Plan for the Session

- Quiz on Constructing Orthogonal Arrays (10 minutes)

- Complete some advanced topics on OAs

- Lecture on Computer Aided Robust Design

- Recitation on HW#5

How to Estimate Error variance in an L_{18}

- Consider Phadke pg. 89

- How would the two unassigned columns contribute to error variance?

- Remember L18(21x37)
  - Has $1+1*(2-1)+7*(3-1) = 16$ DOF
  - But 18 rows
  - Therefore 2 DOF can be used to estimate the sum square due to error
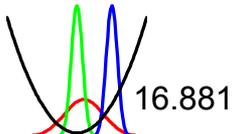
Breakdown of Sum Squares

GTSS

SS due to mean

Total SS

SS due to factor A

SS due to factor B

etc.

SS due to error

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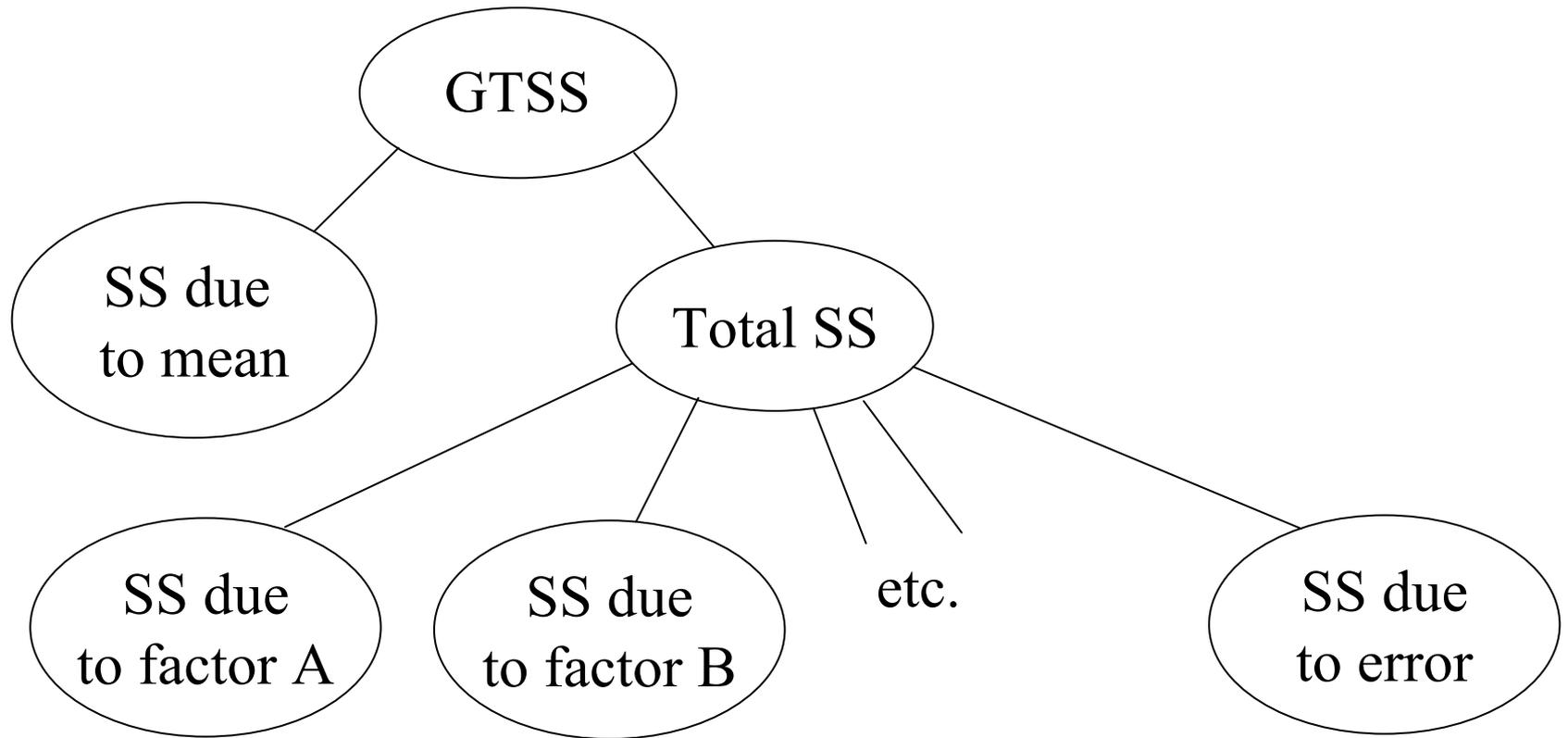
Column Merging

- Can turn 2 two level factors into a 4 level factor

- Can turn 2 three level factors into a six level factor

- Need to strike out interaction column (account for the right number of DOF!)

