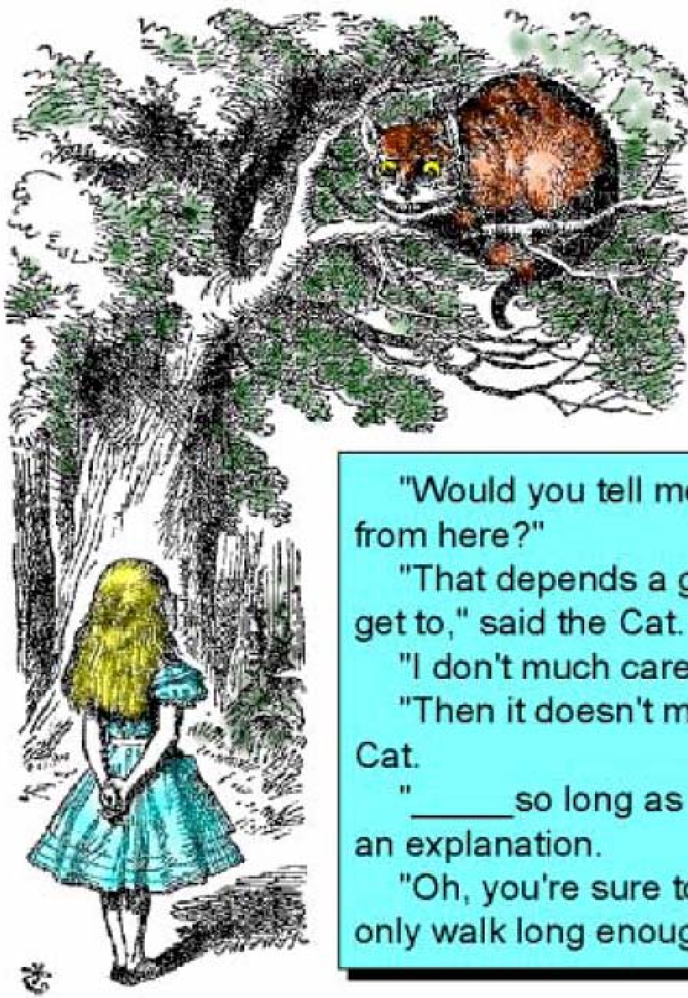
Space Systems Architecture Lecture 4 Utility Theory

Adam Ross
MIT ESD

Space Systems, Policy, and Architecture Research Consortium
A joint venture of MIT, Stanford, Caltech & the Naval War College
for the NRO



Lewis Carroll, *Alice in Wonderland*, 1865.

"Would you tell me, please, which way I ought to go from here?"

"That depends a good deal on where you want to get to," said the Cat.

"I don't much care where_____" said Alice.

"Then it doesn't matter which way you go," said the Cat.

"_____so long as I get *somewhere*," Alice added as an explanation.

"Oh, you're sure to do that," said the Cat, "if you only walk long enough."

<u>Source</u>: Hazelrigg, George A. "Bad Design Decisions: Why Do We Make Them?" Paper presented at the New Design Paradigms, Pasadena, CA, June 14, 2001.

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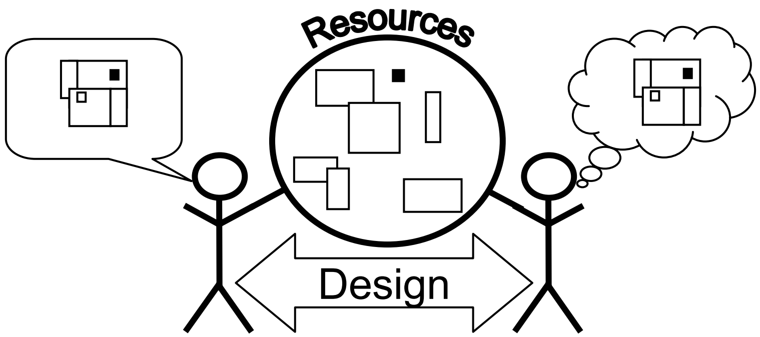
Carroll's Points

