Guidance and Control Methods for Formation Flight

by

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Excerpted from the Ph.D. Thesis Defense
of
Sanghyuk Park
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Presentation Outline

- Background & Motivation
- Parent/Child UAV Project
- Guidance for Phase I
- Estimation
- Flight Test Results
- Summary

Why Formation Flight ?

- **Aerial Refueling**
- **Fuel Efficiency**
- **UAV Landing on Shipboard / Humvee**

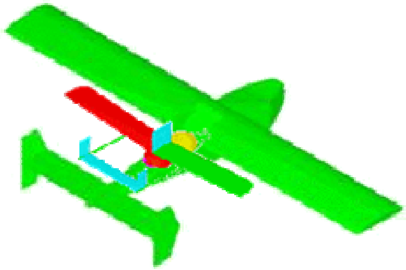
NASA Dryden Flight Research Center Photo Collection
http://www.dfrc.nasa.gov/gallery/photo/index.html
NASA Photo: EC01-0328-12 Date: November 9, 2001 Photo by: Carla Thomas
Smoke generators show the twisting paths of wingtip vortices behind two NASA Dryden F/A-18's used in the Autonomous Formation Flight (AFF) program during flight #743.

If Rendezvous Large UAV + Small UAVs :

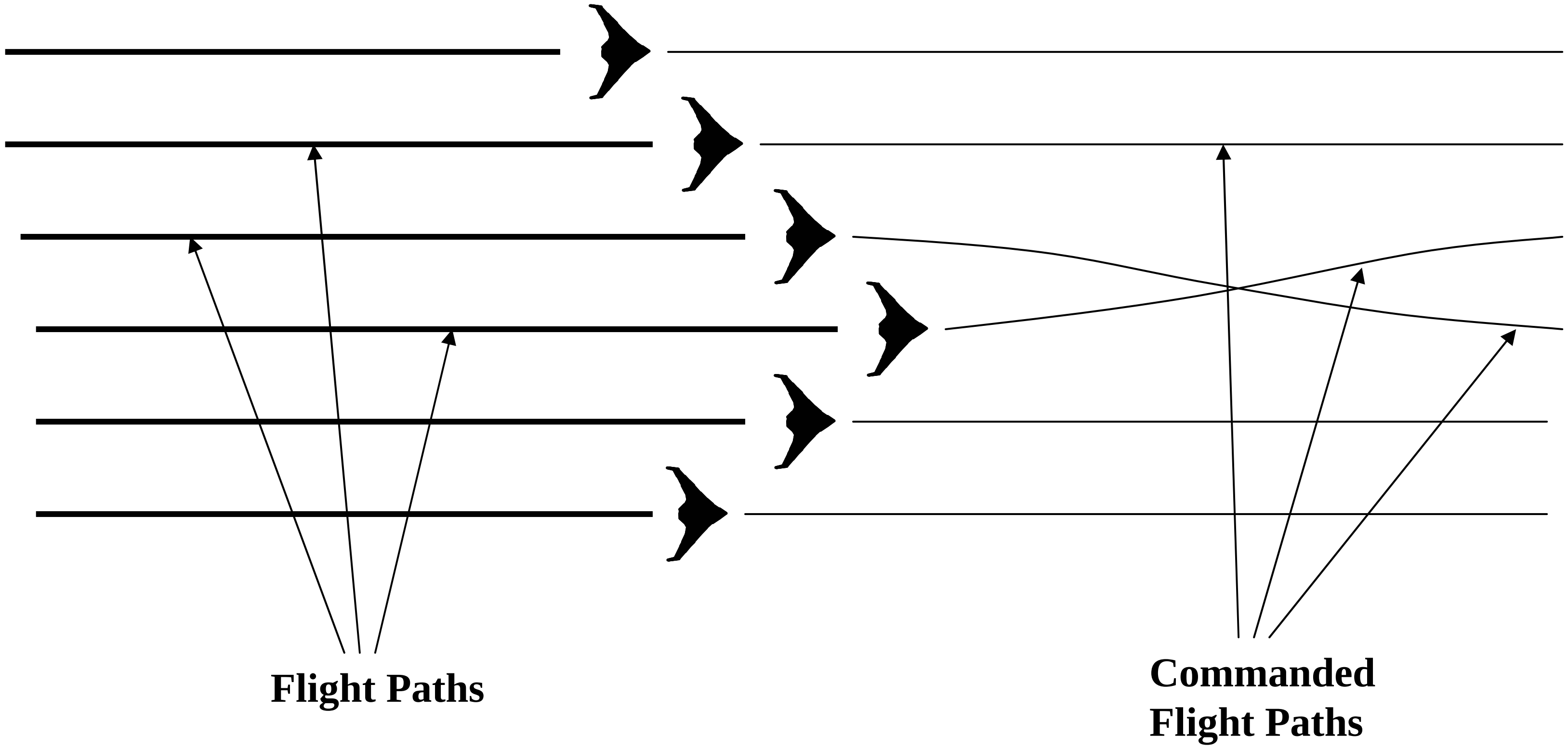
- **Sustained Close-in Surveillance**

  by refueling small UAVs

- **Retrieval of Small UAVs**



A Possible Approach to Formation Flight Guidance



- Central generation of commanded flight paths
- Individual control of path following

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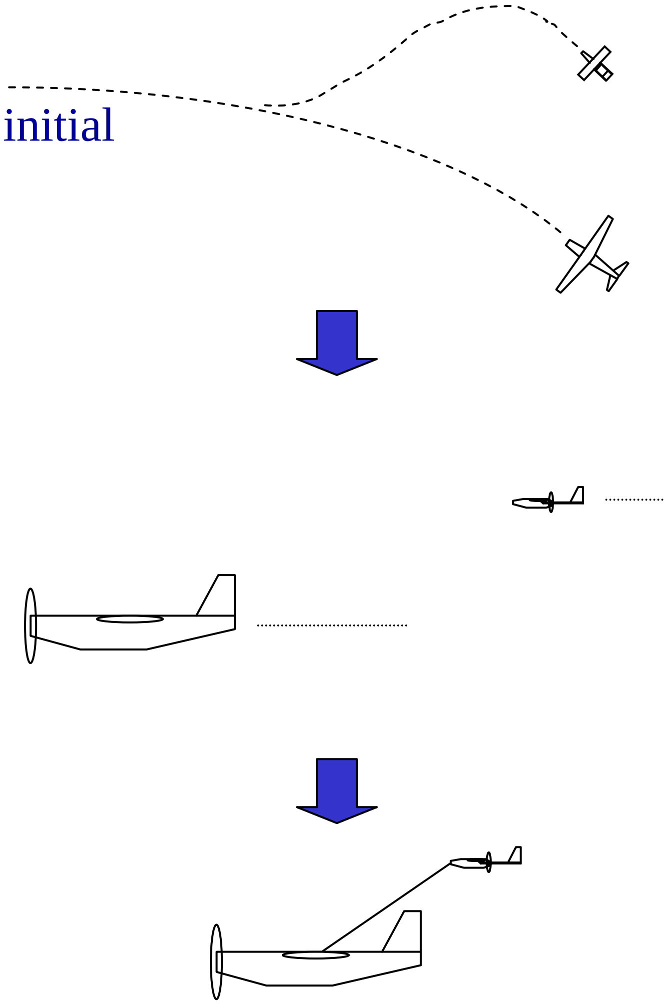
PCUAV Project Objectives

- Guidance and control system development
- Flight tests for mid-air rendezvous of two small UAVs
- Maximum use of inexpensive (off-the-shelf) components

Approach & Challenges

- *Higher Bandwidth (Agile) Vehicles Must Take on Challenging Tasks*
- *Use Control Law Sophistication instead of Costly Instrumentation*



- **PHASE I**

  - MINI UAV approaches Parent to within 20 m from any initial position and flies in formation using stand-alone GPS

  Challenges

  - Path planning
  - Tight control/guidance on the desired trajectory for rendezvous & formation flight

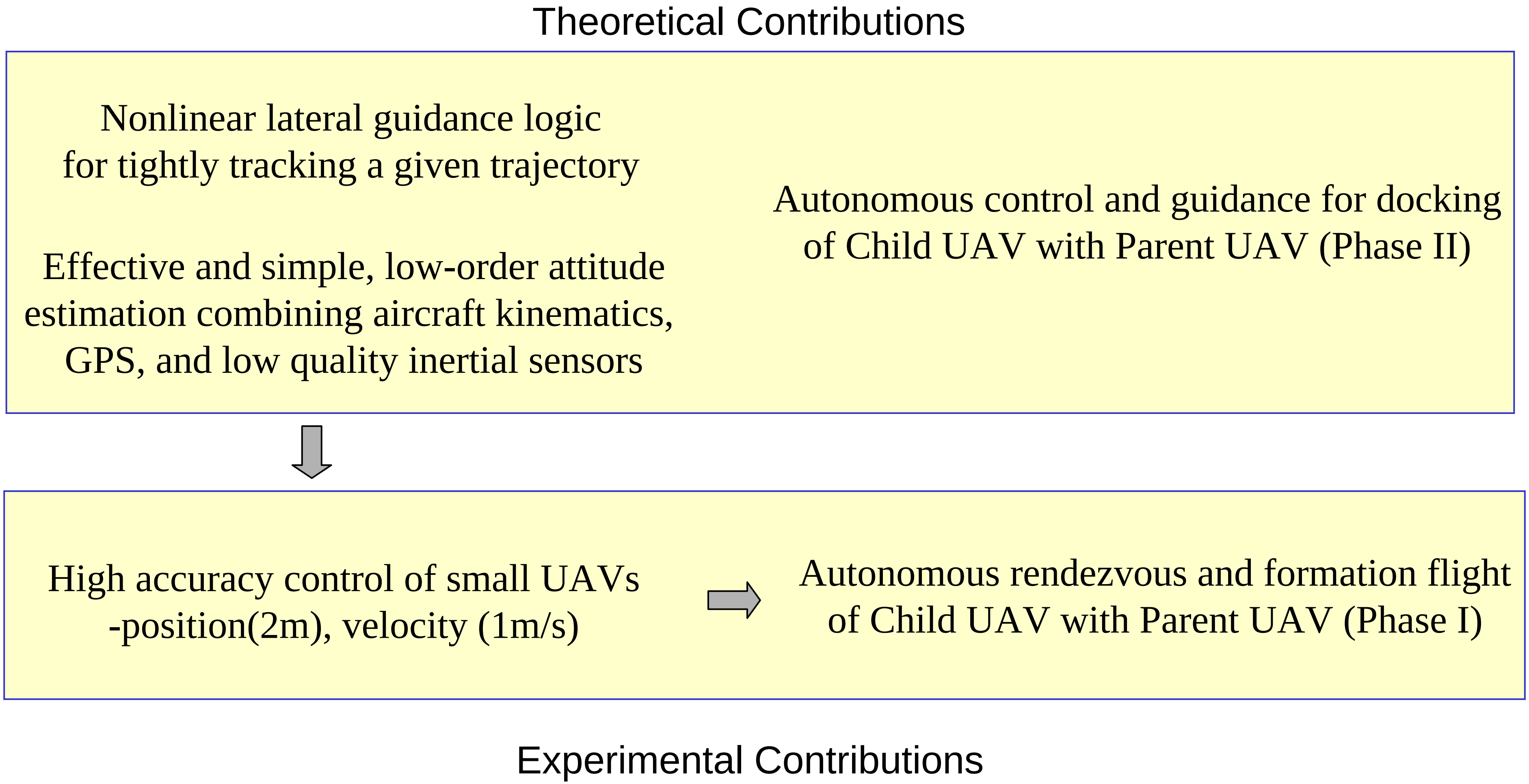
- **PHASE II**

  - Brings two UAVs even closer ~2 m by adding more accurate sensor

  Challenges

  - Accurate sensing/estimation & very tight control

Research Contributions



Theoretical Contributions

Nonlinear lateral guidance logic
for tightly tracking a given trajectory

Effective and simple, low-order attitude
estimation combining aircraft kinematics,
GPS, and low quality inertial sensors