- Example on an L_8

Column Merging in an L$_8$

- Eliminate the column which is confounded with interactions

- Create a new four-level column

Exp no.	Control Factors							η
	A	B	C	D	E	F	G	
1	1	1	1	1	1	1	1	
2	1	1	1	2	2	2	2	
3	1	2	2	1	1	2	2	
4	1	2	2	2	2	1	1	
5	2	1	2	1	2	1	2	
6	2	1	2	2	1	2	1	
7	2	2	1	1	2	2	1	
8	2	2	1	2	1	1	2	

Computer Aided Robust Design
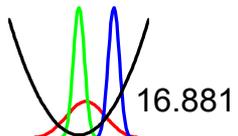
Engineering Simulations

- Many engineering systems can be modeled accurately by computer simulations
  - Finite Element Analysis
  - Digital and analog circuit simulations
  - Computational Fluid Dynamics

- Do you use simulations in design & analysis?

- How accurate & reliable are your simulations?
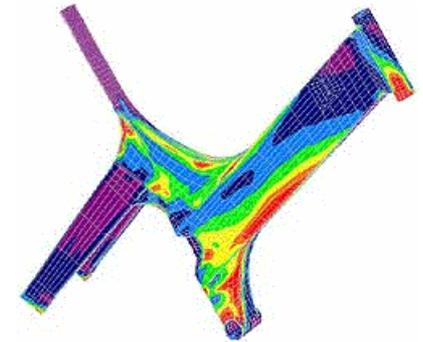
Simulation & Design Optimization

- Formal mathematical form

$$\text{minimize} \quad y = f(\mathbf{x})$$
$$\text{subject to} \quad h(\mathbf{x}) = 0$$
$$g(\mathbf{x}) \leq 0$$

minimize weight
subject to height=23"
max stress<0.8Y

$\mathbf{x}$ → | Simulation | → $f(\mathbf{x})$ $h(\mathbf{x})$ $g(\mathbf{x})$
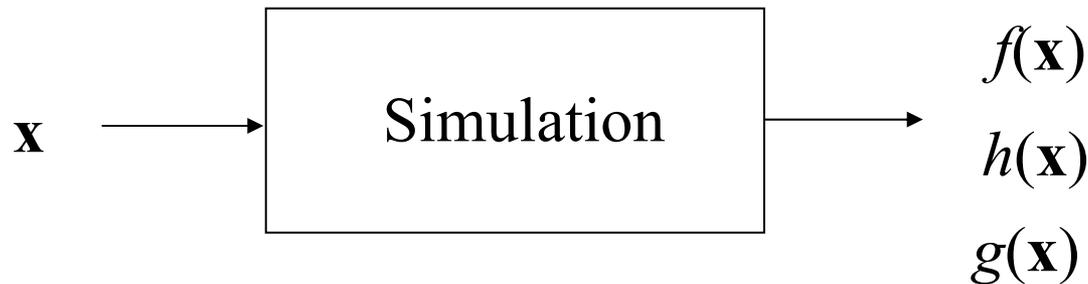
Robust Design Optimization

- Vector of design variables $\mathbf{x}$
  - Control factors (discrete vs continuous)
- Objective function $f(\mathbf{x})$
  - S/N ratio (noise must be induced)
- Constraints $h(\mathbf{x}), g(\mathbf{x})$
  - Not commonly employed
  - Sliding levels may be used to handle equality constraints in some cases
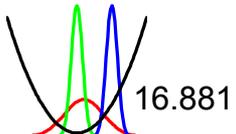
Noise Distributions

- ## Normal
  - Arises when many independent random variables are summed

- ## Uniform
  - Arises when other distributions are truncated

- ## Lognormal
  - Arises when normally distributed variables are multiplied or transformed