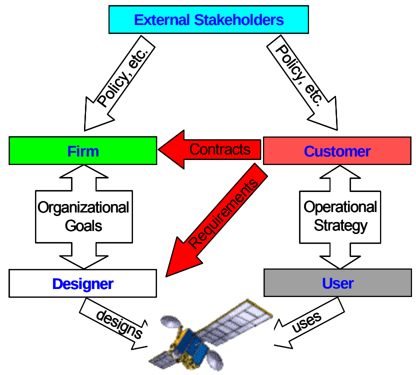
- ***Quote:*** "That depends a good deal on where you want to get to."
  - **Lesson 1:** All decisions are made based on preferences.
  - **Lesson 2:** The (only) preferences that matter are those of the decision maker.
  - **Lesson 3:** Preferences are on outcomes.
- ***Quote:*** "Then it doesn't matter which way you go."
  - **Lesson 4:** Don't try to distinguish between alternatives on the basis of outcomes upon which the decision maker has no preferences.
- ***Quote:*** "Oh, you're sure to do that if only you walk long enough
  - **Lesson 5:** Decisions cannot be avoided. Outcomes will occur.

Source: Hazelrigg, George A. "Bad Design Decisions: Why Do We Make Them?" Paper presented at the New Design Paradigms, Pasadena, CA, June 14, 2001.

