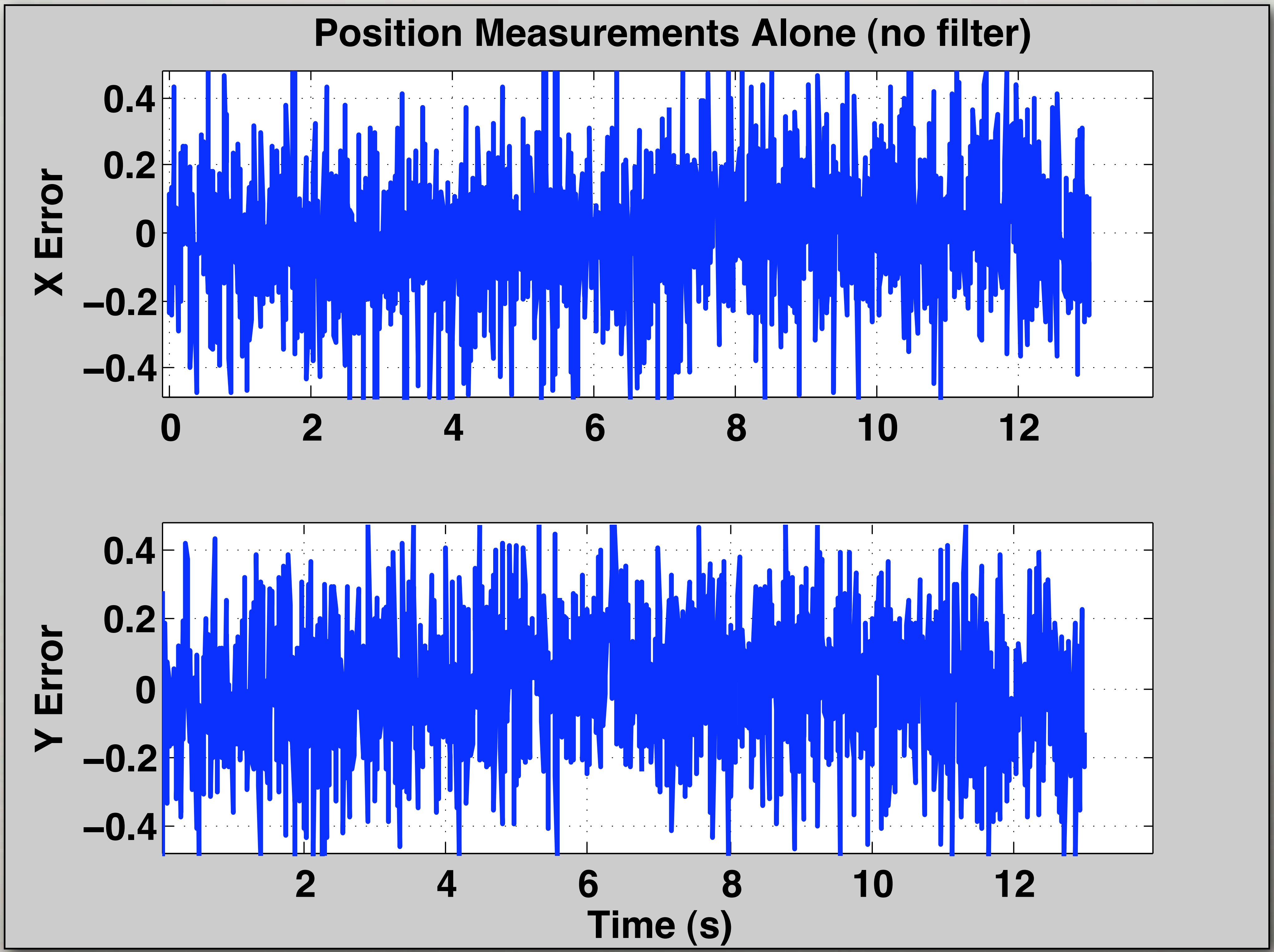
Kalman Measurement Update

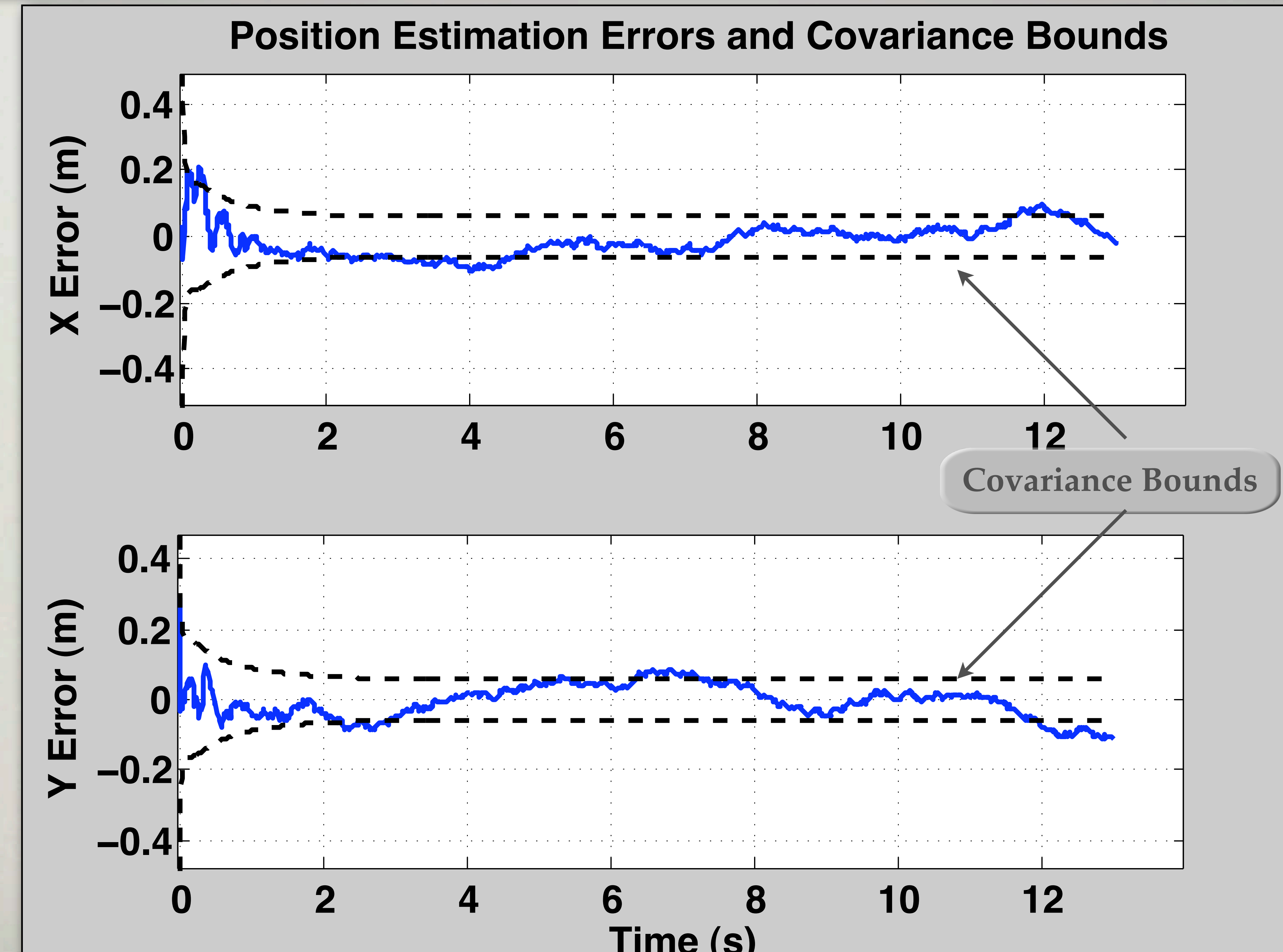
$$
\begin{aligned}
S_k &= H_k P_k^- H_k^T + R_k \\
K_k &= P_k^- H_k^T S_k^{-1} \\
\hat{X}_k^+ &= \hat{X}_k^- + K_k \left(z_k - H_k \hat{X}_k^- \right) \\
P_k^+ &= \left(\mathbf{1} - K_k H_k \right) P_k^-
\end{aligned}
$$

Kalman Time Update

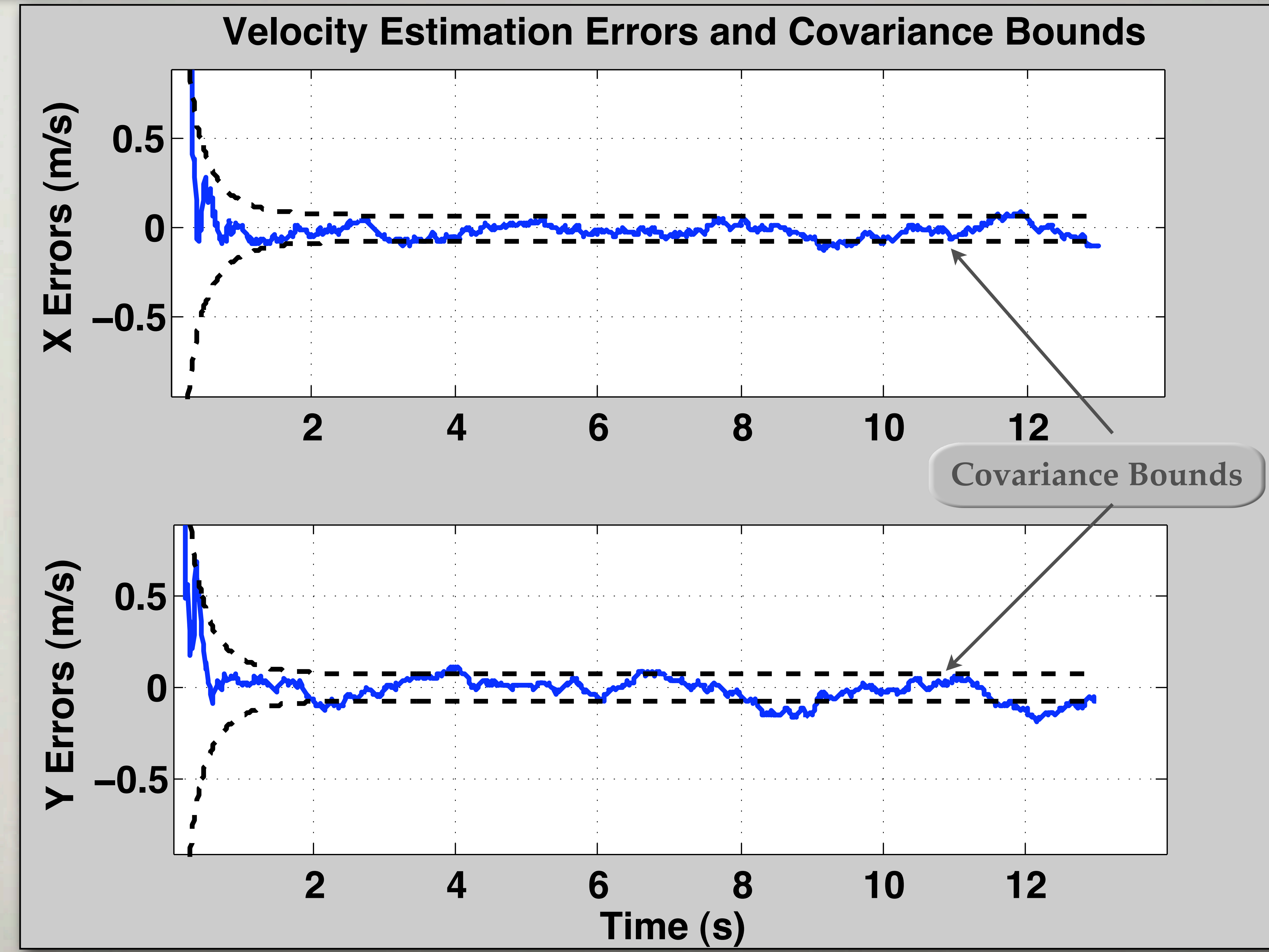
$$
\begin{aligned}
\hat{X}_{k+1}^- &= \Phi_k \hat{X}_k^+ \\
P_{k+1}^- &= \Phi_k P_k^+ \Phi_k^T + Q_k
\end{aligned}
$$

