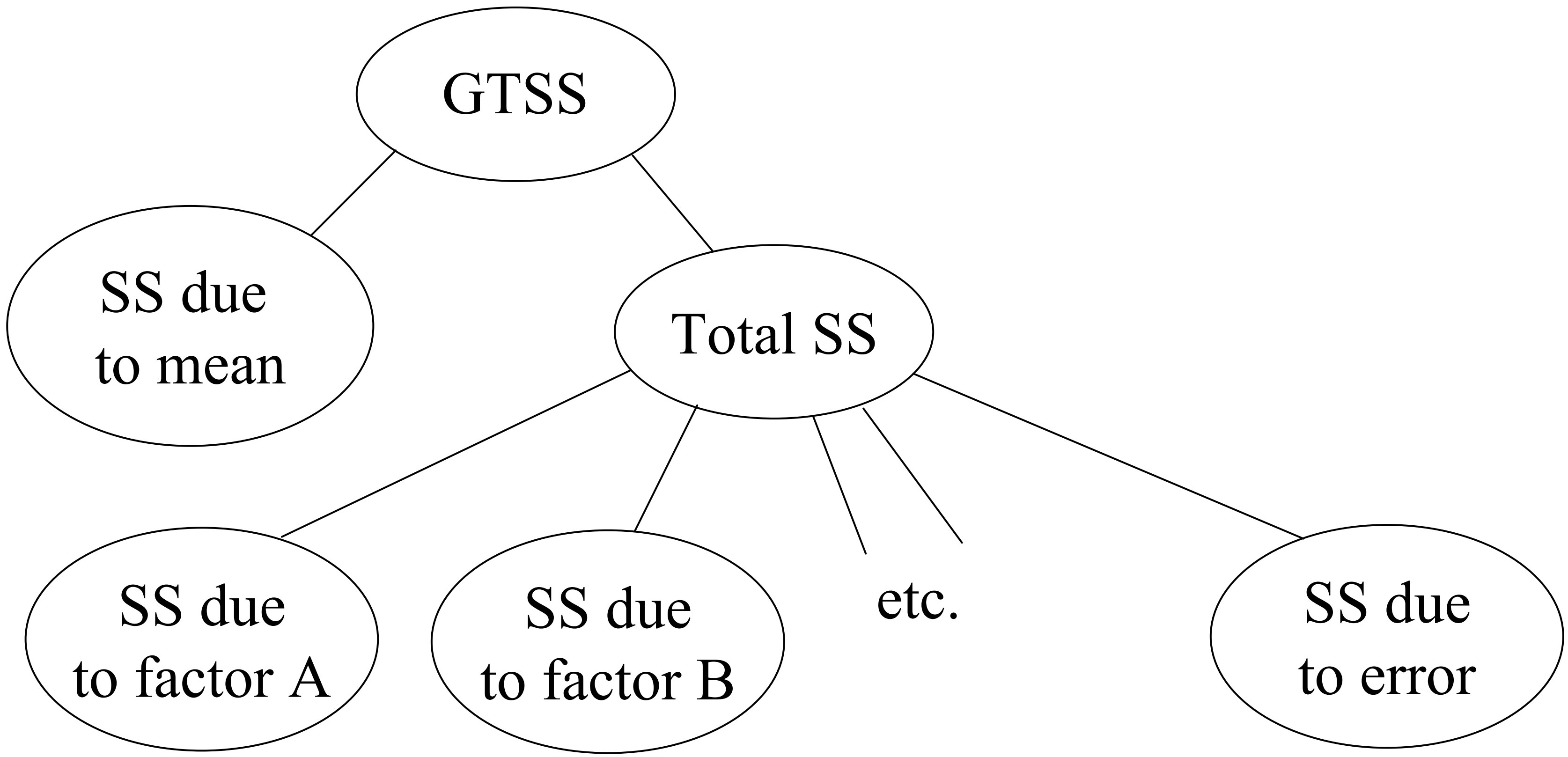
Plan for the Session

- Quiz on Constructing Orthogonal Arrays (10 minutes)
- Complete some advanced topics on OAs
- Lecture on Computer Aided Robust Design
- Recitation on HW#5