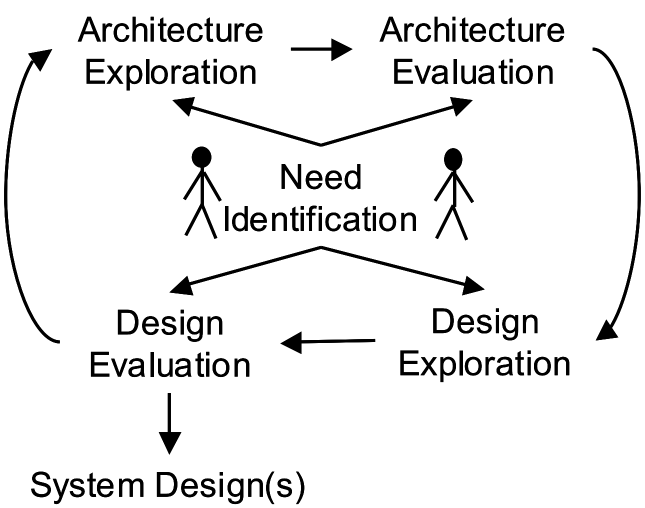
MATE-CON Revisited



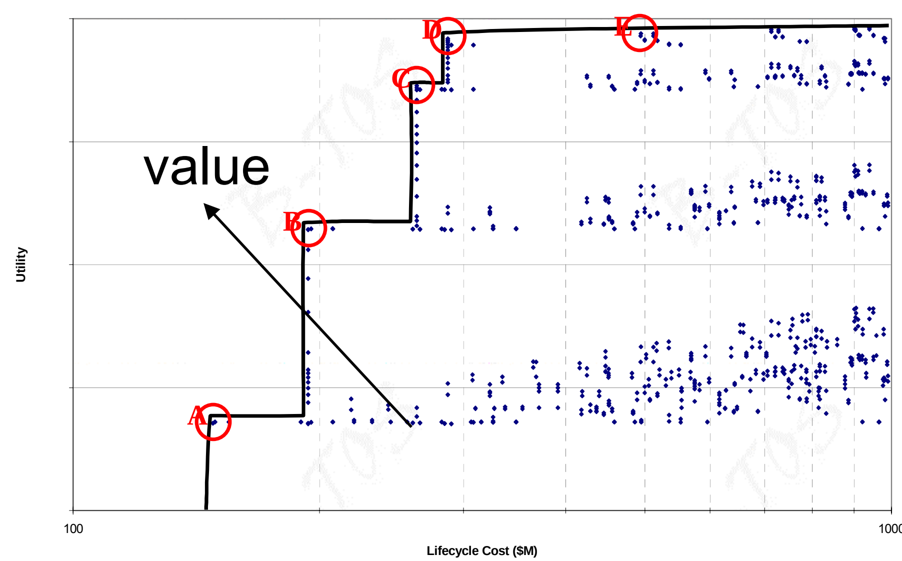
Fundamental X-change



Decision Makers



DM-centric Exploration Process



Tradespace Solution Spaces

Source: Ross, Adam M. "Multi-Attribute Tradespace Exploration with Concurrent Design as a Value-Centric Framework for Space System Architecture and Design." Dual-SM, Massachusetts Institute of Technology, 2003.

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Overview

- Value Functions
- Utility Functions
- Measurement of Utility
  - Process
  - Bracketing
  - CE vs. LEP
- MAUT
  - Motivation
  - Axioms
  - Formula
- Prospect Theory

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Value: Metric of DM Need

- An Attribute…
  - Is a *decision-maker perceived* metric that measures how well a *decision maker-defined* objective is met
- A Value metric…
  - Converts the attributes of a design into a measure of the preference(s) of the decision makers
  - Is quantitative, although often dimensionless and may be relative
  - Aids decision makers in choosing between disparate alternatives

Value Functions

- In General:
  - Preference Measure
  - PM = f(X)
  - where X = vector of attributes
- Semantic Caution: Value
  - Value in Exchange
  - Value in Use
  - "Fair Market Value"

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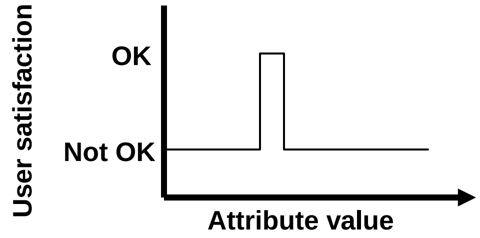
Measures of Value

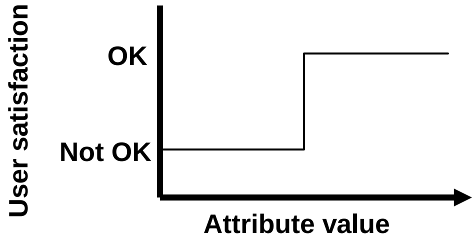
- Functional Requirements (binary - satisfy requirement or not?)
- Single measures of functionality (how much of commodity service is provided?)
- Single-attribute utility (how desirable is a given level of a given attribute?)
- Multi-attribute utility (what is the combined value of a given set of “independent” attributes?)

Progressively more difficult but potentially more useful

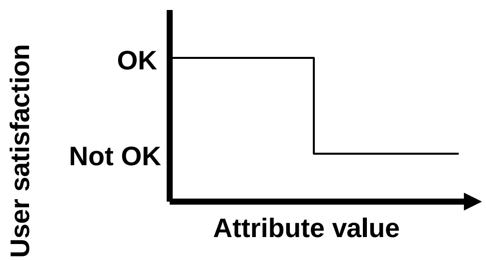
Requirements: Binary Constraint on Attributes



a) Required value (range)