Measurements Only





Position Estimation Errors and Covariance Bounds
X Error (m)
Y Error (m)
Time (s)
0.4
0.2
0
−0.2
−0.4
0
2
4
6
8
10
12
Covariance Bounds



Velocity Estimation Errors and Covariance Bounds
X Errors (m/s)
0.5
0
−0.5
2
4
6
8
10
12
Covariance Bounds
Y Errors (m/s)
0.5
0
−0.5
2
4
6
8
10
12
Time (s)

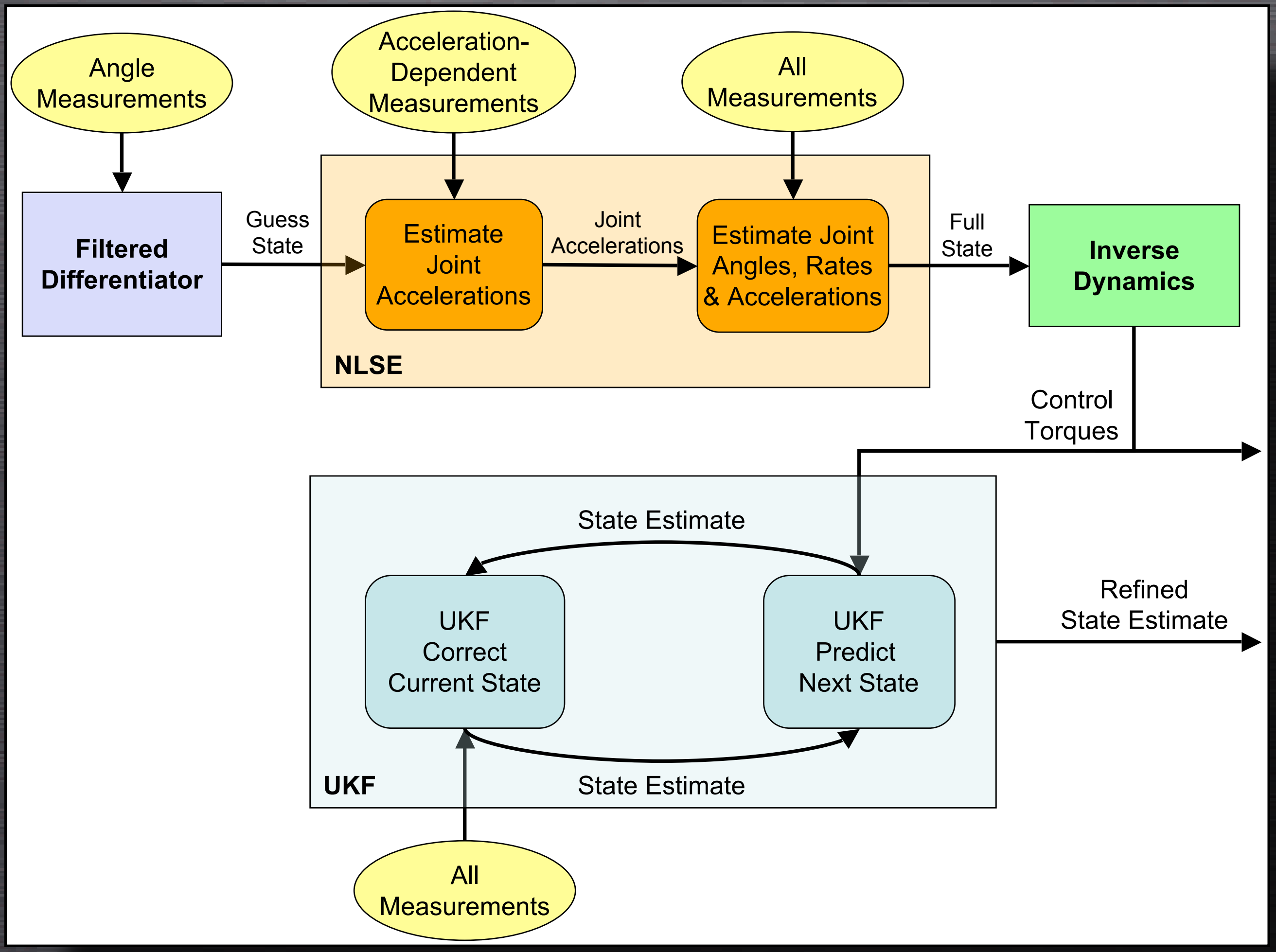
Kalman Filter Basics

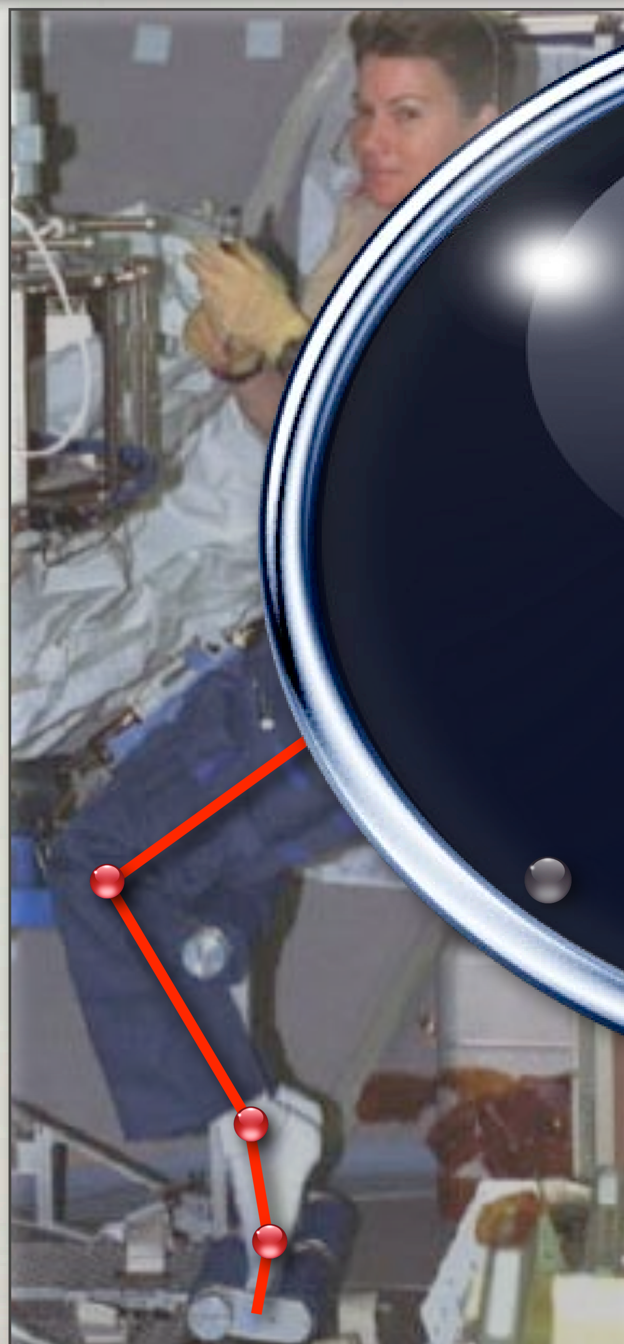
- Assumes measurements and dynamics tainted by Gaussian white noise
  - Need estimates of noise variances before starting
  - Initial process (dynamics) noise variances go on *diagonal* of initial covariance matrix (P)
  - Measurement variances (usually constant) go on *diagonal* of the measurement variance matrix (R)
- Must also start with an initial state estimate (does not need to be very good)
- Dynamics need to be linearized and discretized
  - **State transition matrix**: Takes state at time step i and propagates to time step ($i + 1$)

