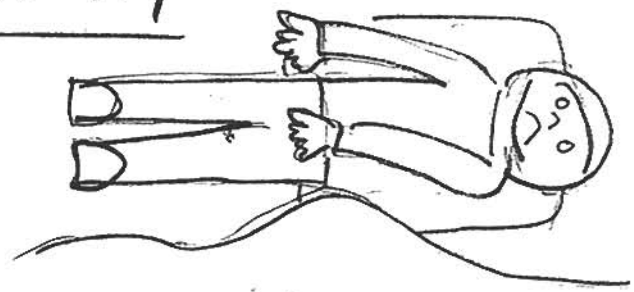
(I)

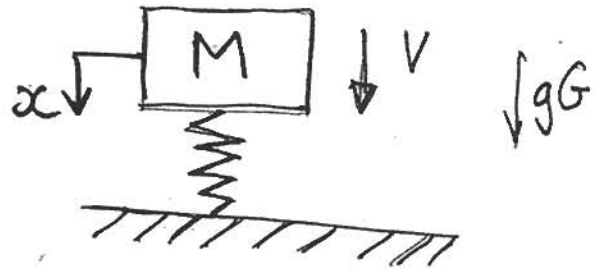
Dynamics – Fall on Hip



$$F_{max} = K x_{max}$$

Diff Eq. $Kx = MgG - M\ddot{x}$

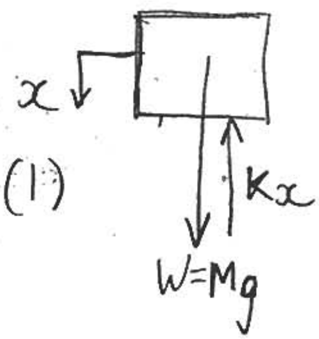
$$\Rightarrow M\ddot{x} + \underbrace{Kx}_{F_{max}} - MgG = 0$$



Solving for x_{max} gives

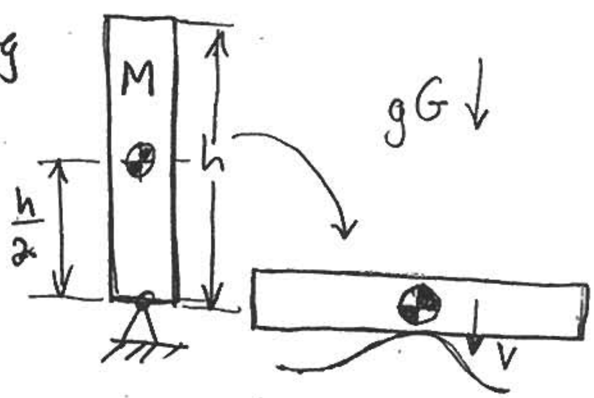
$$F_{max} = MgG\left[\left(\frac{V\omega_0}{gG}\right)\sin(\omega_0 t)_{max} - \cos(\omega_0 t) + 1\right] \quad (1)$$

where $\omega_0^2 = \frac{K}{M}$



Solving for V

model: rigid slender rod pivoting at base



Potential Energy before fall = Kinetic Energy after fall

$$MgG\frac{h}{2} = \frac{1}{2}I\omega^2 \qquad I = \frac{Mh^2}{3}$$

$$V = \frac{h}{2}\omega \Rightarrow \omega^2 = \frac{4V^2}{h^2}$$

$$\Rightarrow MgG\frac{h}{2} = \frac{1}{2}\frac{Mh^2}{3}\frac{4V^2}{h^2}$$

$$\Rightarrow \quad V^2 = \frac{3}{4}gGh$$

$$\Rightarrow \quad \boxed{V = \frac{\sqrt{3gGh}}{2}} \qquad (2)$$

Returning to (1) $\quad F_{max} = MgG\left(\left(\frac{V\omega_0}{gG}\right)\sin(\omega_0 t)_{max} - \cos(\omega_0 t) + 1\right)$

$MV\omega_0 + MgG$

$\omega_0^2 = \frac{K}{M}$, $\omega_0 = \sqrt{\frac{K}{M}}$

and using the fact that $MV\sqrt{\frac{K}{M}}$ $(\omega_0 t)_{max} \approx \frac{\pi}{2}$ rad

we get $\quad \boxed{F_{max} \approx V\sqrt{KM} + MgG}$?