How to Estimate Error variance in an L_{18}

- Consider Phadke pg. 89
- How would the two unassigned columns contribute to error variance?
- Remember L18(21x37)
  - Has 1+1*(2-1)+7*(3-1) = 16 DOF
  - But 18 rows
  - Therefore 2 DOF can be used to estimate the sum square due to error

Breakdown of Sum Squares



Column Merging

- Can turn 2 two level factors into a 4 level factor
- Can turn 2 three level factors into a six level factor
- Need to strike out interaction column (account for the right number of DOF!)
- Example on an L_8

Column Merging in an L_8

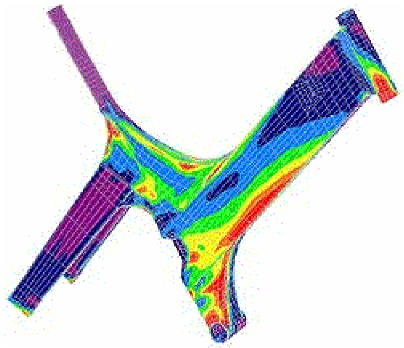
- Eliminate the column which is confounded with interactions
- Create a new four-level column

	Control Factors							
Exp no.	A	B	C	D	E	F	G	η
1	1	1	1	1	1	1	1	
2	1	1	1	2	2	2	2	
3	1	2	2	1	1	2	2	
4	1	2	2	2	2	1	1	
5	2	1	2	1	2	1	2	
6	2	1	2	2	1	2	1	
7	2	2	1	1	2	2	1	
8	2	2	1	2	1	1	2	

Computer Aided Robust Design

Engineering Simulations

- Many engineering systems can be modeled accurately by computer simulations
  - Finite Element Analysis
  - Digital and analog circuit simulations
  - Computational Fluid Dynamics
- Do you use simulations in design & analysis?
- How accurate & reliable are your simulations?

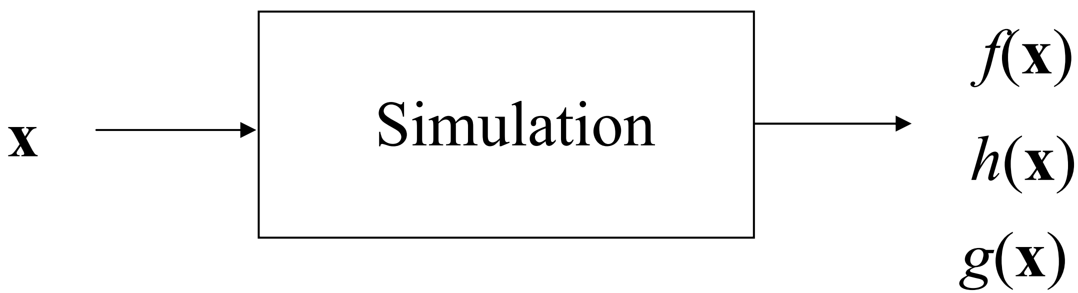


Simulation & Design Optimization

- Formal mathematical form

$$\begin{aligned} &\text{minimize} && y = f(\mathbf{x}) \\ &\text{subject to} && h(\mathbf{x}) = 0 \\ & && g(\mathbf{x}) \le 0 \end{aligned}$$

minimize weight
subject to height=23”
max stress<0.8Y

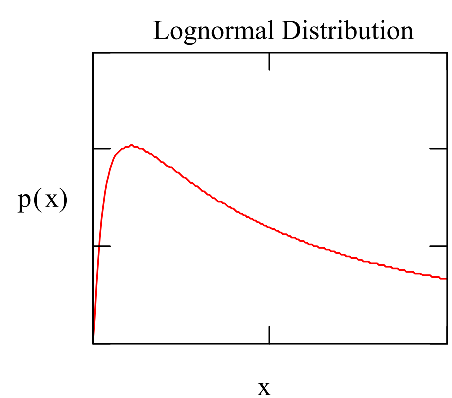


Robust Design Optimization

- Vector of design variables **x**
  - Control factors (discrete vs continuous)
- Objective function $f(\mathbf{x})$
  - S/N ratio (noise must be induced)
- Constraints $h(\mathbf{x})$, $g(\mathbf{x})$
  - Not commonly employed
  - Sliding levels may be used to handle equality constraints in some cases