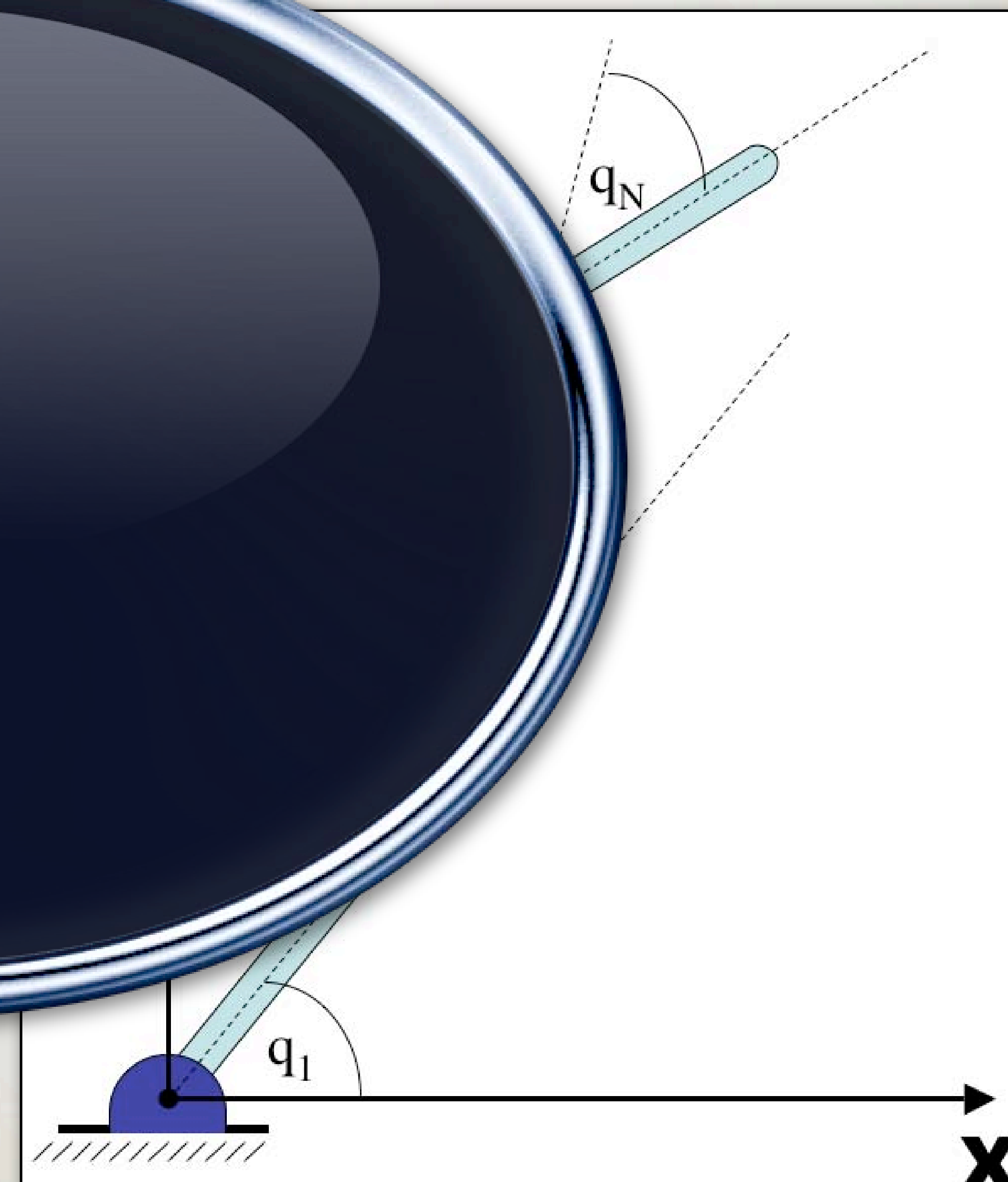
Kalman Steps to Success

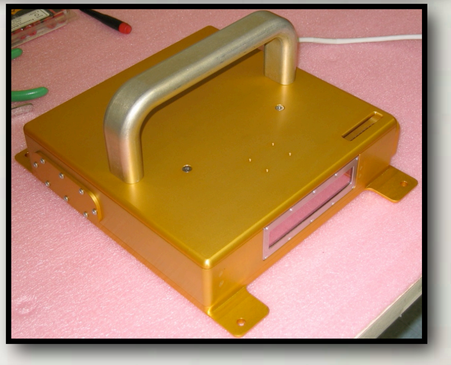
1. Define the state we wish to estimate.
2. Identify process and measurement noise and create P and R matrices.
3. Define, then linearize and discretize state dynamics.
4. Define the measurement equation.
5. Run the Kalman filter in a loop.
6. If running in simulation, plot the estimation error on the same axes as the square roots of the covariance matrix diagonals (to verify proper filter operation).
7. Brag to your friends that you know how to write a Kalman filter.

Advanced Example






q_N
q_1
x



- Forces & Moments (Kinetics)

$$\mathbf{F}_b = \sum_{i=1}^{i=N} m_i \ddot{\mathbf{r}}_i + \mathbf{F}_G$$

Link Mass (m_i), Link Acceleration ($\ddot{\mathbf{r}}_i$), Gravity Forces ($\mathbf{F}_G$)

$$\mathbf{M}_b = \sum_{i=1}^{N} \mathbf{r}_i \times m_i \ddot{\mathbf{r}}_i + I_i \dot{\omega}_i$$

Link Position ($\mathbf{r}_i$), Link Inertia (I_i), Joint Acceleration ($\dot{\omega}_i$)

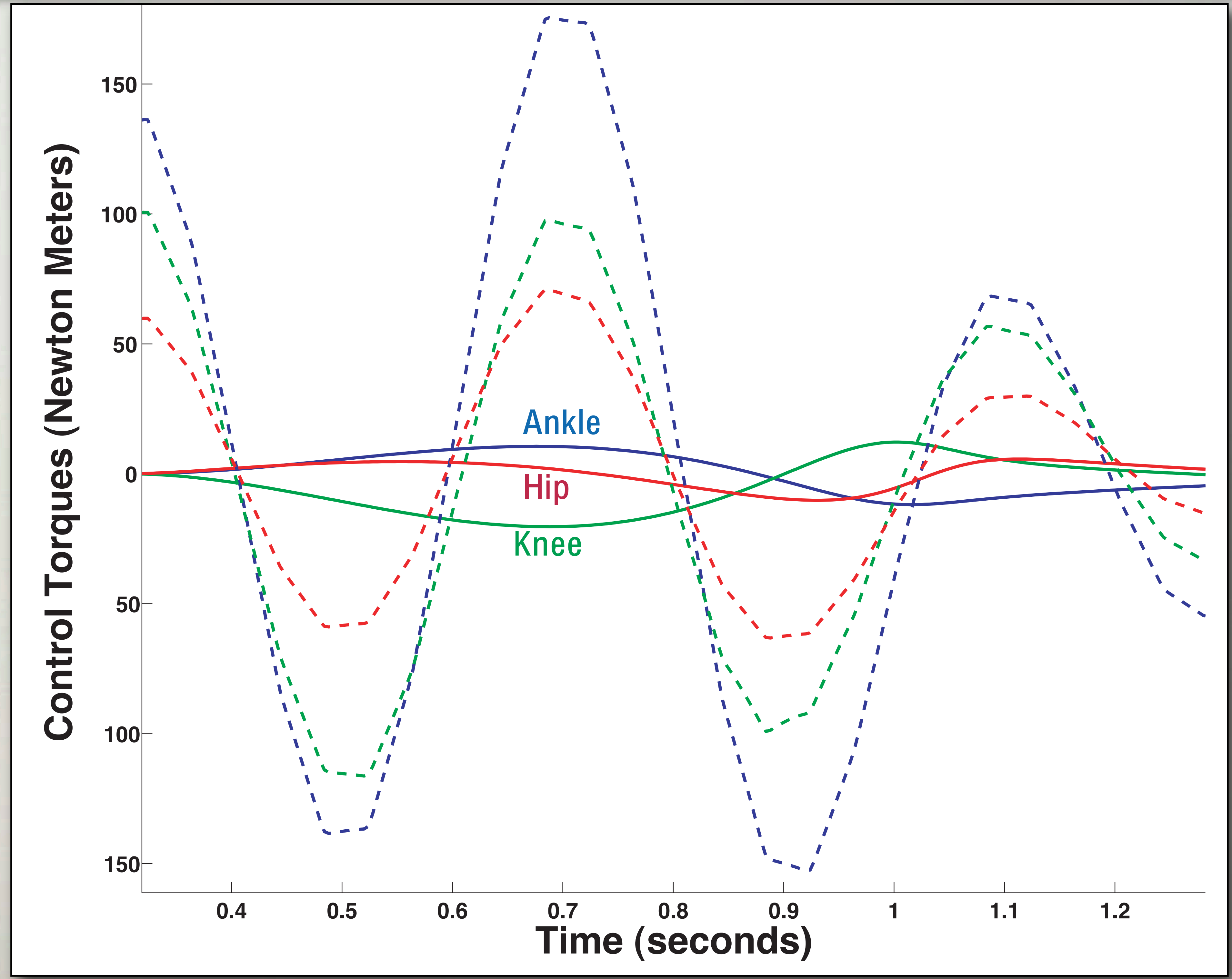
- Joint Angles (Kinematics)
  - Video Analysis

$$\tau = H(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + D(\theta, \dot{\theta})\dot{\theta} + G(\theta)$$

Control Torques = Inertia Torques + Coriolis Torques + Friction Torques + Gravity Torques

- Function of joint angles, rates, accelerations

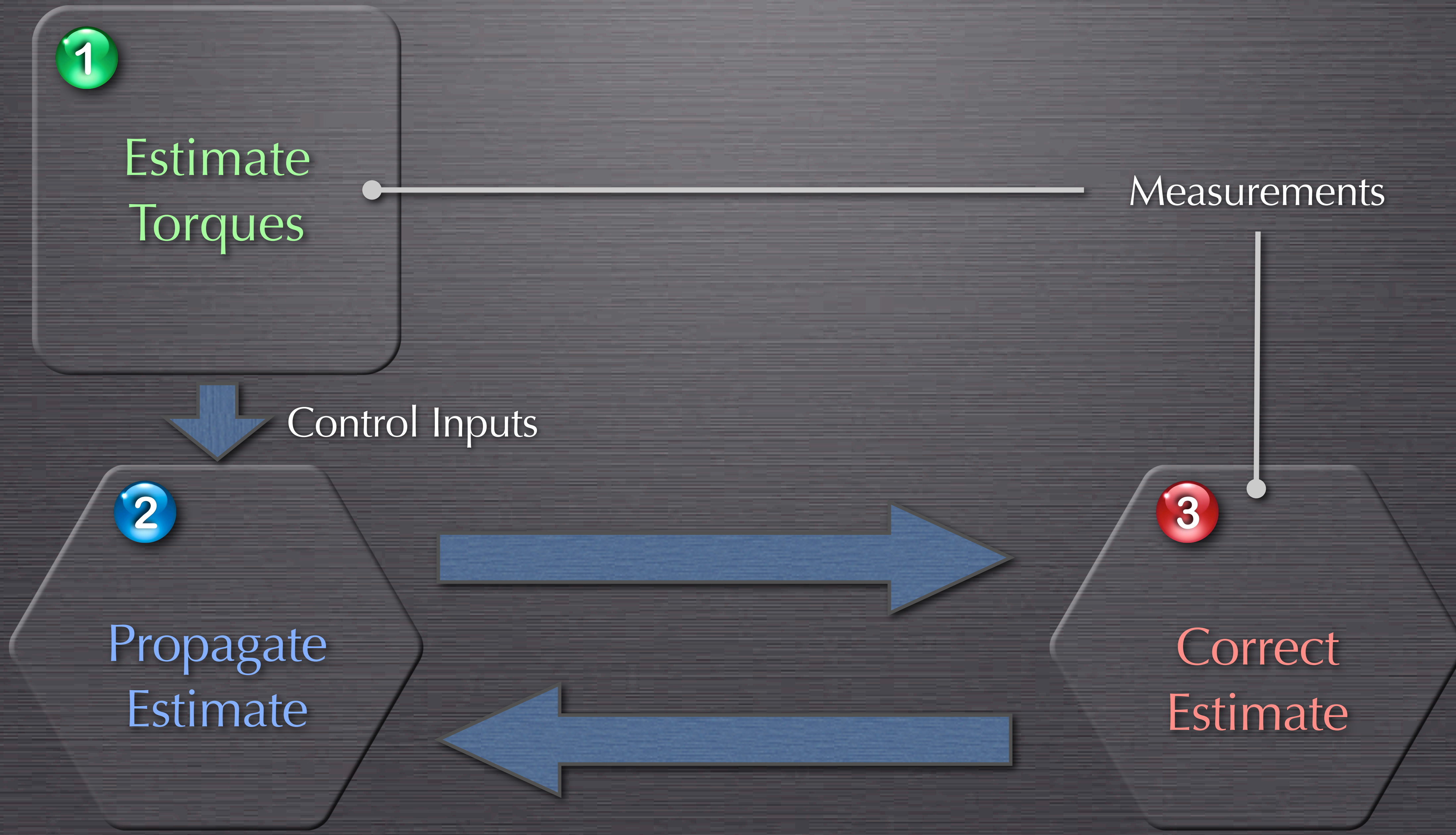
- Nonlinear Dynamics
- Nonlinear Measurements
- Acceleration State Required
  - What are the acceleration dynamics?
- Observability