Lognormal Distribution

p(x)

x

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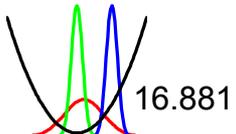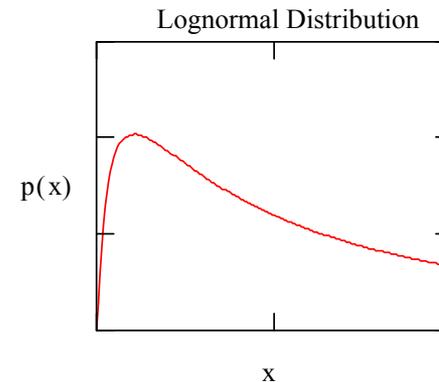
Correlation of Noise Factors

- Covariance

$$COV(x, y) = E((x - E(x))(y - E(y)))$$

$$COV(x, y) \cong \sum_{i=1}^{n} \sum_{j=1}^{m} (x_i - \bar{x})(y_j - \bar{y})$$

Size error, Pin #2 (vertical axis)

Size error, Pin #1 (horizontal axis)
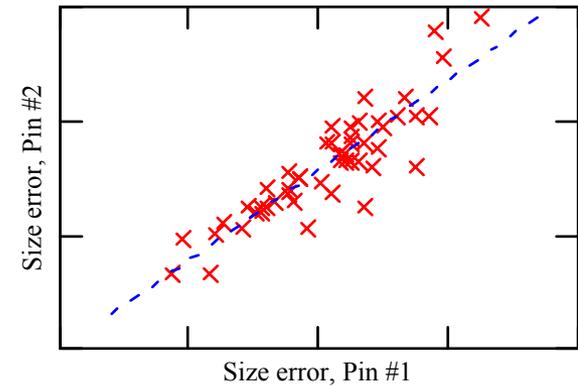
- Correlation coefficient
  - What does k=1 imply?
  - What does negative k imply?
  - What does k=0 imply?

$$k = \frac{COV(x, y)}{\sqrt{VAR(x) \cdot VAR(y)}}$$

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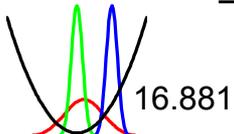
Question About Noise

- Does the distribution of noise affect the S/N ratio of the simulation?

  - If so, under what conditions?

- Does correlation of noise factors affect S/N ratios?

  - If so, in what way? (raise / lower)

Simulating Variation in Noise Factors

- Taylor series expansion
  - Linearize the system response
  - Apply closed form solutions

- Monte Carlo
  - Generate random numbers
  - Use as input to the simulation

- Orthogonal array based simulation
  - Create an ordered set of test conditions
  - Use as input to the simulation
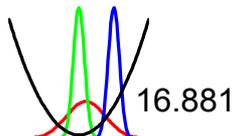
Taylor Series Expansion

- Approximate system response

$$f(x,y) = f(x_o, y_o) + \left.\frac{\partial f}{\partial x}\right|_{x=x_o} \cdot (x - x_o) + \left.\frac{\partial f}{\partial y}\right|_{y=y_o} \cdot (y - y_o) + \text{h.o.t}$$

- Apply rules of probability

$$VAR(aX) = aVAR(X)$$

$$VAR(X+Y) = VAR(X) + VAR(Y) \text{ iff } x, y \text{ independent}$$

- To get

$$\sigma^2(y) = \sum_{i=1}^{n} \frac{\partial y}{\partial x} \sigma^2(X_i)$$

Local Linearity of the System Response Surface wrt Noise

- Holds for
  - Machining (most)
  - CMMs
- Fails for
  - Dimensions of form
  - Dual head valve grinding