Noise Distributions

- Normal
  - Arises when many independent random variables are summed
- Uniform
  - Arises when other distributions are truncated
- Lognormal
  - Arises when normally distributed variables are multiplied or transformed

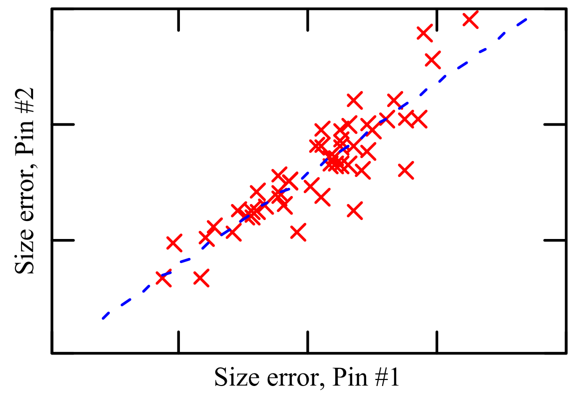


Correlation of Noise Factors

- Covariance

$$COV(x,y) = E((x - E(x))(y - E(y))$$

$$COV(x,y) \cong \sum_{i=1}^{n}\sum_{j=1}^{m}(x_i - \bar{x})(y_j - \bar{y})$$



- Correlation coefficient
  - What does k=1 imply?
  - What does negative k imply?
  - What does k=0 imply?

$$k = \frac{COV(x,y)}{\sqrt{VAR(x) \cdot VAR(y)}}$$

Question About Noise

- Does the distribution of noise affect the S/N ratio of the simulation?
  - If so, under what conditions?
- Does correlation of noise factors affect S/N ratios?
  - If so, in what way? (raise / lower)

Simulating Variation in Noise Factors

- Taylor series expansion
  - Linearize the system response
  - Apply closed form solutions
- Monte Carlo
  - Generate random numbers
  - Use as input to the simulation
- Orthogonal array based simulation
  - Create an ordered set of test conditions
  - Use as input to the simulation

Taylor Series Expansion

- Approximate system response

$$f(x,y) = f(x_o, y_o) + \left.\frac{\partial f}{\partial x}\right|_{x=x_o} \cdot (x - x_o) + \left.\frac{\partial f}{\partial y}\right|_{y=y_o} \cdot (y - y_o) + \text{h.o.t}$$

- Apply rules of probability

$$VAR(aX) = aVAR(X)$$

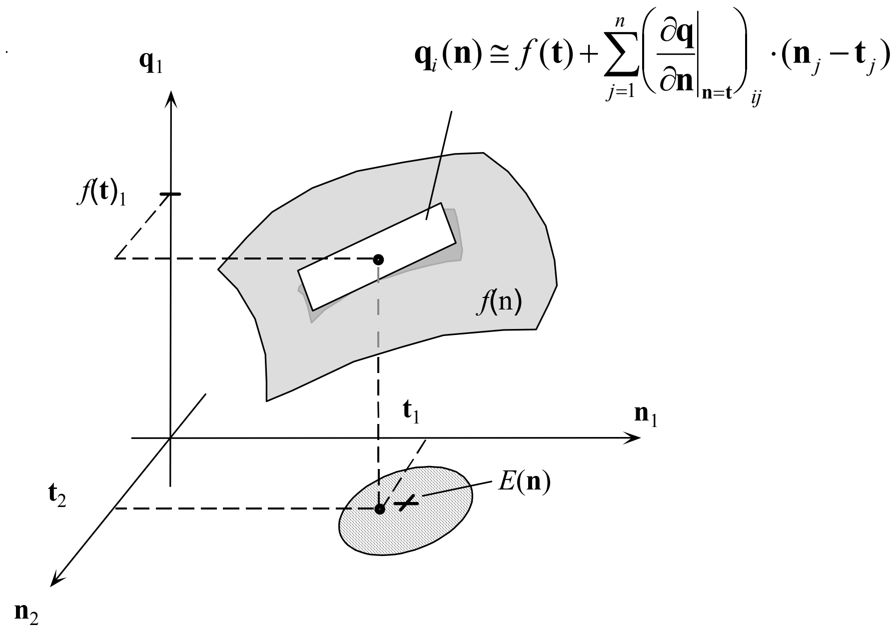
$$VAR(X+Y) = VAR(X) + VAR(Y) \text{ iff x, y independent}$$