Control Torques (Newton Meters)
150
100
50
0
50
100
150
Ankle
Hip
Knee
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
Time (seconds)

Strategy

- Dynamic equation known
  - But not control inputs!
- “One-Step” torque estimator required
- Estimate accelerations → compute torques
- Torques can be used in 2-step estimator to improve angle and rate estimates


1
Estimate Torques
Measurements
Control Inputs
2
Propagate Estimate
3
Correct Estimate

- Non-linear Least-Squares

$$J = (\mathbf{y} - h(X))^T R^{-1} (\mathbf{y} - h(X))$$

Nonlinear Measurements

- Solution is a function of state
  - Require iterative solution technique

Newton-Raphson

Newton-Raphson

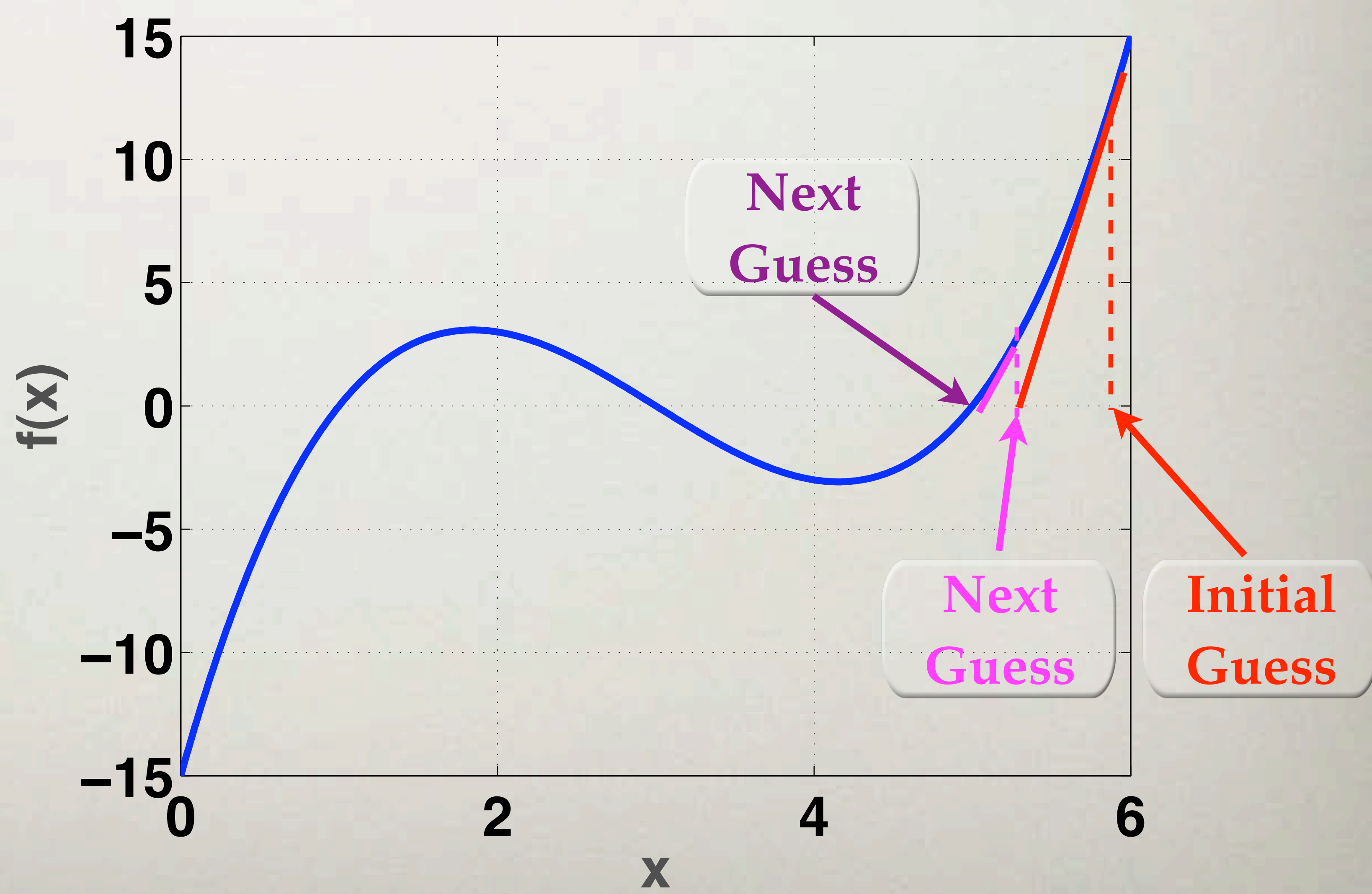
1

Taylor Series

$$f\left(x_0+\Delta x\right)=f\left(x_0\right)+\left.\frac{df\left(x\right)}{dx}\right|_{x_0}\Delta x+\left.\frac{d^2f\left(x\right)}{dx^2}\right|_{x_0}\frac{\left(\Delta x\right)^2}{2!}+\ldots$$

Update

$$\Delta x=-\frac{f\left(x_0\right)}{\left.\frac{df(x)}{dx}\right|_{x_0}}$$



Observability

- Measurement *Information Content*

$$Y = H^T R^{-1} H$$

  - Shows up in Least Squares Estimator

- Full state observability requires **full rank**
  - Increase rank?
    - Increase measurements
    - Decrease state size

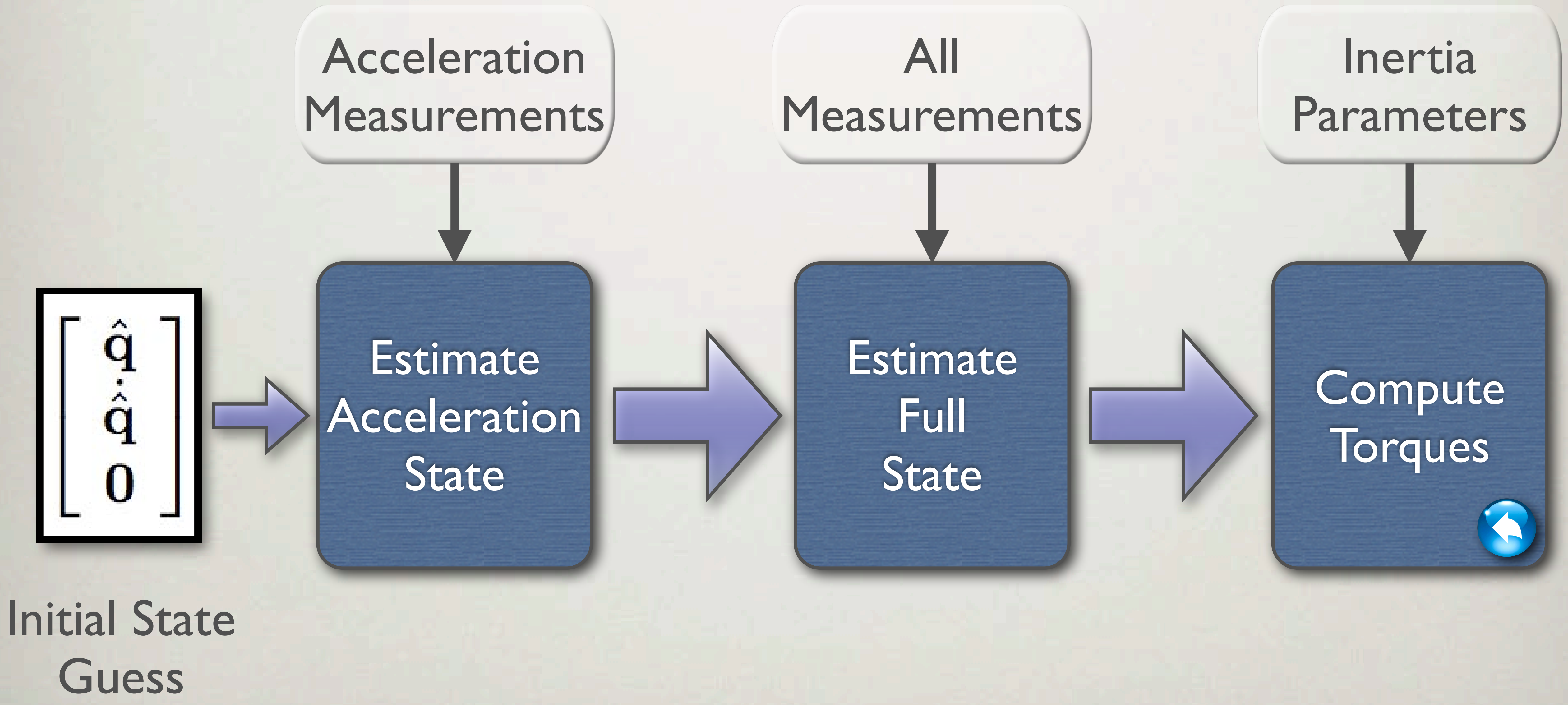
Prior Information

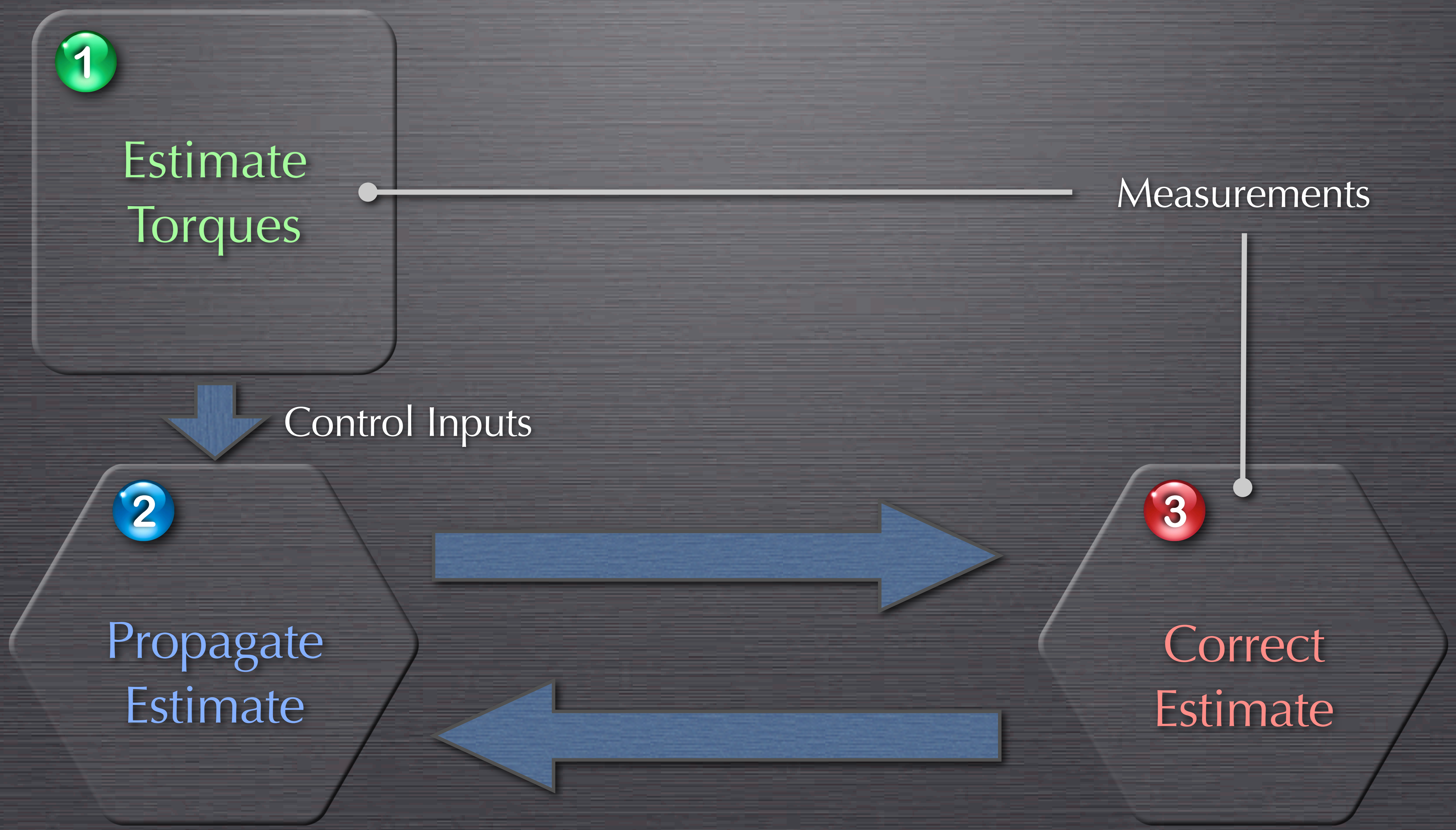
- Augment cost to include prior estimates