$$\mathbf{q}_i(\mathbf{n}) \cong f(\mathbf{t}) + \sum_{j=1}^{n}\left(\left.\frac{\partial \mathbf{q}}{\partial \mathbf{n}}\right|_{\mathbf{n}=\mathbf{t}}\right)_{ij} \cdot (\mathbf{n}_j - \mathbf{t}_j)$$
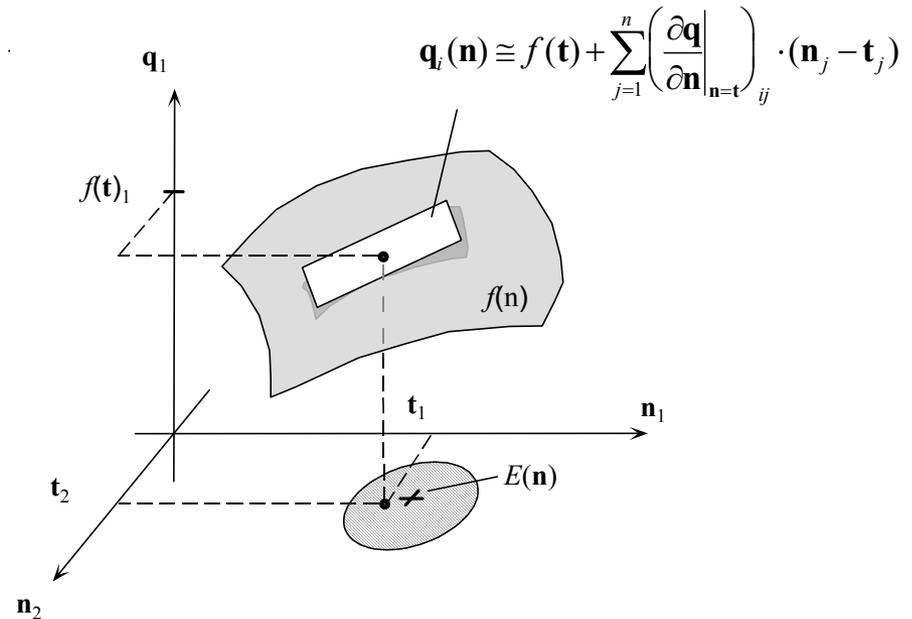
Key Limitations of
Taylor Series Expansion

- System response must be approximately *linear* w.r.t. noise factors

  - Linear over what range?

  - What if it isn't quite linear?

- Noise factors must be statistically independent

  - How common is correlation of noise?

  - What happens when you neglect correlation?
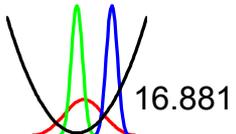
Monte Carlo Simulation

2.01

$$y_{trial} = f(x_1, x_2)$$

1.59
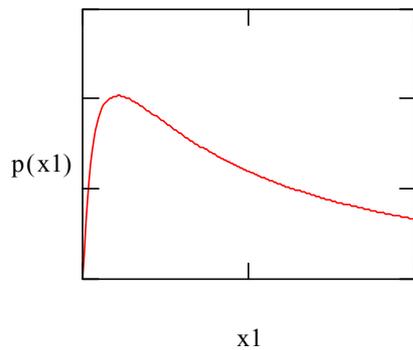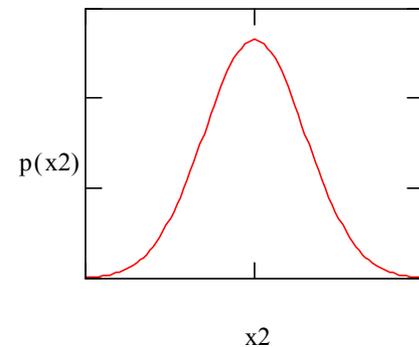
$$\sigma^2{}_y \cong \frac{1}{trials-1} \sum_{i=1}^{trials} (y_{trial} - \bar{y})^2$$

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Monte Carlo Simulation Pros and Cons

- No assumptions about system response $f(\mathbf{x})$

- You may simulate correlation among noises
  - How can this be accomplished?

- Accuracy not a function of the number of noises

$$95\% \text{ confidence interval} = \pm \frac{1.96\sigma}{\sqrt{\text{trials}}}$$ It's easy too!

- It takes a large number of trials to get very accurate answers
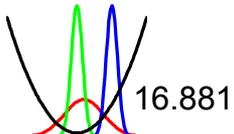
Othogonal Array Based Simulation

- Define noise factors and levels

- Two level factors

  – Level 1 = $\mu_i - \sigma_i$ Level 2 = $\mu_i + \sigma_i$

- Three level factors

  – Level 1 = $\mu_i - \sqrt{\dfrac{3}{2}}\sigma_i$ Level 2 = μ_i Level 3 = $\mu_i + \sqrt{\dfrac{3}{2}}\sigma_i$

- Choose an appropriate othogonal array

- Use as the outer array to induce noise
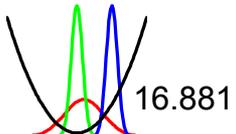
Setting Levels for Lognormal Distributions