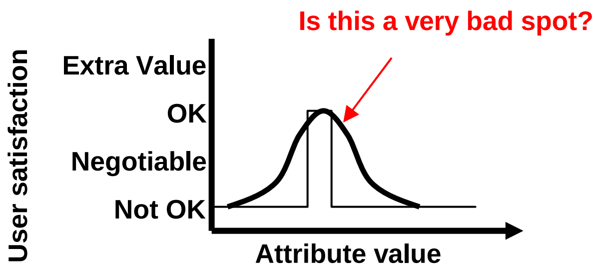
b) Must exceed



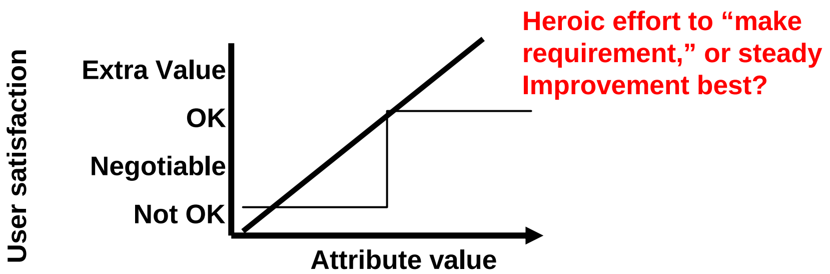
c) Must not exceed

If needs really are binary,
model as constraints, not attributes

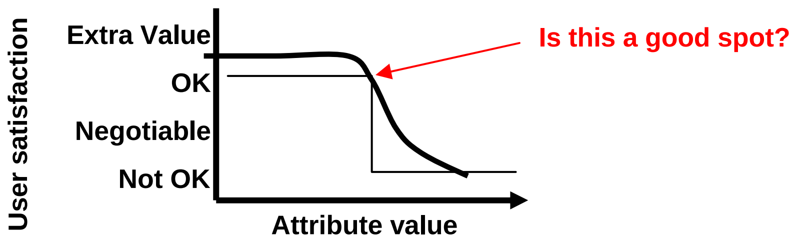
Real Needs May Not be Captured



a) Soft target



b) More is better



c) Soft threshold

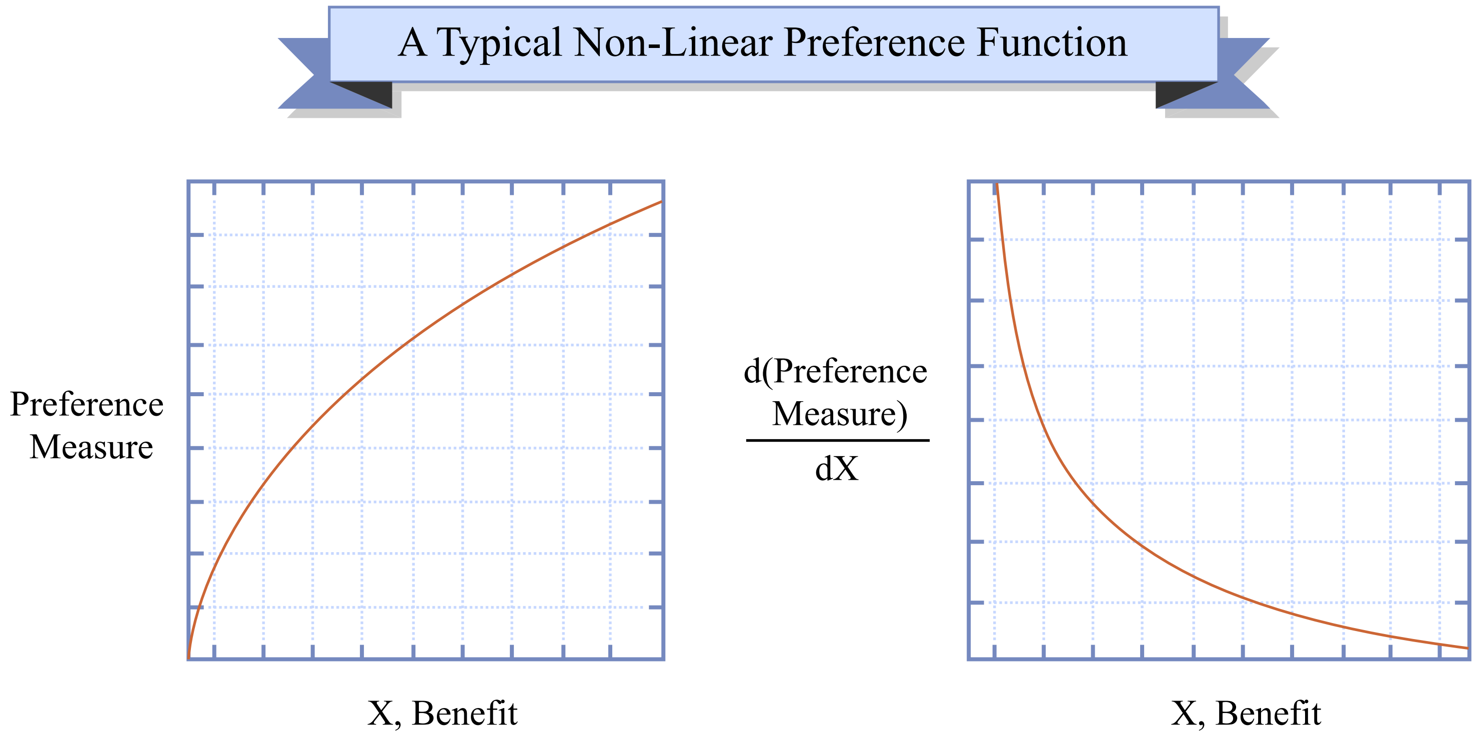
Premature requirements definition may mask real trades