$$J = (X - X_0)^T P_0^{-1} (X - X_0) + (y - h(X))^T R^{-1} (y - h(X))$$

Nonlinear Least Squares
with
Prior Information

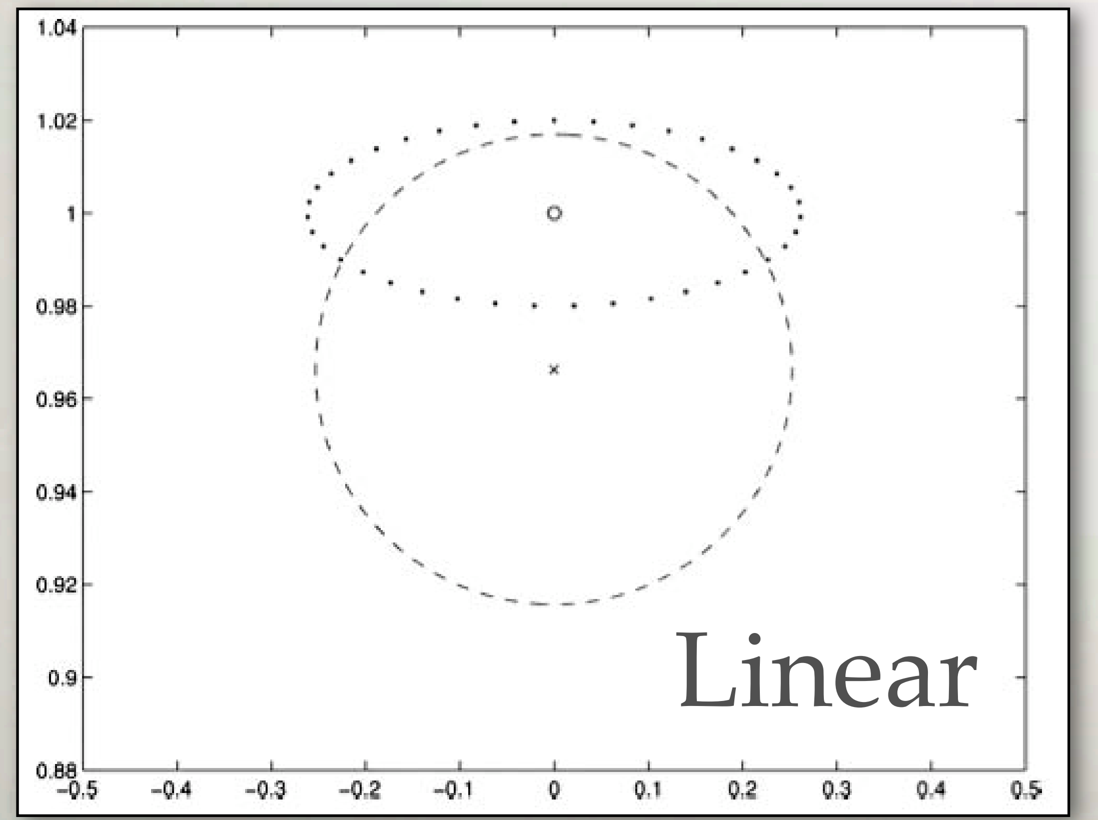
Torque Estimator

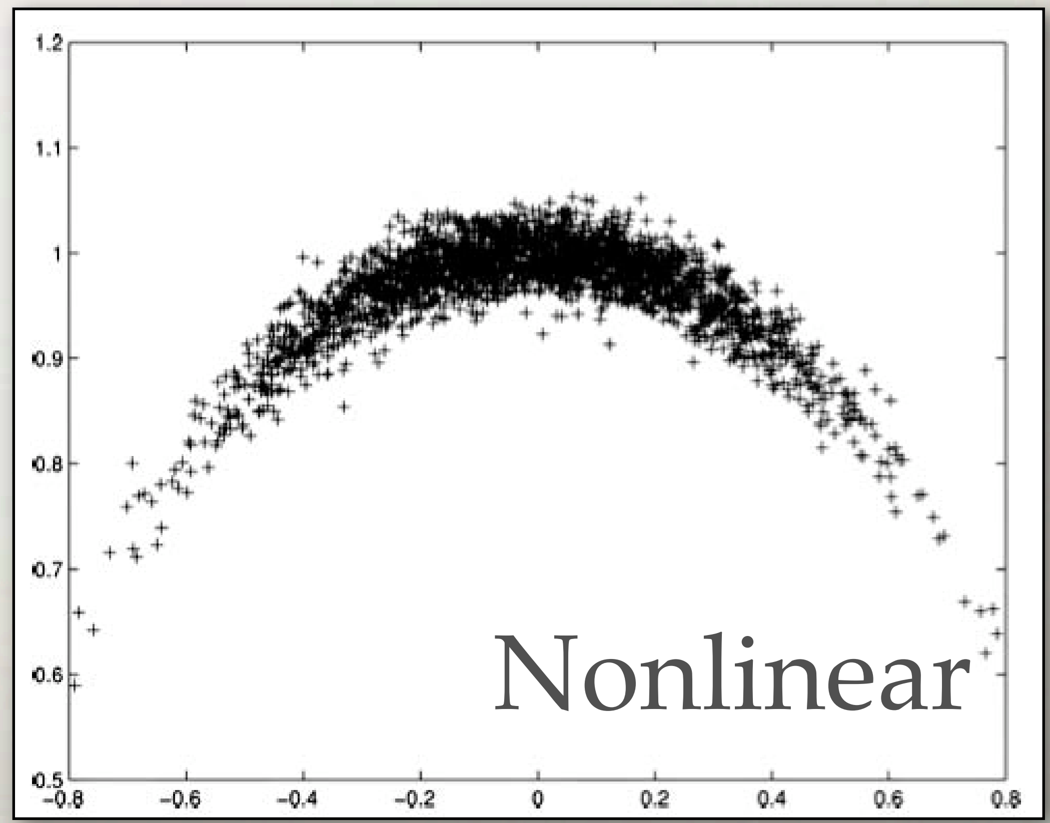



1
Estimate Torques
Measurements
Control Inputs
2
Propagate Estimate
3
Correct Estimate

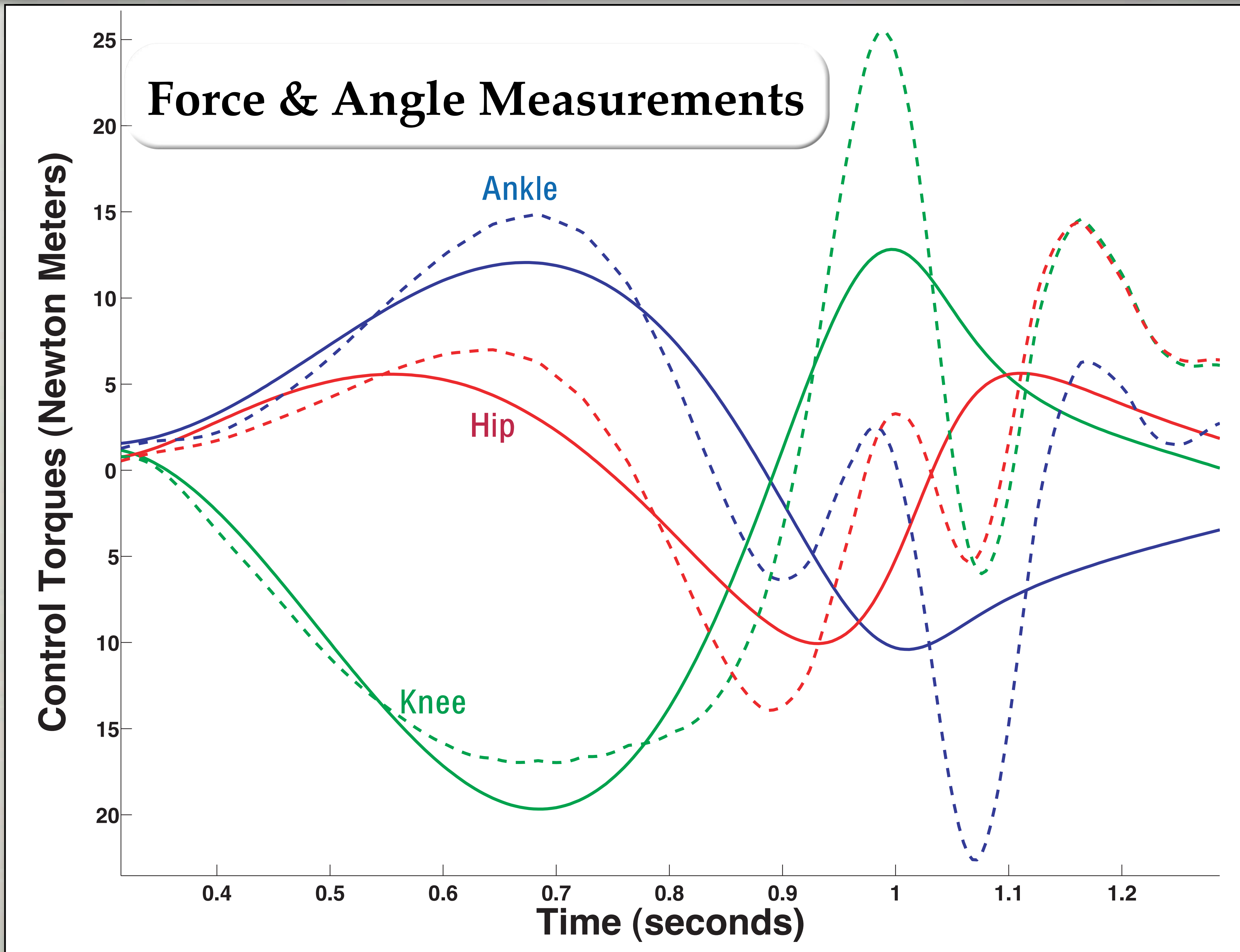
Unscented Kalman Filter

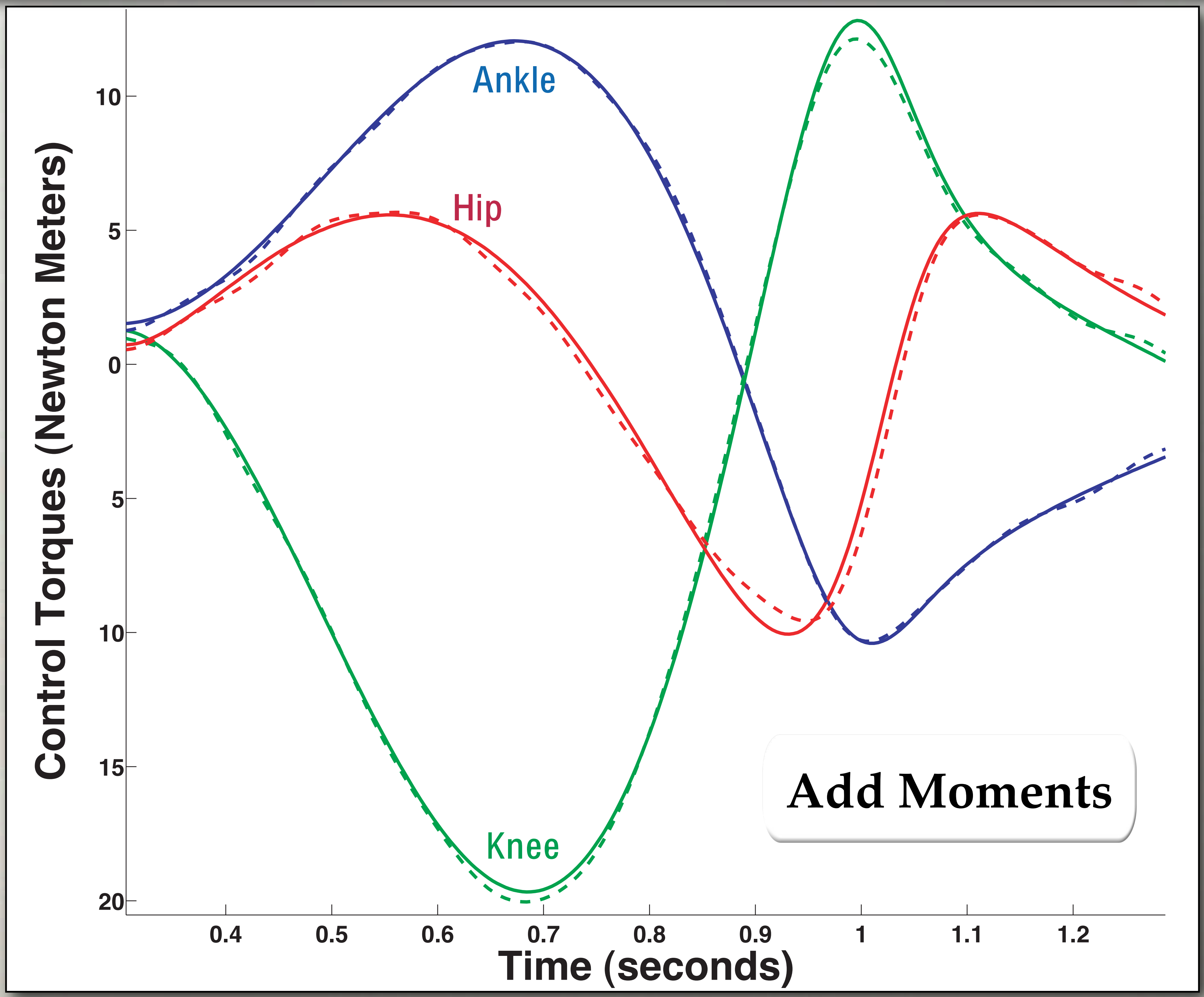