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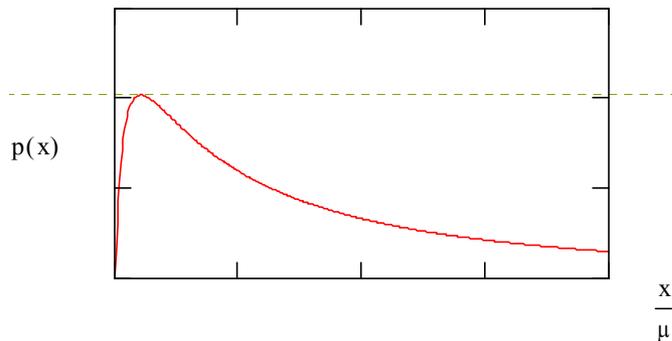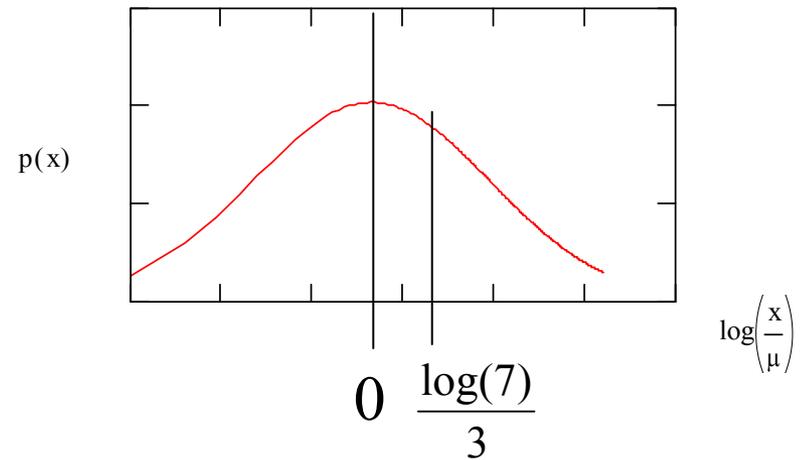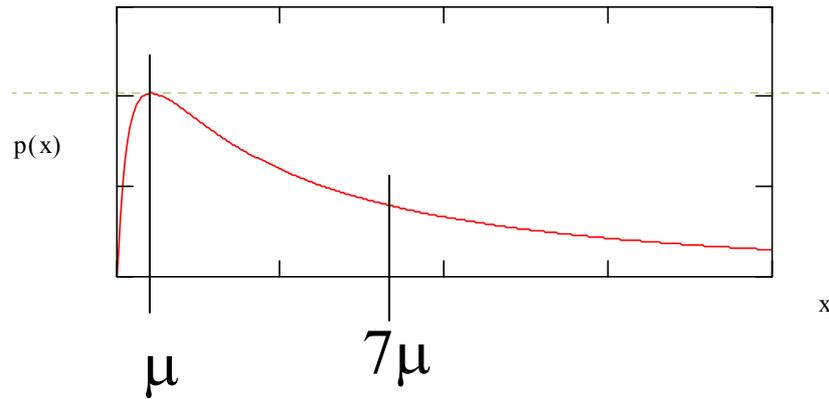
Using Sliding Levels to Simulate Correlation

- Try it for RFP

- Mean is defined as RFM

- Tolerance is 2%

- Fill out rows 1 and 19 of the noise array

Run the Noise Array

- At the baseline control factor settings

- Run the simulation at all of the noise factor settings

- Find the system response for each row of the array

- Perform ANOVA on the data

  - Percent of total SS is percent contribution to variance in system response
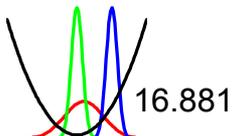
Othogonal Array
Pros and Cons

- Can handle some degree of non-linearity

- Can accommodate correlation

- Provides a direct evaluation of noise factor contributions

- Usually requires orders of magnitude fewer function evaluations than OA simulation

Optimization

- Choose control factors and levels
- Set up an inner array of control factors
- For each row, induce noise from the outer array
- Compute mean, variance, and S/N
- Select control factors based on the additive model
- Run a confirmation experiment
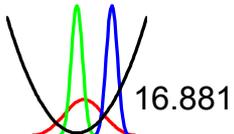
Next Steps

- Homework #8 due on Lecture 13

- Next session

  - Read Phadke Ch. 9 -- "Design of Dynamic Systems"

  - No quiz tomorrow

- Lecture 13- Quiz on Dynamic Systems