Single Functions and CPF

- Some systems may have only a single, "commodity" function (e.g. Comm. Bandwidth)
- Value can be expressed directly in terms of this function (Bandwidth is good)
- Providing this function at low cost becomes single metric for evaluation - Cost Per Function (CPF)
- May be used for non-commodity attributes (e.g. cost per image of science missions)
  - Could be over-simplifying: What sort of images? Quality? etc.

Value Function V(x)

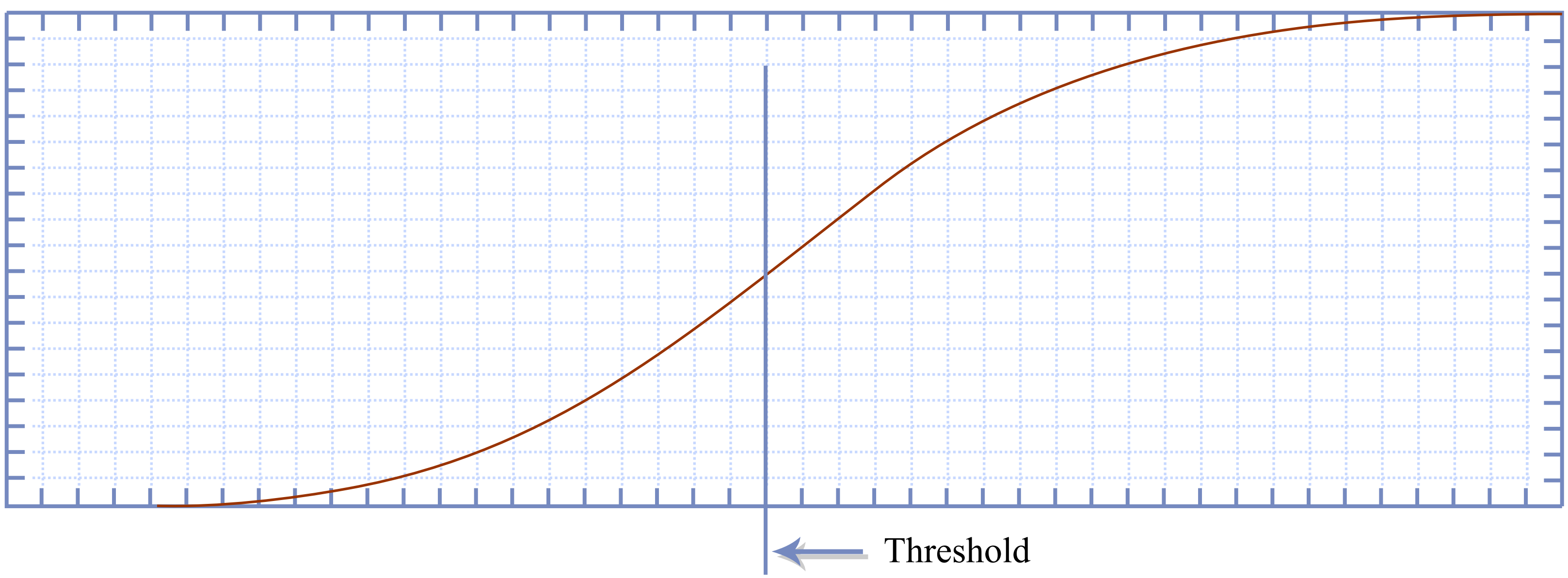
Definition: V(X) is a means of ranking the relative preference of an individual for a bundle on consequences, X