Autonomous control and guidance for docking
of Child UAV with Parent UAV (Phase II)

High accuracy control of small UAVs
-position(2m), velocity (1m/s)

Autonomous rendezvous and formation flight
of Child UAV with Parent UAV (Phase I)

Experimental Contributions

Demonstration Vehicles

Ref. Master's Theses of Francois Urbain and Jason Kepler

MINI Child

- Wingspan = 2.5 m
- Gross Weight = 10 kg
- GA-15 Airfoil
- .91 cu. in. O.S. Engine, Pusher Prop.
- Vertical fin (direct side force)
- Large area flaperons (direct lift)

Parent

- Wingspan = 4.5 m, Tailspan = 6.1 m
- Gross Weight = 20 kg
- NACA 2412 Airfoil
- Moki 2.10 cu. in. Engine (5 hp max)
- Outboard Horizontal Stabilizer (OHS)

→ Open space behind, Aerodynamic efficiency

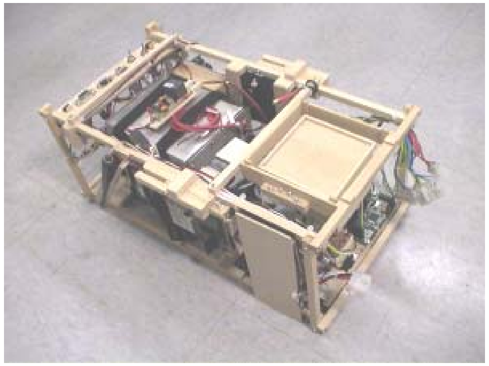
Avionics

- PC/104 Computer Stack
  - CPU module, Analog Data module, Utility module : $ 2,200
- GPS : Marconi, Allstar GPS Receiver : $ 1,000
- Inertial Sensors
  - Crossbow 3-axis Accelerometer (MINI) : $ 350
  - Tokin Ceramic Gyro (MINI) – ***Note : drift by 3~5 deg/min*** : $ 150
  - Crossbow IMU (OHS) : $ 3,500
- Pitot Static Probe : hand-made with Omega, Pressure Sensor : $ 75
- Altitude Pressure Sensor (for high frequency estimation) : $ 75
- Communication : Maxstream, 9XStream Transceiver : $ 200

Avionics ~ Mini : $ 4,000
OHS : $ 7,500

OHS Avionics Box

MINI Avionics Box



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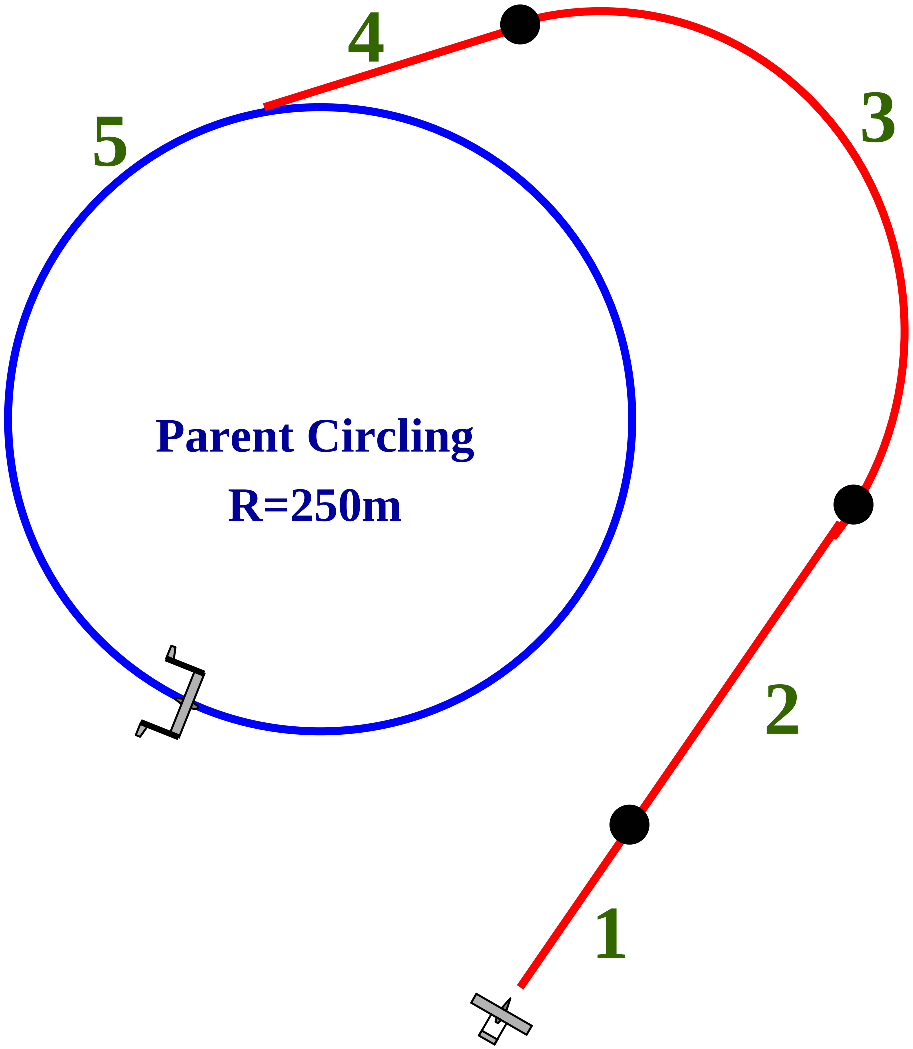
Phase I Flight Path

Ref. Master's Thesis of Damien Jourdan



- **Parent is maintained on circle**
- **Parent transmits its path to the Mini, which generates a path plan to follow the parent**

  1 : Climb
  2 : Straight (synchronization)
  3 : Turn (R=250m)
  4 : Straight
  5 : Formation Flight

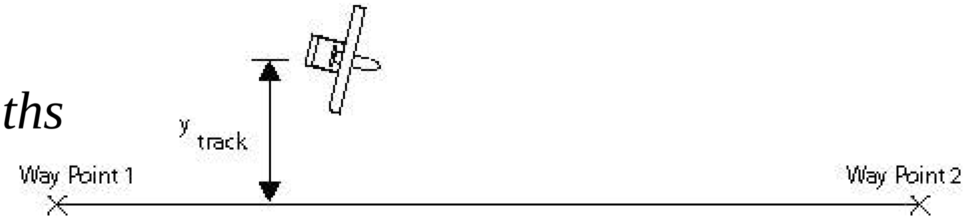
- **Relative longitudinal position control by Mini during 5**

Previous Work on Outer-Loop Guidance

- **Cross-track Error Guidance (typically PD controller)**

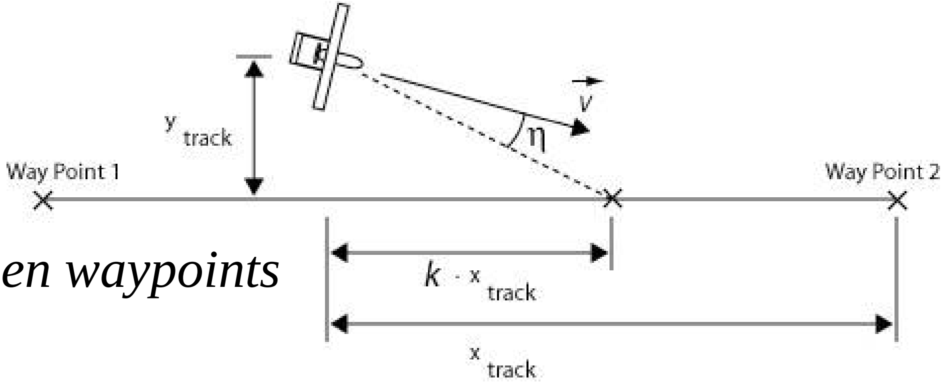
Limitation

Performance degrades on curved paths



- **M. Niculescu (2001)**

$$a_{cmd} = K_a\left(k\,x_{track}\,\dot{y}_{track} - y_{track}\,\dot{x}_{track}\right)$$



Limitation

Flies straight line trajectories between waypoints

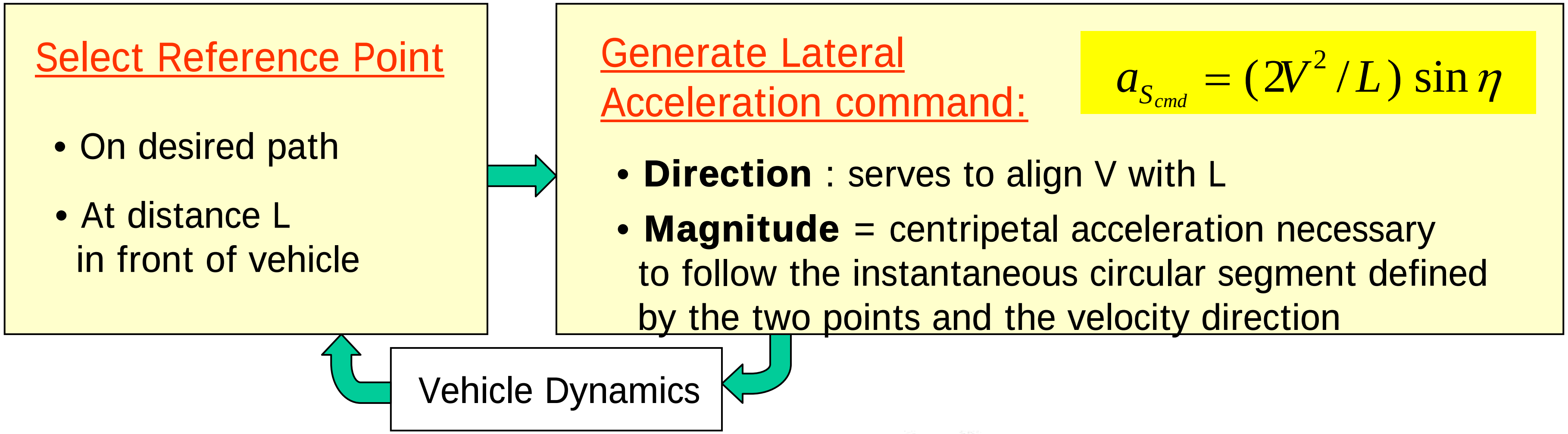
- **Guidance Laws for Tactical Missiles**

  - Line-of-sight guidance
  - Pursuit guidance
  - Proportional Navigation
  - Optimal linear guidance

Limitation

Cut corners on curved trajectories

New Guidance Logic for Trajectory Following



Select Reference Point

- On desired path
- At distance L in front of vehicle

Generate Lateral Acceleration command:

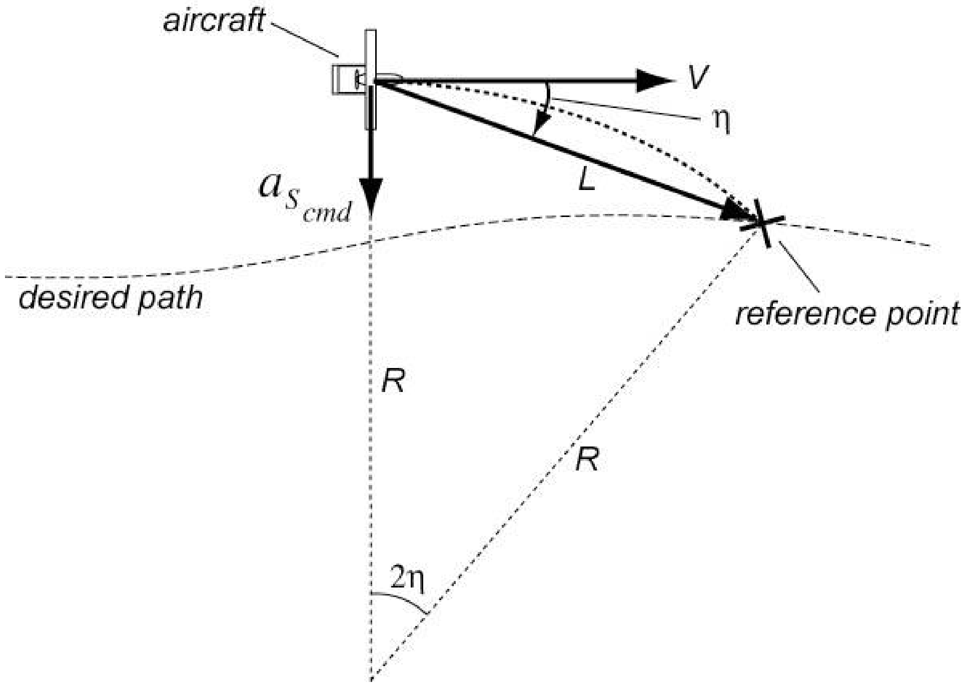
$$a_{S_{cmd}} = (2V^2/L)\sin\eta$$

- **Direction** : serves to align V with L
- **Magnitude** = centripetal acceleration necessary to follow the instantaneous circular segment defined by the two points and the velocity direction

Vehicle Dynamics

$$a_{S_{cmd}} = \begin{matrix}\text{centripetal}\\ \text{acceleration}\end{matrix} = \frac{V^2}{R}$$

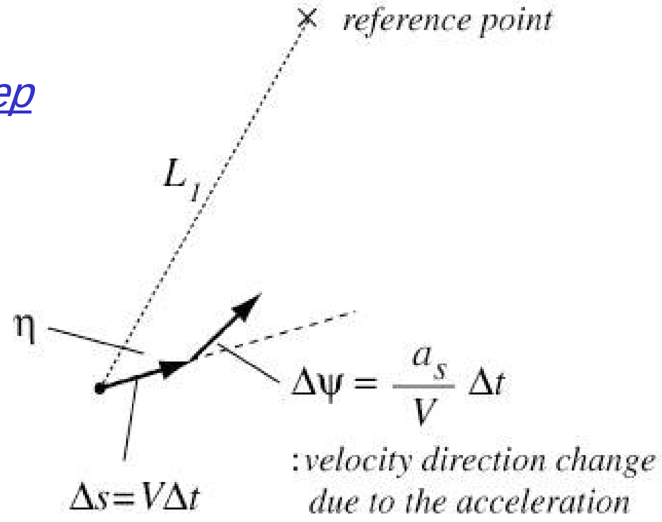
$$= \frac{V^2}{\left(\dfrac{L}{2\sin\eta}\right)} = 2\frac{V^2}{L}\sin\eta$$



Mechanism of the Guidance Logic

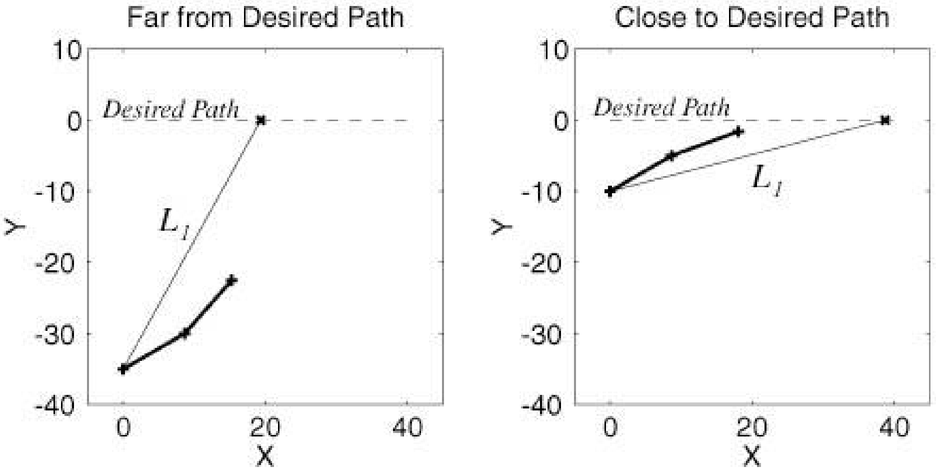
Reference Point Selection
+
Acceleration Command
⬇
Convergence to Desired Path

One Step



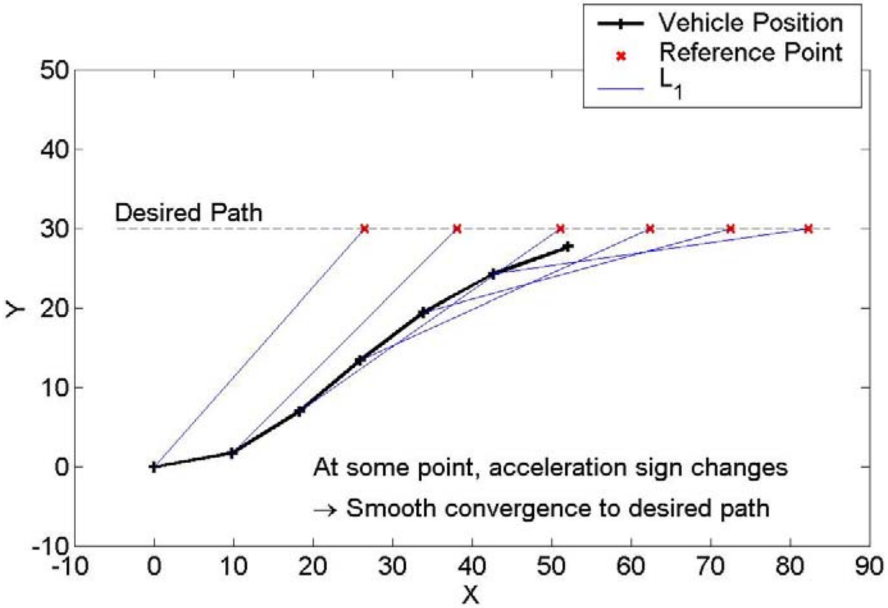
: velocity direction change due to the acceleration

Far vs. Close



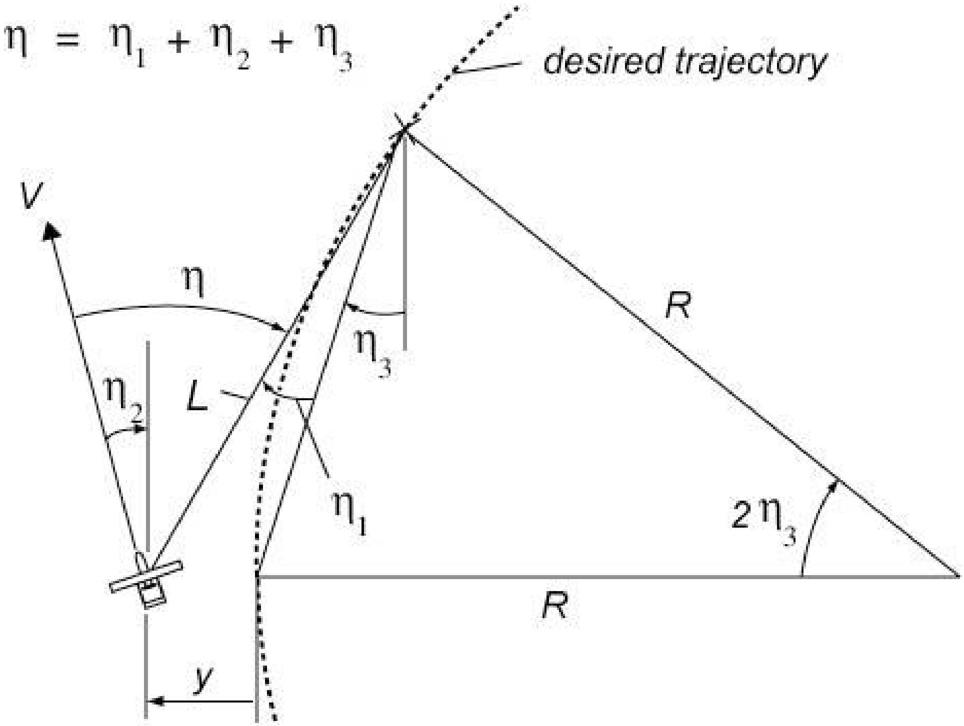
Note : same L_1 (=40) distance
$\Delta t=1$, $V=10$ are used in the simulations

Step by Step



At some point, acceleration sign changes
→ Smooth convergence to desired path

Decomposition of the Bearing Angle (η)



$$a_{S\,cmd} = 2\frac{V^2}{L}\sin\eta \approx 2\frac{V^2}{L}\sin(\eta_1 + \eta_2 + \eta_3)$$

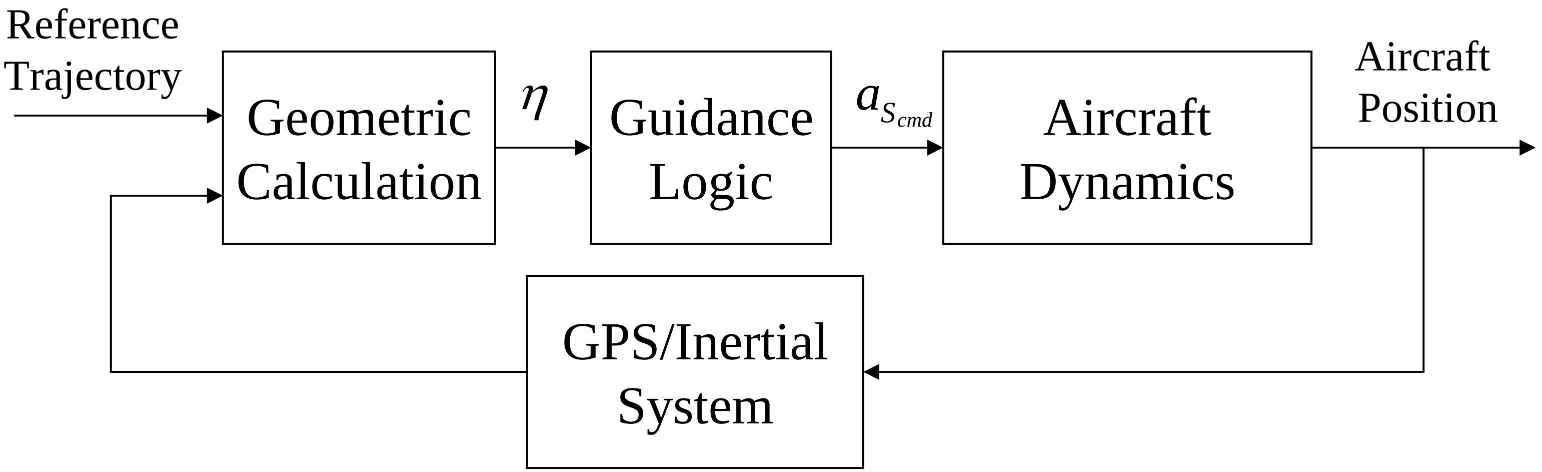
anticipate curved path

feed back heading error

feed back displacement error

System Configuration

- *A system block diagram is-*



Reference Trajectory

Geometric Calculation

η

Guidance Logic

$a_{S_{cmd}}$

Aircraft Dynamics

Aircraft Position

GPS/Inertial System

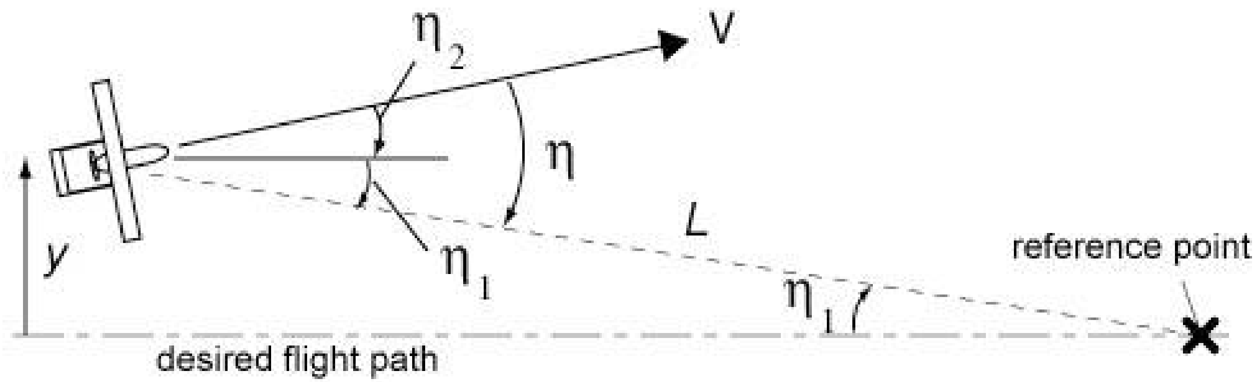
- *Both the geometry and the guidance logic are nonlinear*