- Extended Kalman Filter permits nonlinear measurements & dynamics
  - Still requires linearization of dynamics for covariance propagation
- Unscented Kalman Filter incorporates nonlinear covariance propagation



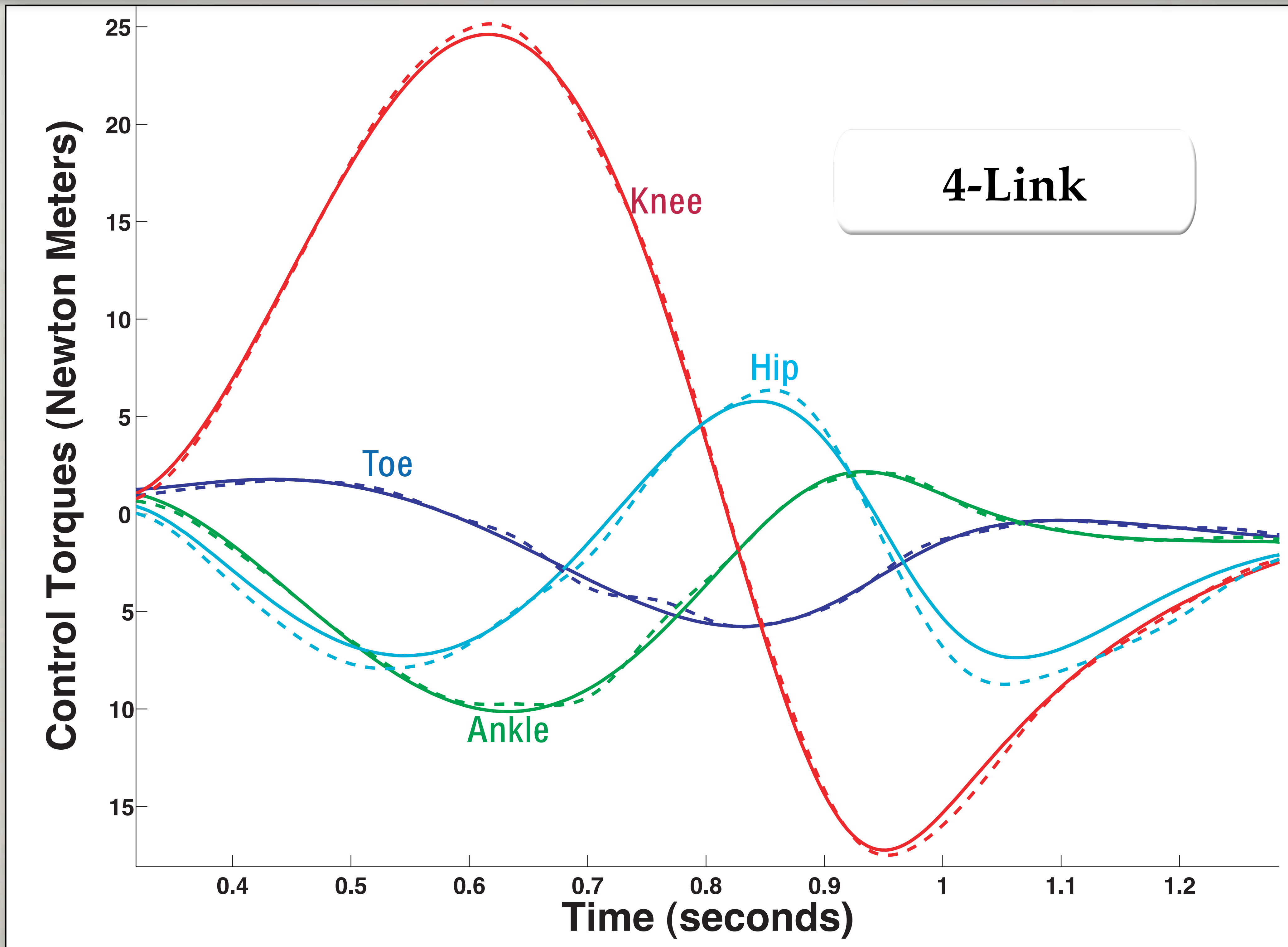


Julier, '04

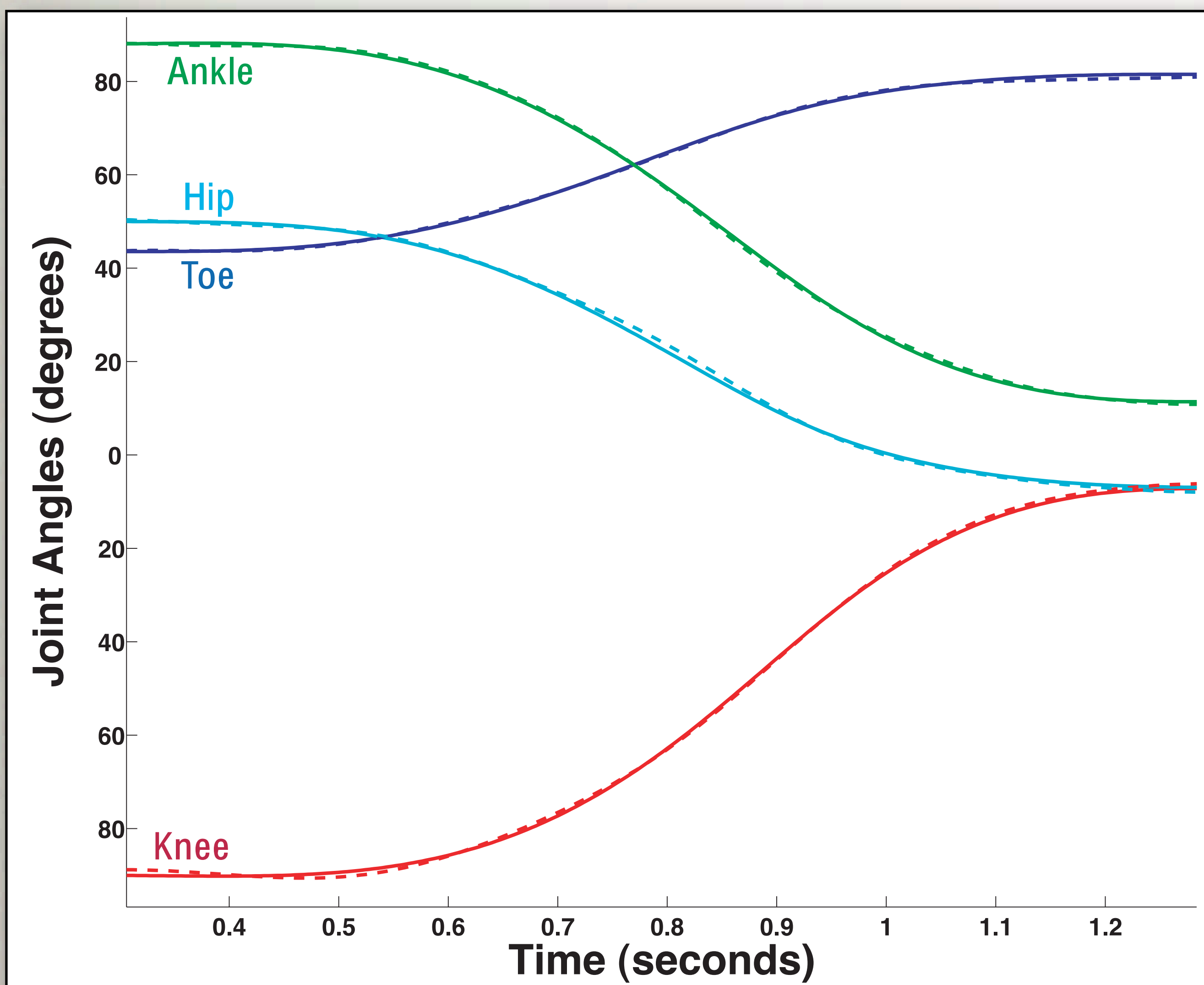

Force & Angle Measurements
Ankle
Hip
Knee
Control Torques (Newton Meters)
25
20
15
10
5
0
5
10
15
20
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
Time (seconds)