- To get

$$\sigma^2(y) = \sum_{i=1}^{n} \frac{\partial y}{\partial x} \sigma^2(X_i)$$

Local Linearity of the System Response Surface wrt Noise

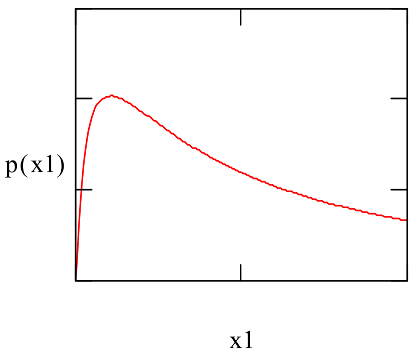
- Holds for
  - Machining (most)
  - CMMs
- Fails for
  - Dimensions of form
  - Dual head valve grinding



Key Limitations of Taylor Series Expansion

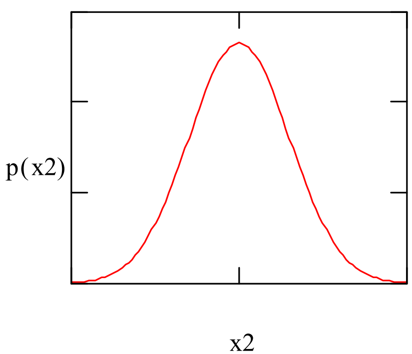
- System response must be approximately *linear* w.r.t. noise factors
  – Linear over what range?
  – What if it isn't quite linear?
- Noise factors must be statistically independent
  – How common is correlation of noise?
  – What happens when you neglect correlation?

Monte Carlo Simulation



2.01

$$y_{trial} = f(x_1, x_2)$$



1.59

$$\sigma^2_y \cong \frac{1}{trials - 1} \sum_{i=1}^{trials} (y_{trial} - \bar{y})^2$$

Monte Carlo Simulation Pros and Cons

- No assumptions about system response $f(\mathbf{x})$
- You may simulate correlation among noises
  - How can this be accomplished?
- Accuracy not a function of the number of noises

$$95\% \text{ confidence interval} = \pm \frac{1.96\sigma}{\sqrt{\text{trials}}}$$

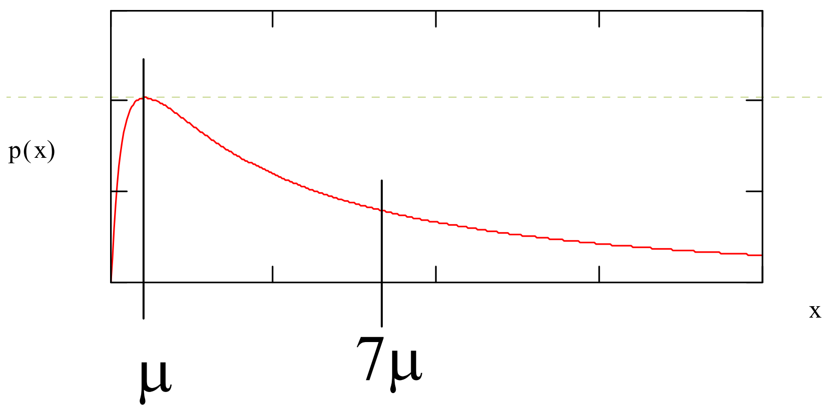
It's easy too!