- *A small perturbation linear analysis can provide important insights*

Linear Properties of Guidance Law

Assumptions

- *Aircaft is close to a desired straight line path*
- *Heading angle is close to path heading*



$$\left.\begin{aligned} \sin\eta \approx \eta = \eta_1 + \eta_2 \\ \eta_1 \approx \frac{y}{L}, \eta_2 \approx \frac{\dot{y}}{V} \end{aligned}\right\} \Rightarrow a_{S_{cmd}} = 2\frac{V^2}{L}\sin\eta \approx 2\frac{V}{L}\left(\frac{V}{L}y + \dot{y}\right)$$

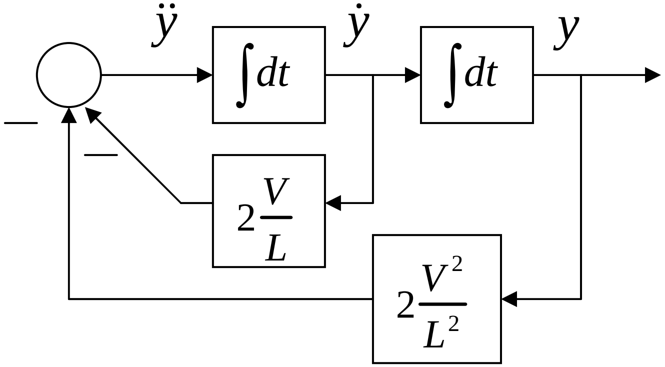
Proportional +Derivative (PD) control

Linear Properties of Guidance Law (cont.)

The linearized guidance equation for lateral motion is

$$\ddot{y} = -a_{S_{cmd}} = -2\frac{V^2}{L}\sin\eta \approx -2\frac{V}{L}\left(\dot{y} + \frac{V}{L}y\right)$$

and a block diagram of the linearized system is-



Linear Properties of Guidance Law (cont.)

The system is linear, constant coefficient, and second order

$$\ddot{y} + \frac{2V}{L}\dot{y} + \frac{2V^2}{L^2}y = 0$$

Its characteristic equation can be written as

$$s^2 + \frac{2V}{L}s + \frac{2V^2}{L^2} = s^2 + 2\varsigma\omega_n s + \omega_n^2 = 0$$

which yields

$$\omega_n^2 = \frac{2V^2}{L^2} \qquad 2\varsigma\omega_n = \frac{2V}{L}$$

so the undamped natural frequency and damping ratio are

$$\omega_n = \frac{\sqrt{2}V}{L} \qquad \varsigma = .707$$

Linear Properties of Guidance Law (cont.)

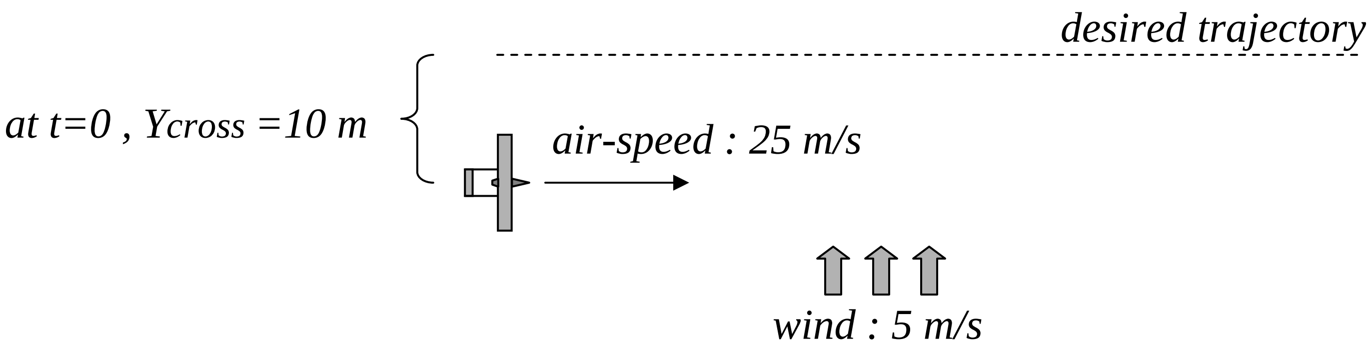
For small perturbations about the desired trajectory

- *The system is approximately linear and second order*
- *Its damping ratio is always .707*
- *The undamped natural frequency (bandwidth) is proportional to velocity (V) and inversely proportional to the trajectory reference point distance (L)*

Thus, if the aircraft is traveling at 200 m/s and the desired system bandwidth is 0.5 rad/sec, then the trajectory reference point distance must be

$$L = \frac{\sqrt{2}V}{\omega_n} = \frac{1.41 \cdot 200}{0.5} = 564m \approx 0.56km$$