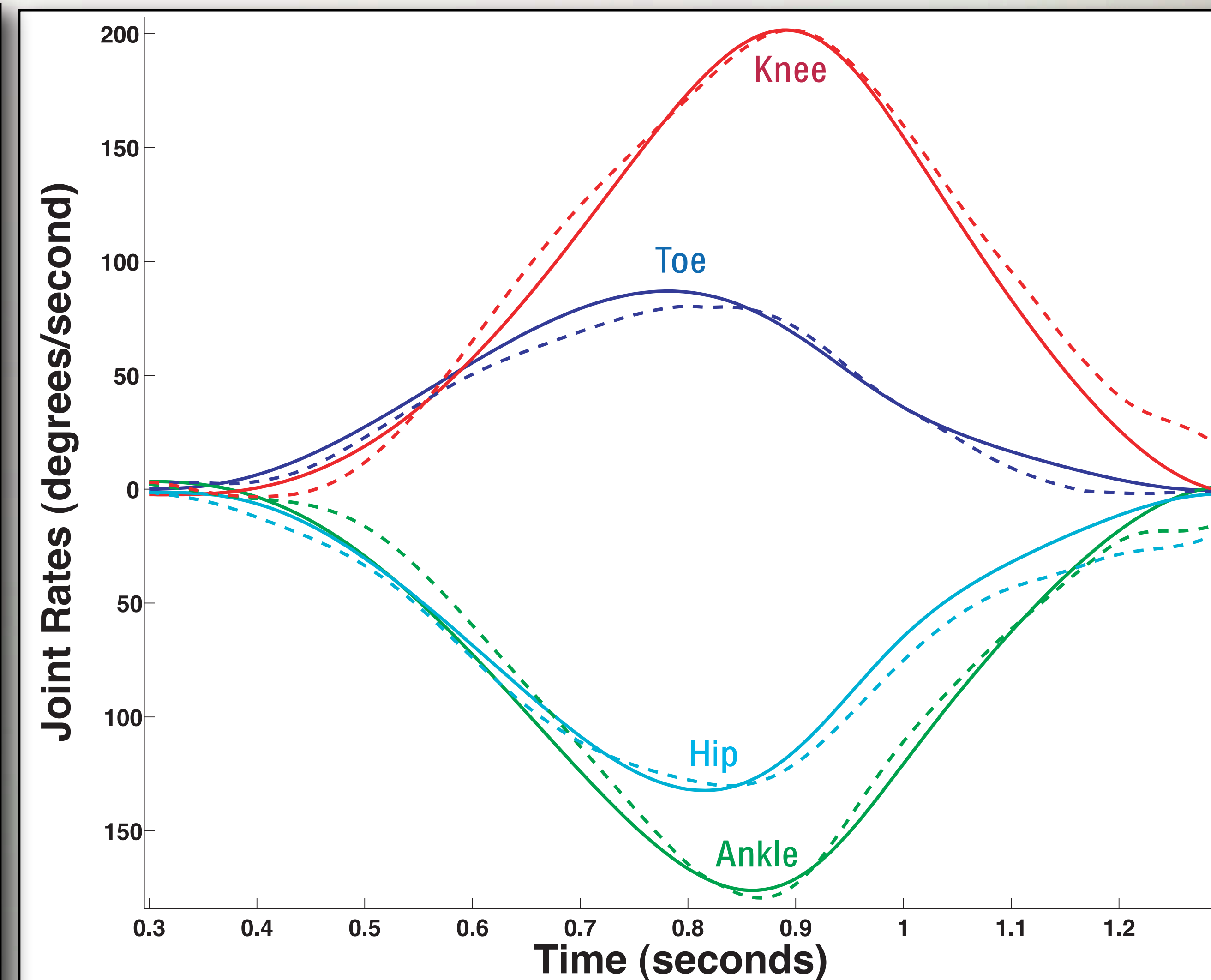


Ankle
Hip
Knee
10
5
0
5
10
15
20
Control Torques (Newton Meters)
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
Time (seconds)
Add Moments


4-Link
Knee
Hip
Toe
Ankle
Control Torques (Newton Meters)
25
20
15
10
5
0
5
10
15
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
Time (seconds)

Angles and Rates





Astronaut Rotation Dynamics

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Adapted from SBE-2003 term project by P. Ferguson and K. Bethke

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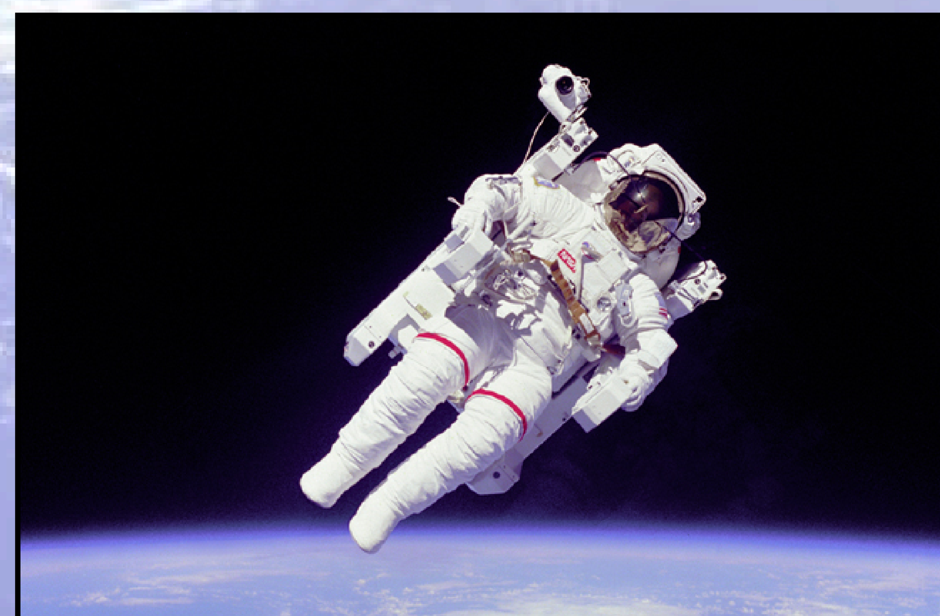
Motivation: Why Self-Rotation?

- EVA separation danger
  - 1 emergency predicted for every 1,000 hours of EVA
- Attitude-hold thrusters are inadequate
- IVA training / adaptation

Propulsive Devices

Ed White using HHMMU (NASA)



Bruce McCandless using MMU (NASA)

SAFER (NASA)

Background

- Astronauts experimenting with self-rotation for years! (largely undocumented).
- Kulwicki outlined several body motions in 1962.
- Kane provided an “optimal” analysis in 1968.
- Frohlich provided further insight, linking divers, cats and astronauts in 1980.
- No substantial scientific link between astronaut EVA operations and dynamics.

Momentum

- Linear
  - Velocity vector always parallel to momentum
  - Zero momentum implies zero velocity
  - Mass dependent
  - Remains constant if no force (and constant mass)

$$\mathbf{p} = m\mathbf{v}$$

- Angular
  - Velocity vector not always parallel to momentum
  - Zero momentum does not imply zero velocity
  - Inertia dependent
  - Remains constant if no torque (and constant mass)

$$\mathbf{h} = I\omega$$

Some Mathematics ...

- Rotation Matrix
  - Relates two coordinate frames at different rotations
  - Permits us to easily express a vector in multiple frames

$$R(\theta) = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$$

$$\mathbf{v}_a = R_{ab}\mathbf{v}_b$$

Vector v expressed in frame a = Rotation matrix rotating frame b to frame a * Vector v expressed in frame b

NOTE: A vector does NOT depend on the frame it is expressed in! Using rotation matrices, a single vector can be expressed in ANY reference frame.

More Mathematics ...

- Vector Calculus
  - Often convenient to express quantities in rotating frames
  - But, Newton's laws must be applied in "Newtonian" (inertial) frames
    - Non accelerating, non rotating
  - Must express vector derivatives in inertial frames

$$\dot{\mathbf{v}} = \mathring{\mathbf{v}} + (\omega \times \mathbf{v})$$

Derivative in inertial frame

Derivative as seen in rotating frame

Rotational rate of rotating frame with respect to inertial frame

Equations of Motion

- Linear Motion
  - Consider astronaut floating in space
  - How can the astronaut translate?
  - Where can external forces come from?

$$\dot{\mathbf{p}} = \mathbf{F}$$

$$m\dot{\mathbf{v}} = \mathbf{F}$$

  - No assumptions made on reference frames
  - What frame makes the most sense to report the forces in?
    - What if the astronaut is rotating?