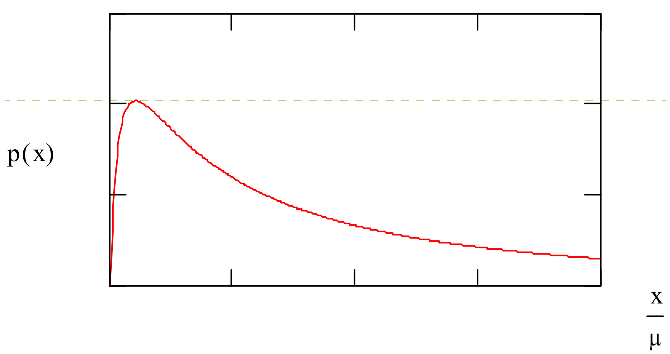
- It takes a large number of trials to get very accurate answers

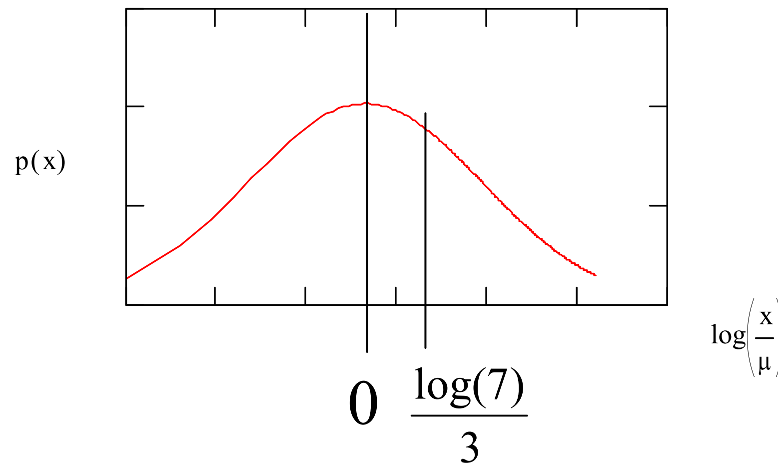
Othogonal Array Based Simulation

- Define noise factors and levels
- Two level factors
  - Level 1 = $\mu_i - \sigma_i$ Level 2 = $\mu_i + \sigma_i$
- Three level factors
  - Level 1 = $\mu_i - \sqrt{\frac{3}{2}}\sigma_i$ Level 2 = μ_i Level 3 = $\mu_i + \sqrt{\frac{3}{2}}\sigma_i$
- Choose an appropriate othogonal array
- Use as the outer array to induce noise

Setting Levels for Lognormal Distributions







Using Sliding Levels to Simulate Correlation

- Try it for RFP
- Mean is defined as RFM
- Tolerance is 2%
- Fill out rows 1 and 19 of the noise array

Run the Noise Array

- At the baseline control factor settings
- Run the simulation at all of the noise factor settings
- Find the system response for each row of the array
- Perform ANOVA on the data
  - Percent of total SS is percent contribution to variance in system response

Othogonal Array Pros and Cons

- Can handle some degree of non-linearity
- Can accommodate correlation
- Provides a direct evaluation of noise factor contributions
- Usually requires orders of magnitude fewer function evaluations than OA simulation

Optimization

- Choose control factors and levels
- Set up an inner array of control factors
- For each row, induce noise from the outer array
- Compute mean, variance, and S/N
- Select control factors based on the additive model
- Run a confirmation experiment

Next Steps

- Homework #8 due on Lecture 13
- Next session tomorrow
  - Read Phadke Ch. 9 -- “Design of Dynamic Systems”
  - No quiz tomorrow
- Quiz on Dynamic Systems