Comparison - *Straight Line* Following



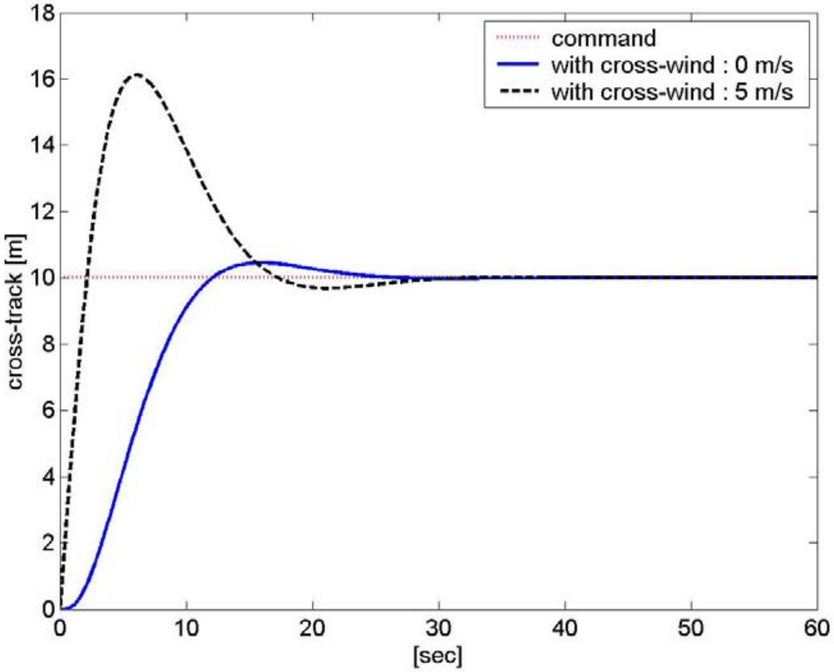
desired trajectory

at t=0 , Ycross =10 m

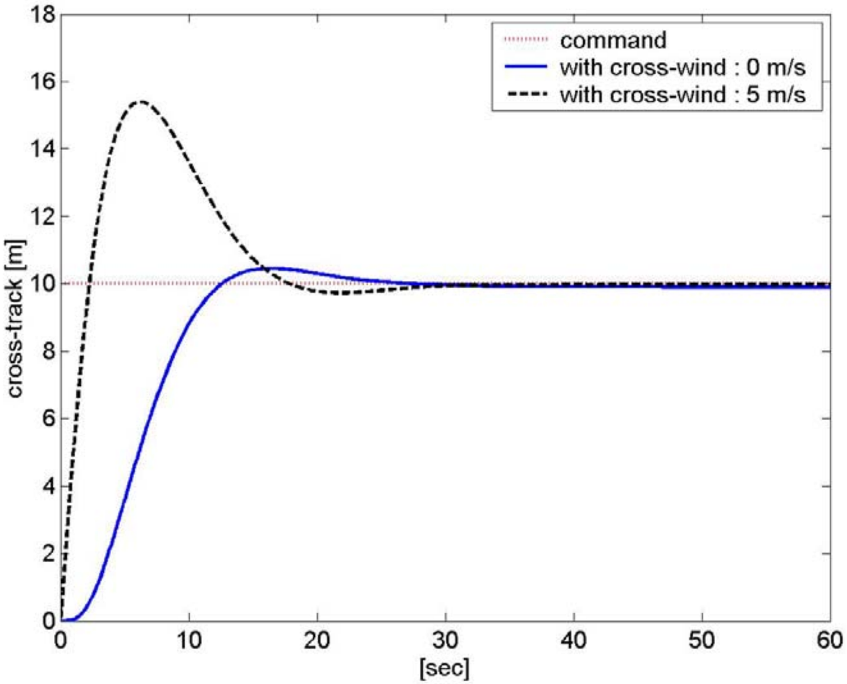
air-speed : 25 m/s

wind : 5 m/s

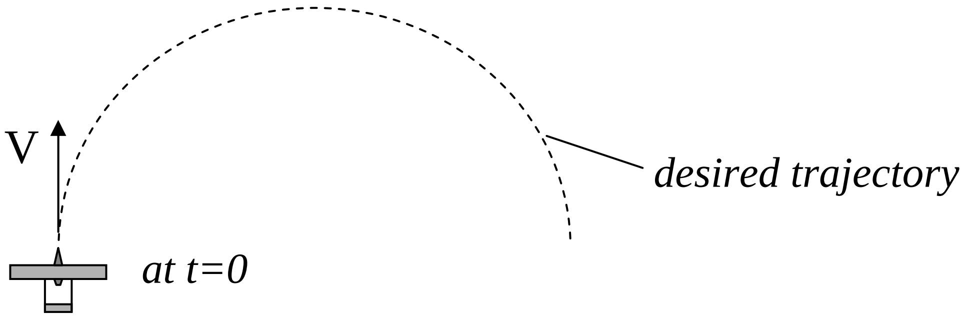
PD Linear Control

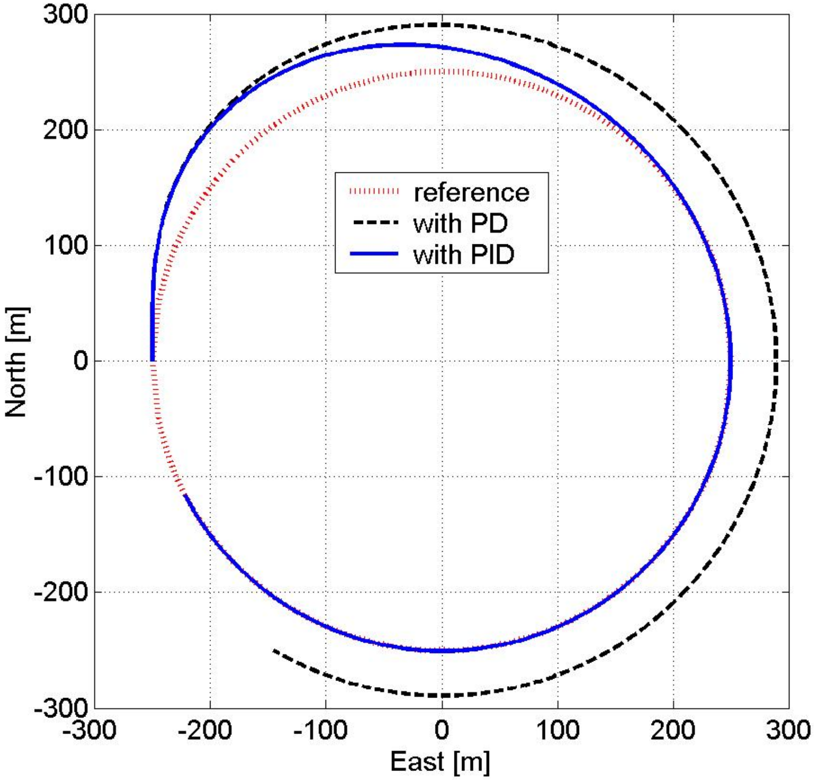


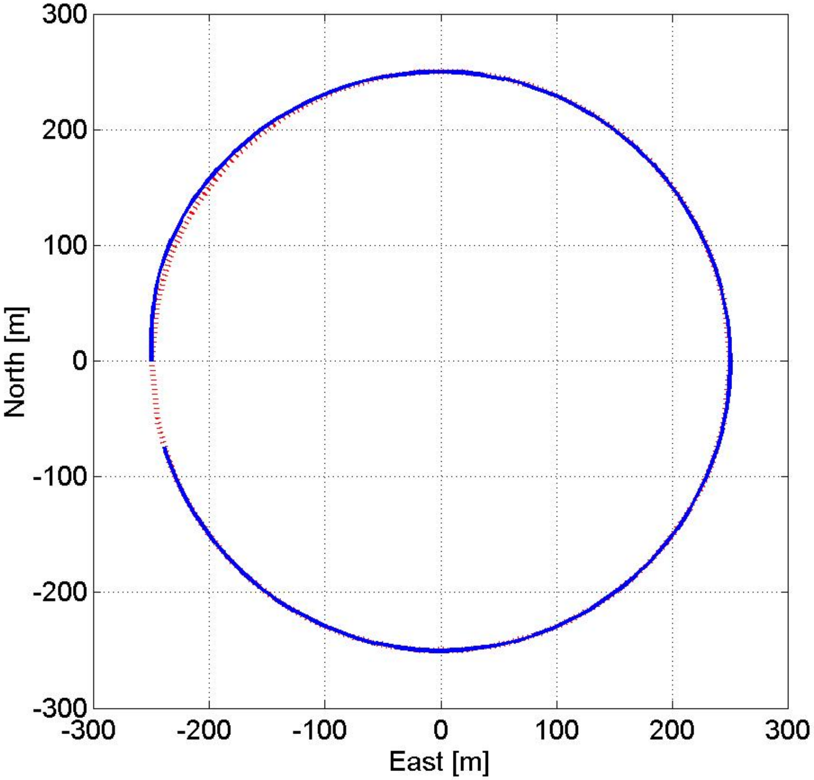
New Guidance Logic



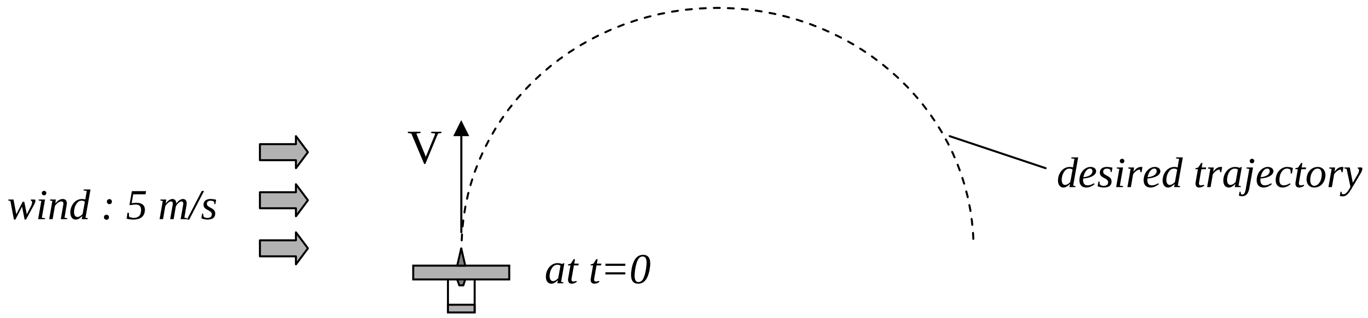
Comparison – *Curved Line* Following



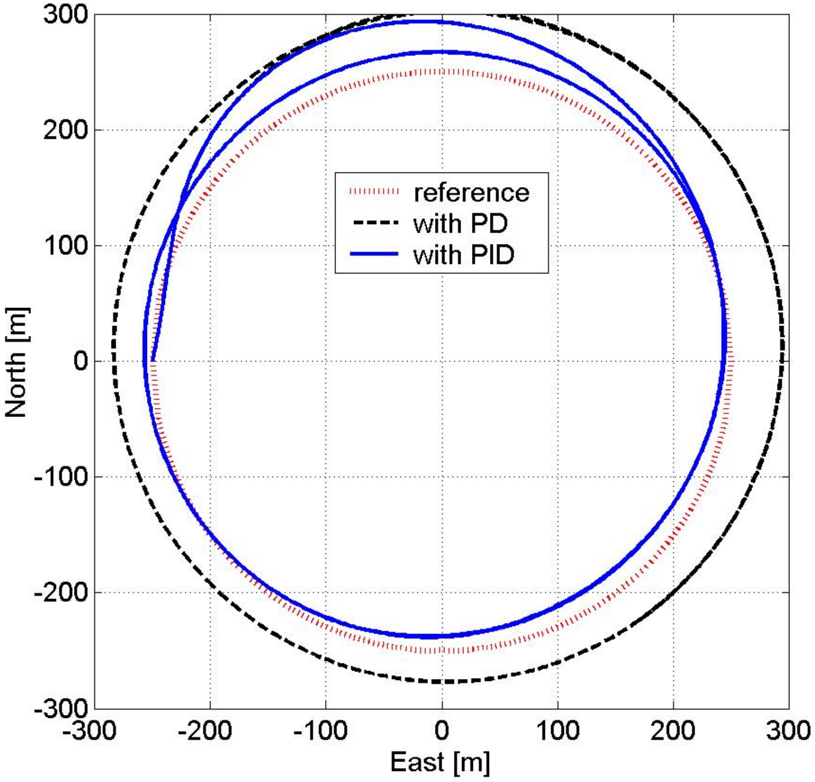

PD, PID Linear Control


New Guidance Logic

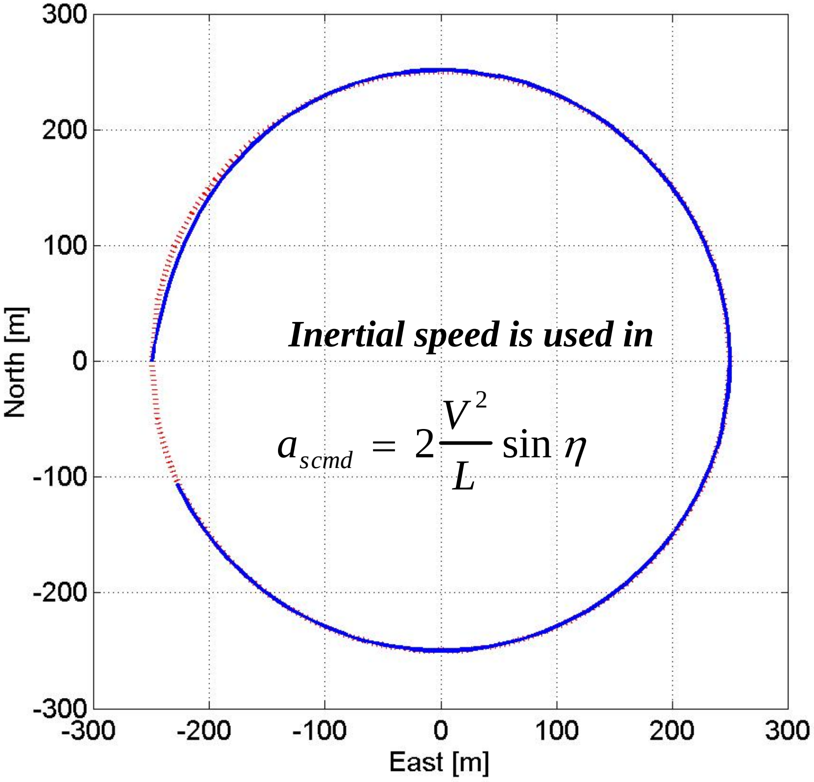
Comparison – Curved Line Following *with Wind*



PD, PID Linear Control



New Guidance Logic



Inertial speed is used in

$$a_{scmd} = 2\frac{V^2}{L}\sin\eta$$

Summary of the Lateral Guidance Logic

Superior performance of the nonlinear guidance logic comes from :

1. The feedback angle η anticipates the future trajectory to be followed
2. Use of inertial speed in the computation of acceleration makes the system adaptive to changes in vehicle speed due to external disturbances such as wind
3. The nominal trajectory is a circular arc so the system doesn’t cut corners on curved trajectories
4. Small perturbation behavior is second order with a damping ratio of .707
5. The lateral displacement from the reference trajectory converges asymptotically to zero

Sensitivity to Bank Angle Biases

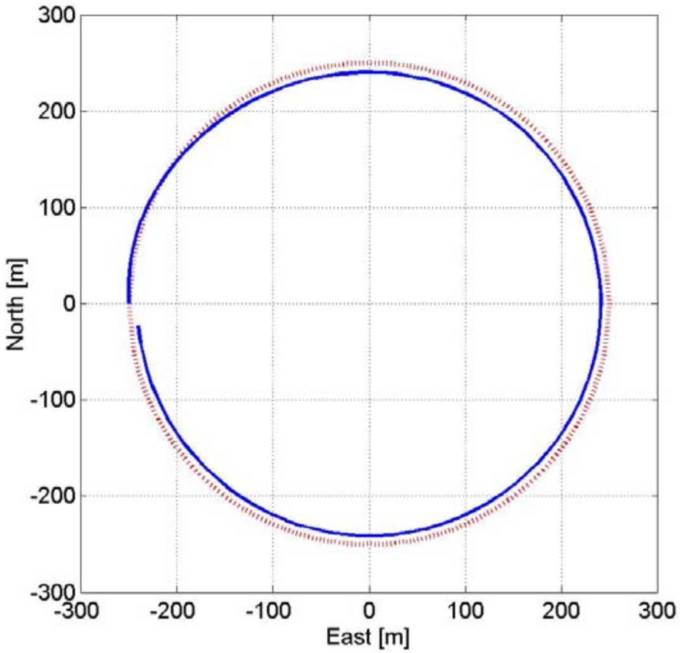
- Aircraft bank angle is used to generate lateral acceleration
- Bank angle biases result in lateral acceleration biases
- The guidance law will correct these accelerations but a trajectory bias error will result

No integral control element

→ *Not robust to the bias in lateral acceleration*

Simulation with 3 degree bank angle bias

: ~ 9 m steady cross-track error

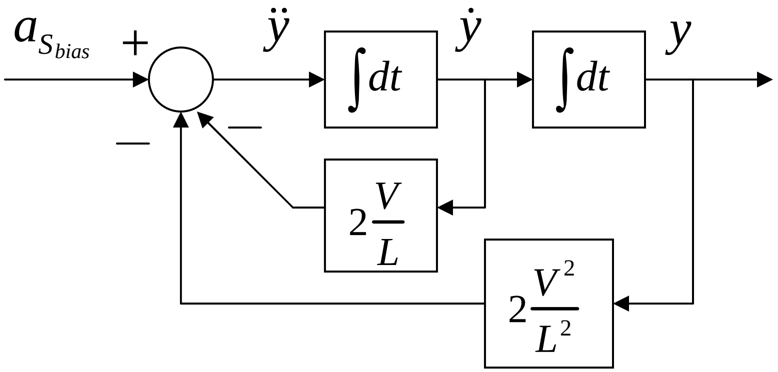


Sensitivity to Bank Angle Biases (cont.)