Equations of Motion

- Rotational Motion
  - Always express rotation rate in ***body frame***
    - *I.e. Frame that rotates w.r.t. the astronaut*

$$\dot{h} = \tau$$
$$\dot{I}\omega + I\dot{\omega} = \tau$$

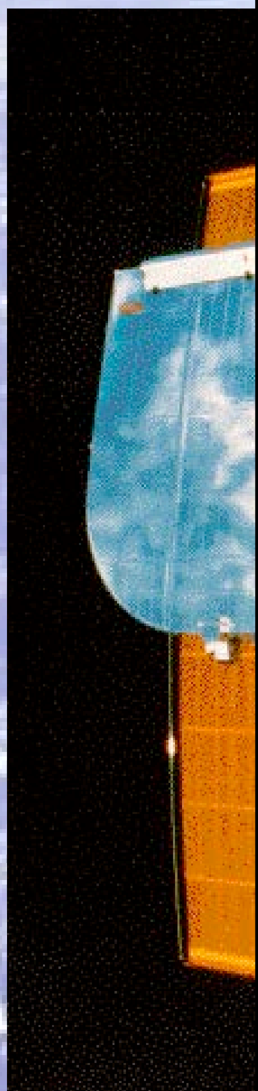
  - Now, assume all quantities are in the body frame

$$\mathring{I}_{body}\omega_{body} + I_{body}\mathring{\omega}_{body} + \omega_{body} \times (I_{body}\omega_{body}) = \tau_{body}$$

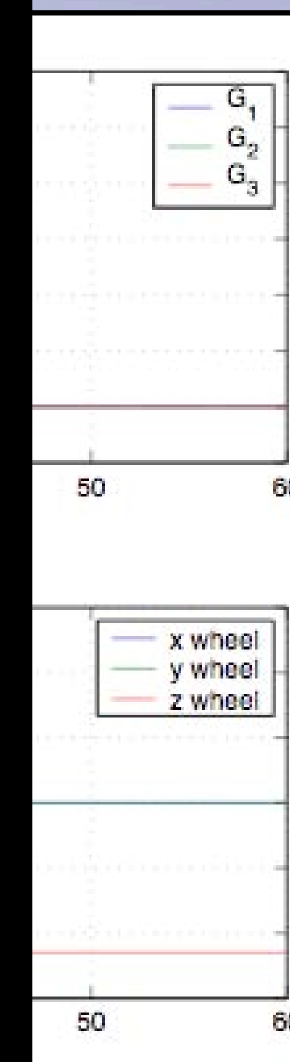
IMPORTANT: In a given expression, once a reference frame has been chosen, ALL terms MUST be expressed in the same frame!

Conservation of Angular Momentum

Basic Attitude Regulation



But astronauts have no momentum wheels,
so how can they do this?



Also, what about h=0?

- Easy to imagine re-orientations when starting with non-zero momentum
- How do astronauts do this when they start from rest?
  - If h=0, how can ω be anything but 0?

$$\mathbf{h} = I\omega$$

- Answer: I is a *tensor* and the human body is made of several attached segments!

Astronaut Pitch

Astronaut Yaw

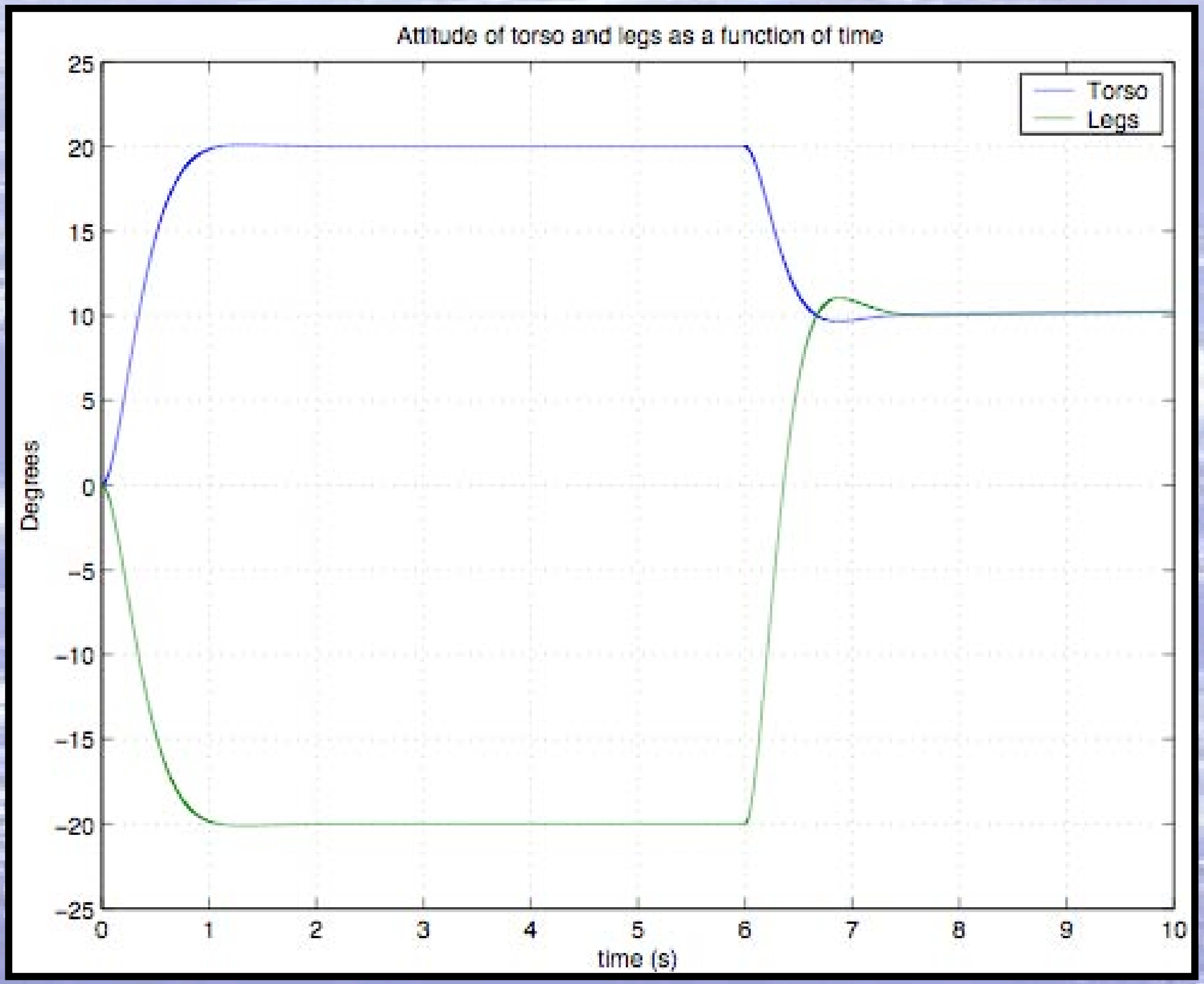
How did they do that?

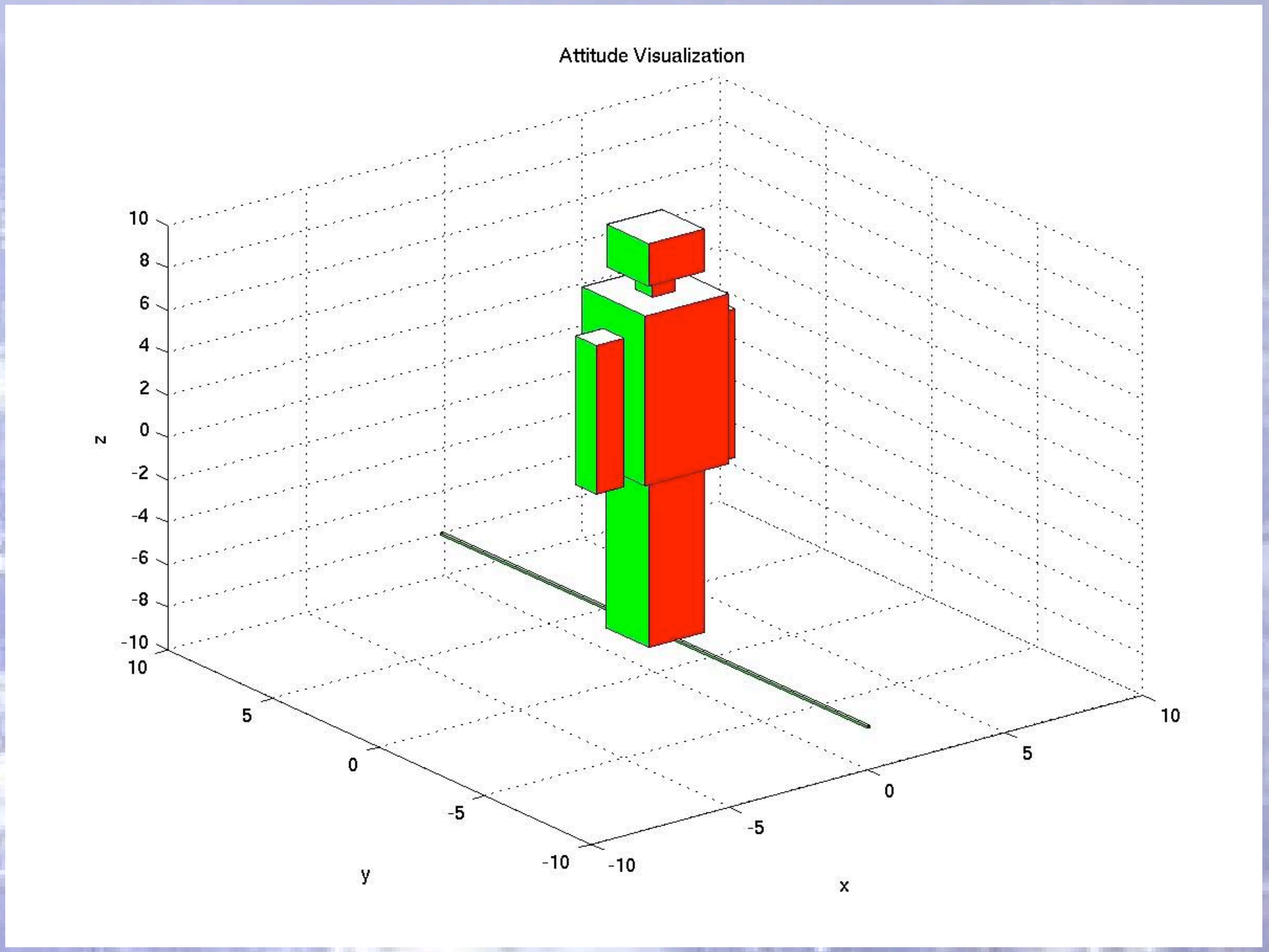
- Two separate techniques:
  - Change of Inertia tensor
    - Makes one body segment resistant to rotation
    - Astronaut then applies torque between segments

$$\theta_{\Delta} = (\theta_t - \theta_l)\left[\frac{I_{l_1}}{I_{t_1} + I_{l_1}} - \frac{I_{l_2}}{I_{t_2} + I_{l_2}}\right]$$

  - Mimic momentum wheels
    - Common because similar to swimming
    - Uses both arms and legs
    - Constant motion required
      - Practical for EVA? (stay tuned)

Self-Rotation Simulation





Other Z-axis motions

Discrete Inertia Changes	Continuous Momentum-wheel Motion
"Cat Reflex"	"Lasso"
Twist torso, feet pointed opposite shoulders. Increase upper moment of inertia by extending arms straight out. Untwist torso to achieve net displacement. Lower arms.	Extend one or both arms straight up over head. Continuously revolve arm(s) about the z-axis. Arm(s) trace cone with shoulder at apex. Make cone as wide as possible.
"Bend and Twist"	"Pinwheel"
Bend at waist to one side and extend arms overhead. Rotate upper body to other side, keeping back horizontal and arms out. Draw arms down and unbend at waist.	With body straight and hands on hip, continuously rotate the entire upper part of the body in a conical motion. The upper body draws out a cone with its apex at the waist.