Values:

K (estimated for soft tissue + spacesuit) ≈ 60 kN/m

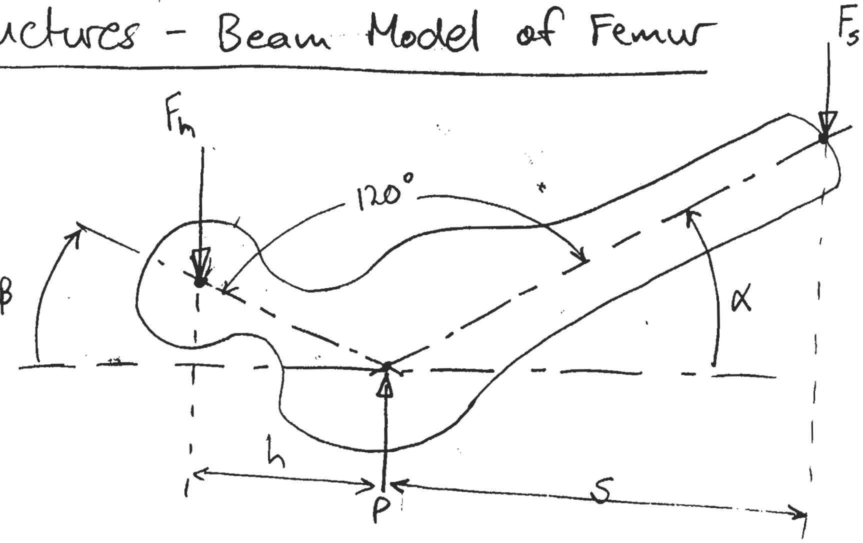
M (including spacesuit + PLSS) (113 kg) ≈ 183 kg

$h = 1.76$ m $\qquad g = 9.81\ m/s^2 \qquad G = 3/8$

(2) $\longrightarrow$ $V = 2.20$ m/s

$$\Rightarrow \boxed{F_{max} \approx 7.96\ kN}$$

II Structures – Beam Model of Femur



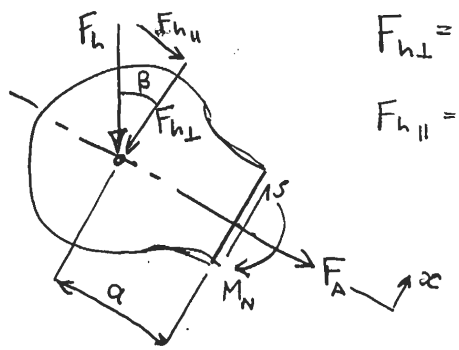
$$+\uparrow \sum F_v = 0 : \quad P - F_h - F_s = 0 \qquad (1)$$

$$+\circlearrowright \sum (M)_p = 0 : \quad (F_h)h - (F_s)s = 0 \qquad (2)$$

$$(1) \rightarrow \quad F_h = P - F_s$$

$$\rightarrow (2) \rightarrow \quad \boxed{F_s = P\left(\frac{h}{s+h}\right)}$$

$$\Rightarrow \quad \boxed{F_h = P\left(\frac{s}{s+h}\right)}$$



$$F_{h\perp} = F_h \cos\beta$$

$$F_{h\parallel} = F_h \sin\beta$$

$$+\circlearrowleft \sum M_N = 0 : \quad M_N - (F_{h\perp})a = 0 \quad \Rightarrow \quad \boxed{M_N = P\left(\frac{a\,s}{s+h}\right)\cos\beta}$$

$$\xrightarrow{+} \sum F = 0 : \quad F_A + F_{h\parallel} = 0 \quad \Rightarrow \quad \boxed{F_A = -P\left(\frac{s}{s+h}\right)\sin\beta}$$

Normal Stress

Due to Moment: $\sigma_M = \frac{M_N \cdot x}{I}$

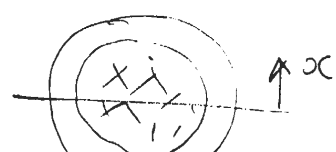
Due to axial force: $\sigma_F = \frac{F_A}{A}$

Total Stress in Neck: $\sigma_N = \sigma_M + \sigma_F = \frac{M_N \cdot x}{I} + \frac{F_A}{A}$

$$\Rightarrow \boxed{\sigma_N = P\left(\frac{s}{s+h}\right)\left(\frac{ax\cos\beta}{I} - \frac{\sin\beta}{A}\right)}$$

Values:

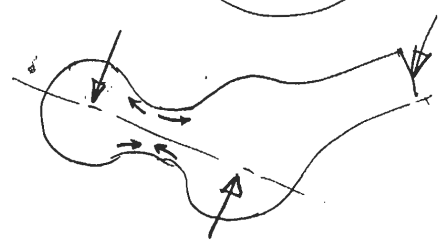
$I = 1\times10^{-8}\,m^4$ $A = 1.25\times10^{-4}\,m^2$



$h = .03\,m$ $s = .18\,m$

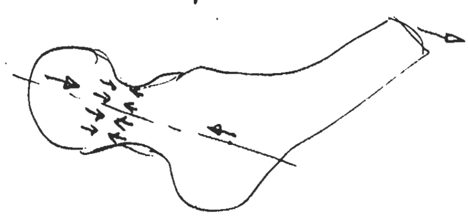
$a = .025\,m$ $\beta = 50°$

$P = F_{max} = 7.96\ kN$



Tension Side $x = +.015\,m$

$$\boxed{(\sigma_N)_t = 123\ MPa}$$



From literature:

Compression Side $x = -.015\,m$

$$\boxed{(\sigma_N)_c = -206.\ MPa}$$

$\sigma_{ut} = 135\ MPa$

$\sigma_{uc} = 199\ MPa$

∴ FAILS ON COMPRESSION SIDE