- Assuming the bank angle bias is small the linear system can be used to understand its effect

- The bank angle bias produces a lateral acceleration bias

- The lateral acceleration bias produces a bias in lateral position



$a_{S_{bias}}$ = lateral acceleration bias

- GPS/Inertial information can be used to effectively eliminate lateral acceleration bias

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Previous Attitude Estimation Methods

- Traditional AHRS with INS

  - Integration of rate gyros → Euler angle

  - Roll/Pitch correction by accelerometers (gravity aiding), Heading correction by compass

  Drawback : requires high quality, low drift inertial sensors

- INS/GPS Integration Methods

  - Many integration architectures: uncoupled/loosely/tightly coupled

  - trade-off (cost, constraints, performance)

  Drawback : high cost, complexity

- Multi-Antenna GPS-Based Attitude Determination (Cohen, 1996)

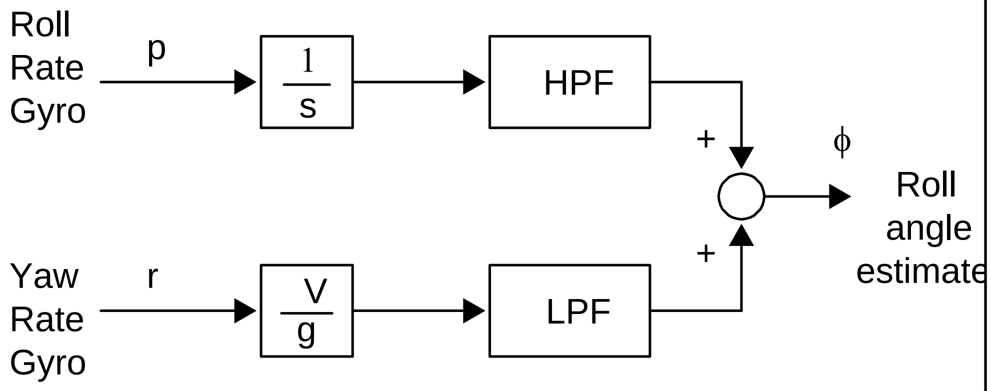
  - Use multiple antenna (typically at least 3), carrier phase differences

  - The attitude solution can be combined with inertial sensors in complementary filte

Drawback : multi-path, integer ambiguity, performance depends on baseline length

Previous Attitude Estimation Methods Using Aircraft Kinematics

Complementary filter with roll and raw gyros



Roll Rate Gyro

p

$\frac{1}{s}$

HPF

+

ϕ

Roll angle estimate

Yaw Rate Gyro

r

$\frac{V}{g}$

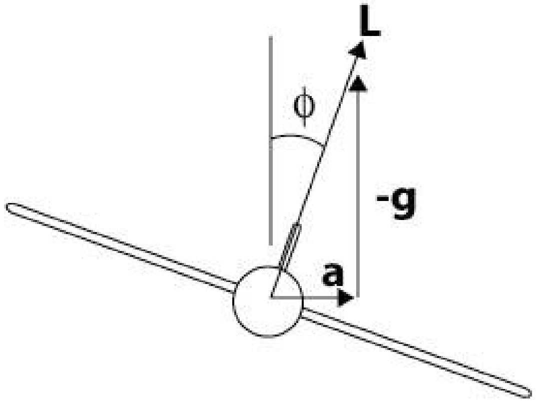
LPF

+

Drawback : biased estimate

Single-Antenna GPS Based Aircraft Attitude Determination

- *Richard Kornfeld, Ph.D.* (1999)



Drawback : sampling rate limit (GPS), typical filter time constant ~ 0.5 sec.

$$\boxed{a_s \approx g \cdot \phi \approx V \cdot r \qquad \dot{\phi} \approx p}$$

Estimation of Bank Angle & Roll / Yaw Rate Gyro Biases

Kalman Filter Setup

from Rate Gyros

p_{meas} →

r_{meas} →

$(a_s)_{est}$ →

from GPS Kalman Filter

Measurement Equations

$$p_{meas} = p + bias_p + \nu_2$$

$$r_{meas} = \frac{g}{V}\phi + bias_r + \nu_3$$

$$a_s = g\phi + \nu_1$$

Filter Dynamics

$$\frac{d}{dt}\phi = p + \omega_1$$

$$\frac{d}{dt}p = \omega_2$$

$$\frac{d}{dt}bias_p = \omega_3$$

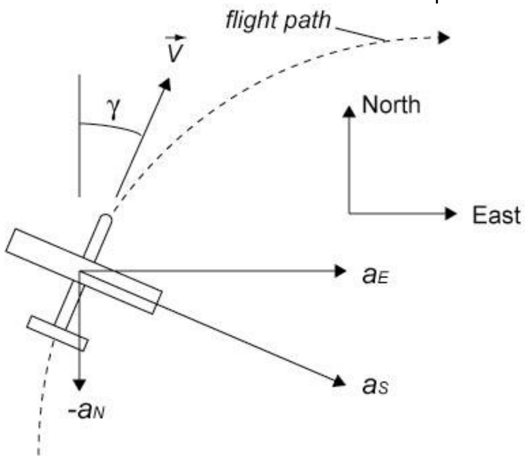
$$\frac{d}{dt}bias_r = \omega_4$$

→ ϕ_{est}

→ p_{est}

→ $(bias_p)_{est}$

→ $(bias_r)_{est}$



ϕ : bank angle V : velocity

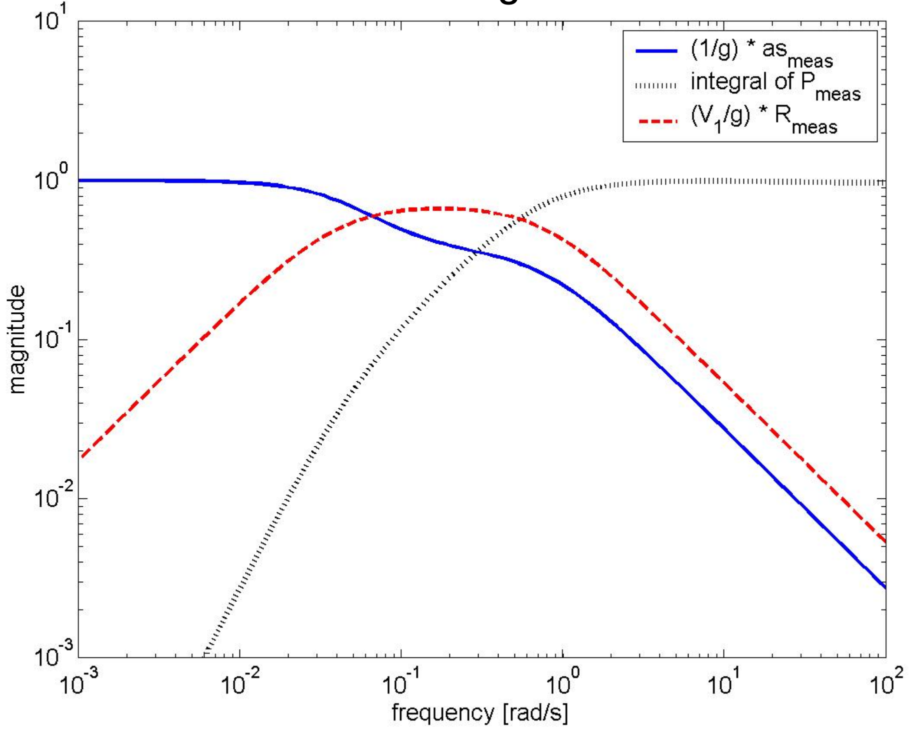
a_s : acceleration in sideways direction

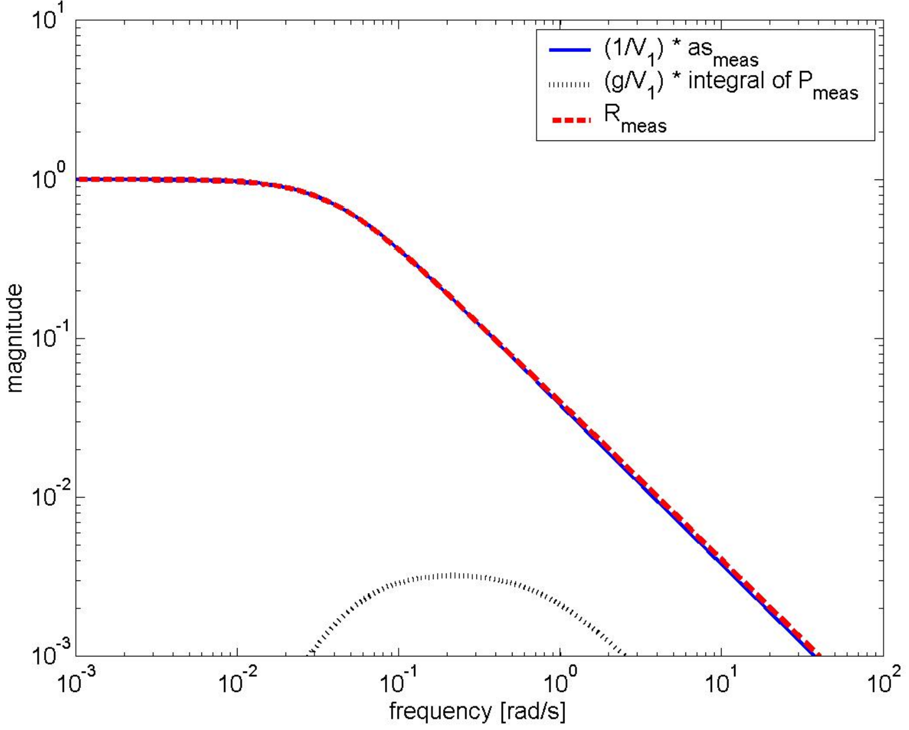
p : roll rate r : yaw rate

ν_i , ω_i : white noises

Contributions of Measurements on Estimates

(*Examples : Estimates of Bank Angle & Yaw-Rate Gyro Bias*)


for bank angle estimate


for yaw-gyro bias estimate

Bank Angle Estimate

low freq. : GPS acceleration

mid freq. : yaw rate gyro + GPS acceleration

high freq. : roll rate gyro

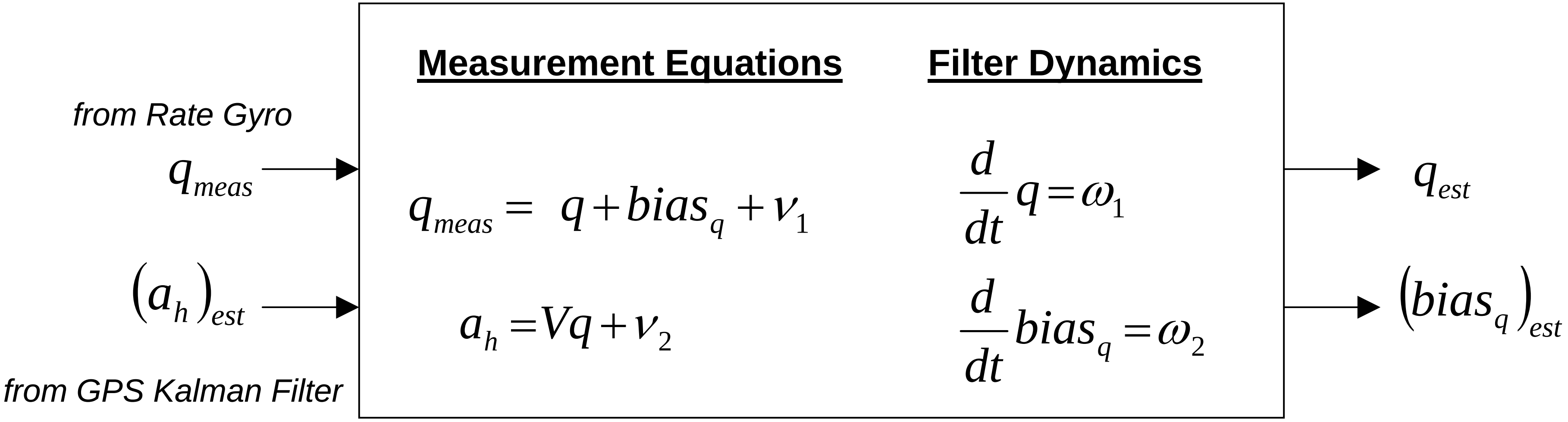
Yaw-Rate Gyro Bias Estimate

roll rate gyro doesn’t have effect

GPS acceleration, yaw gyro: 180 deg. phase diff.

Estimation of Pitch Rate Gyro Bias

Kalman Filter Setup



from Rate Gyro

q_{meas}

$(a_h)_{est}$

from GPS Kalman Filter

Measurement Equations

$$q_{meas} = q + bias_q + \nu_1$$

$$a_h = Vq + \nu_2$$

Filter Dynamics

$$\frac{d}{dt} q = \omega_1$$

$$\frac{d}{dt} bias_q = \omega_2$$

q_{est}

$(bias_q)_{est}$

a_h : acceleration in height (vertical) direction

V : velocity

q : pitch rate

ν_i , ω_i : white noises

Note: for turning with large bank angles, replace $(a_h)_{est}$ with $(a_h)_{est} + Vr|\tan\phi|$

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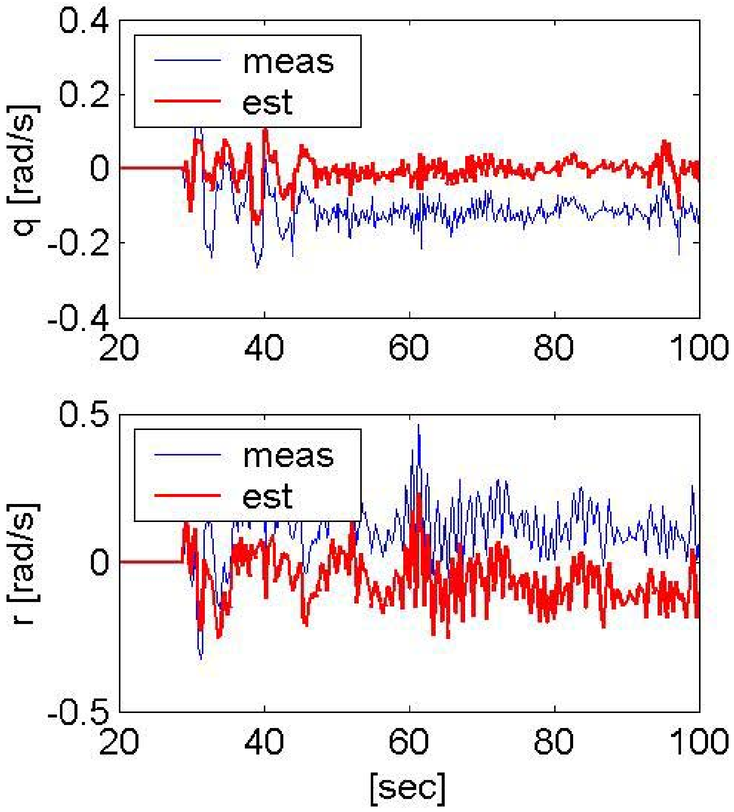
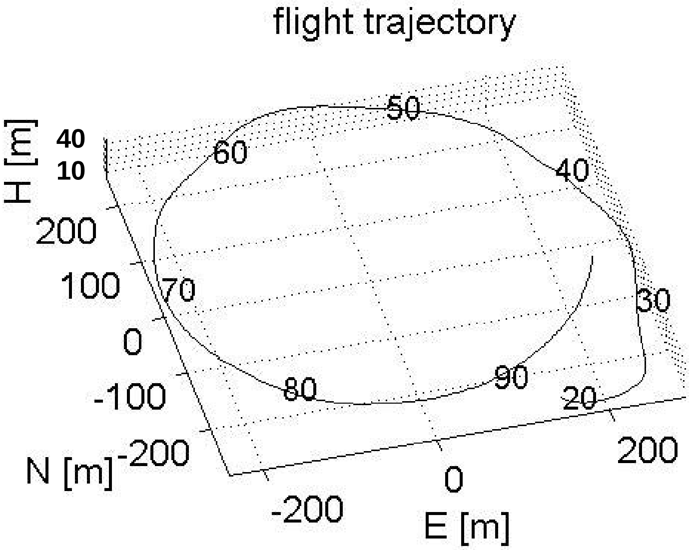
Flight Test Data : Rate Gyro Bias Estimation

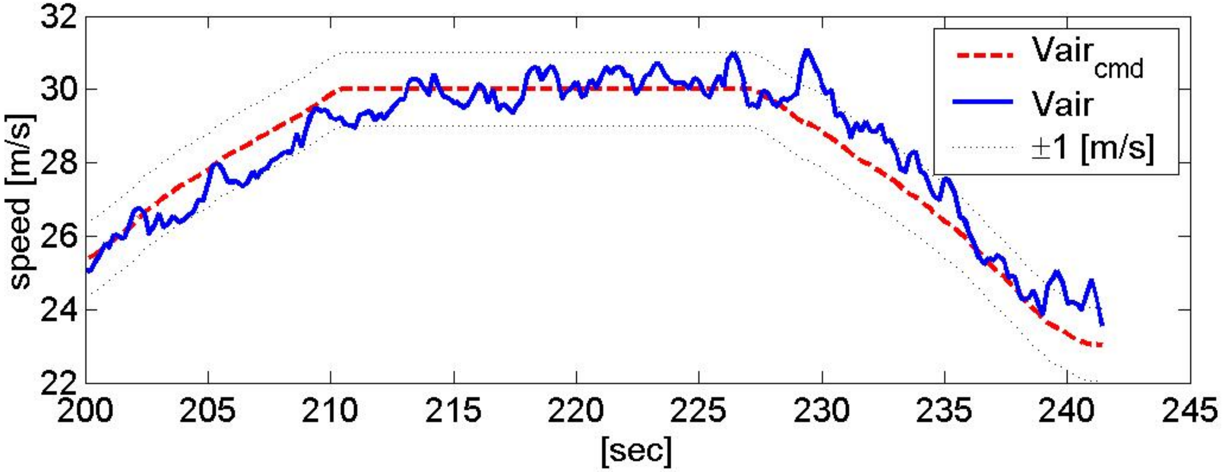


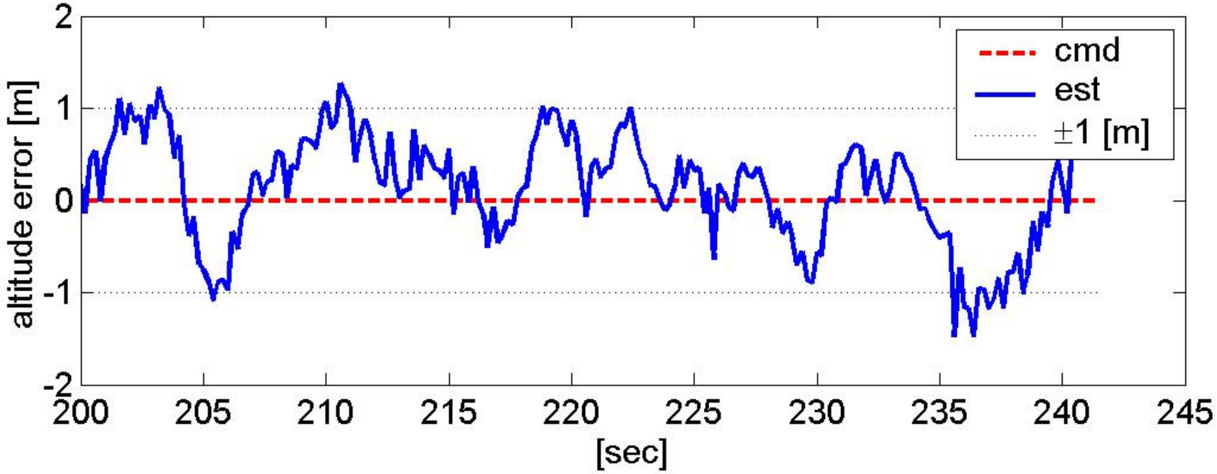


Flight Test Data : Longitudinal Control

Phase I Controller

Example : Mini





Flight Time Percentages
Within Error Bounds

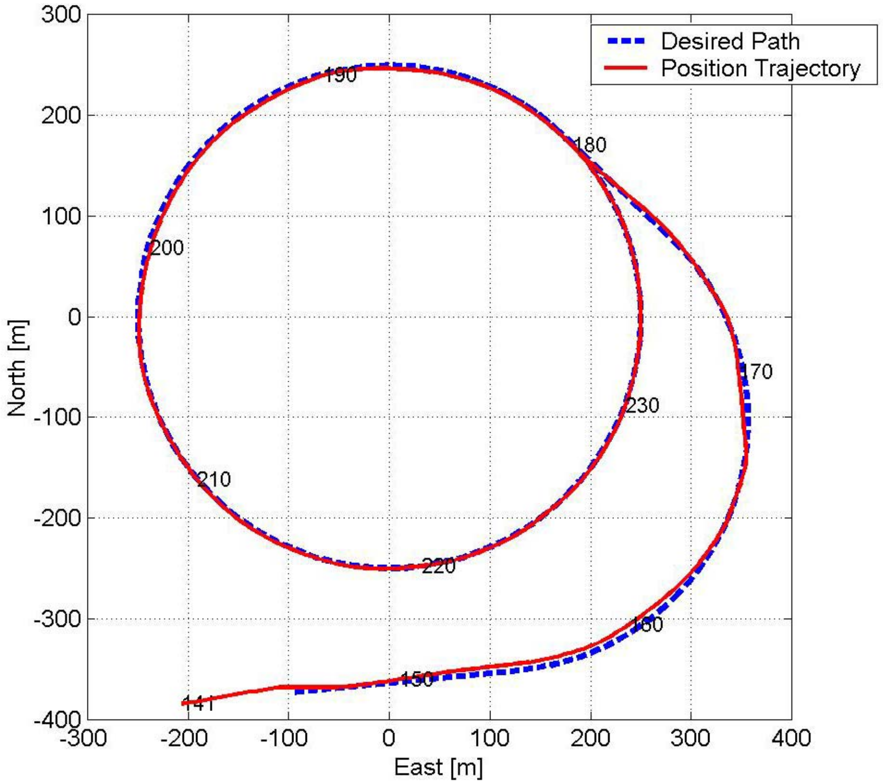
Error	Mini
Altitude	< 1m for 90%
Air Speed	< 1m/s for 88%

Error	OHS Parent
Altitude	< 2m for 97%
Air Speed	< 1m/s for 86%

Flight Test Data - Lateral Trajectory Following

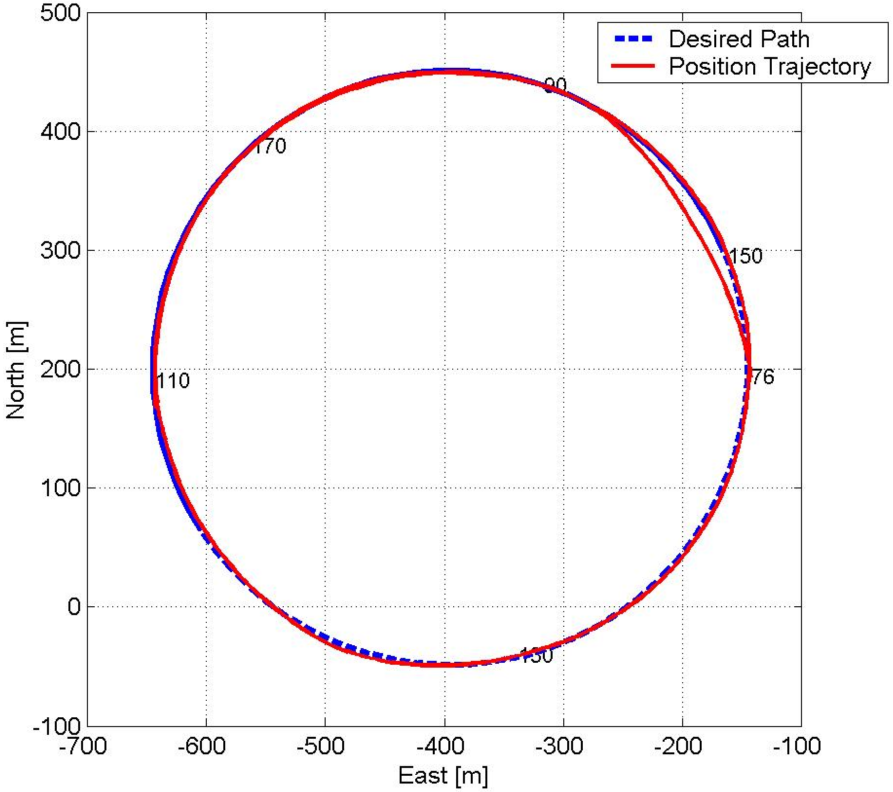
Phase I Controller

MINI



Displacement Error (during on circle) :
< 2 m for 75 %, < 3 m for 96 % of flight time

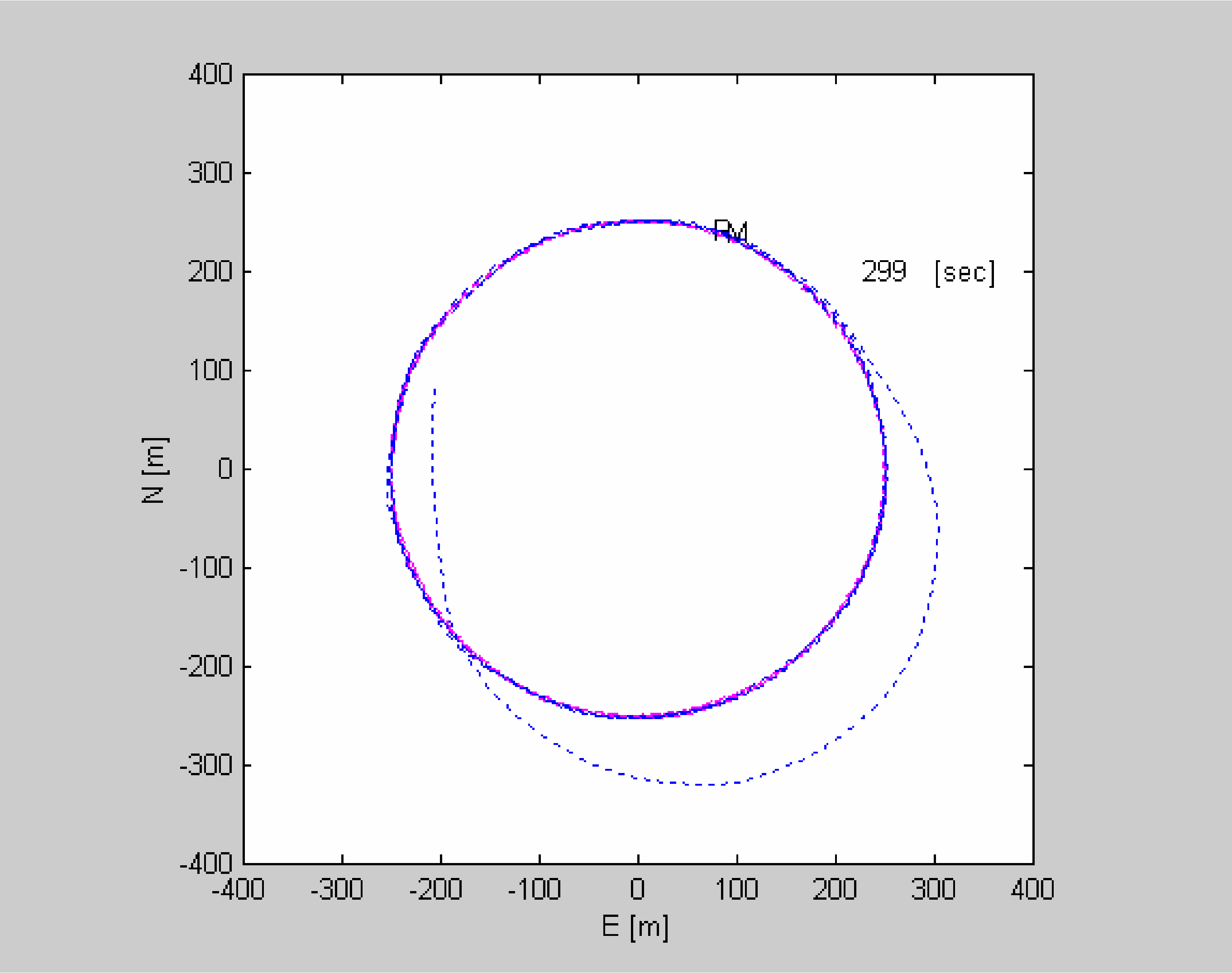
OHS Parent



Displacement Error (after initial transition) :
< 2 m for 78 %, < 3 m for 97 % of flight time

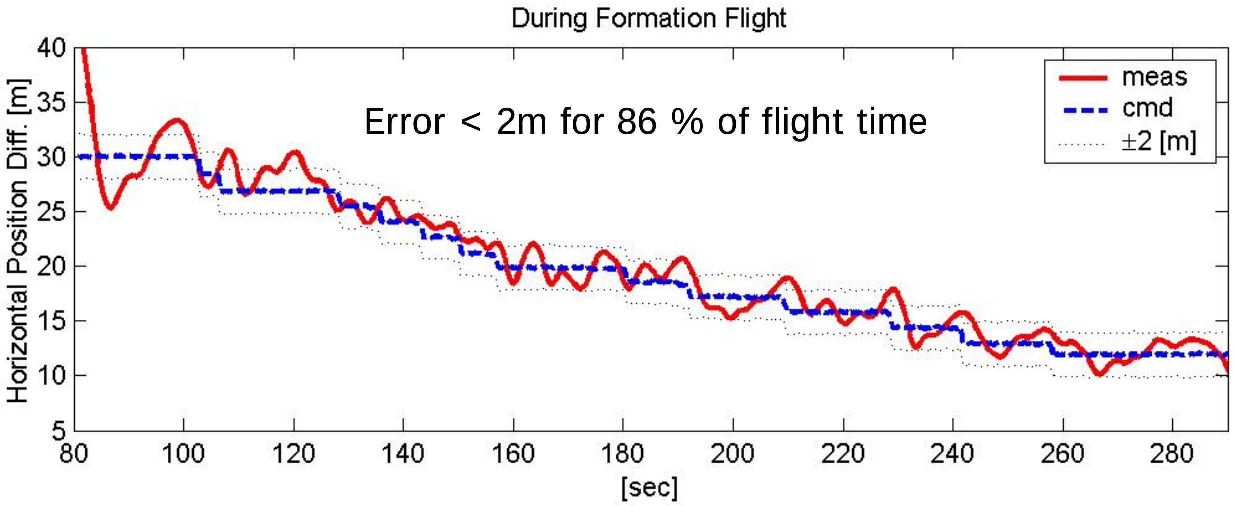
Flight Test Data – Phase I

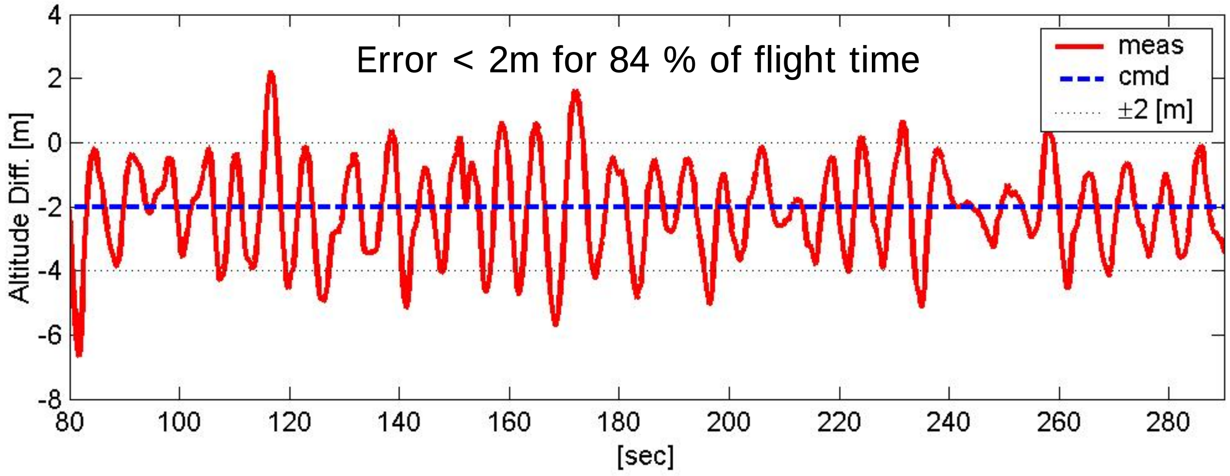
P : OHS Parent
M : Mini



Flight Test Data – Phase I

Relative Position Difference during Formation Flight


Error < 2m for 86 % of flight time


Error < 2m for 84 % of flight time

Summary of Contributions

- **Lateral Guidance Logic for Trajectory Following**
  - Tight Tracking for Arbitrary Curved Path Trajectories
  - Adaptive to Speed Changes due to Wind Disturbances

- **Estimation using Aircraft Kinematics + GPS + Low Quality Gyros**
  - Simple & Low-Order
  - Provides a Means for Non-biased Lateral Acceleration Determination

- **Trajectory Following for Two UAVs**
  - Implementation & Flight Demonstration of Guidance & Estimation Methods
  - Precise Control in the Presence of Wind Speed Disturbances ~ 5 m/s (> 20% of Flight Speed)

- **Phase I Rendezvous Flight Demonstration**
  - Most precise Control of Relative Positions of Two UAVs Demonstrated To Date