Adapted from: de Neufville, Richard. *Applied Systems Analysis: Engineering Planning and Technology Management.* New York: McGraw-Hill Co., 1990.

Preference Function

- Risk Preference Also Observed
- (There is no "Law of Diminishing Marginal Utility") However, May Instead Reflect "Threshold Effect" (Asymmetric Behavior About A Threshold Value)



Adapted from: de Neufville, Richard. *Applied Systems Analysis: Engineering Planning and Technology Management.* New York: McGraw-Hill Co., 1990.

$V(X)$: Basic Axioms

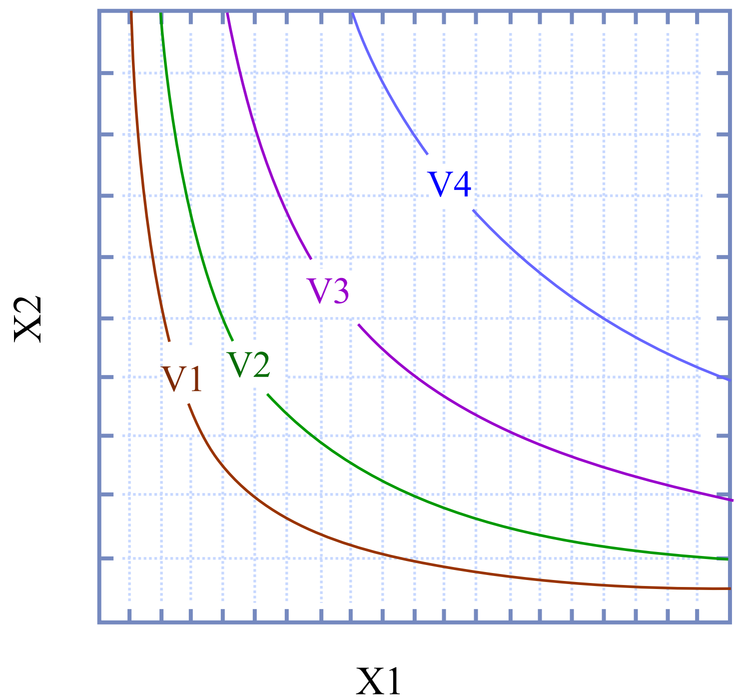
- Completeness or Complete Preorder
- People have preferences over all X_i
- Transitivity
  - If X_1 is preferred to X_2 ; and X_2 is preferred to X_3; Then X_1 is preferred to X_3
  - Caution: Assumed True for Individuals; NOT Groups
- Monotonicity or Archimedean Principle
  - For any X_i ($X_* < X_i < X^*$) there is a w ($0 < w < 1$) such that $V(X_i) = w\, V(X^*) + (1 - w)\, V(X_*)$
  - That is, More is Better (or Worse)

V(X): Consequences

- Consequence of V(*X*) Axioms
  - Existence of V(*X*)
  - Ranking only
- Strategic equivalence of many forms of V(*X*)
- Any monotonic transform of a V(*X*) is still an equivalent V(X)
- For example:
  - $\mathbf{V}(X_1, X_2) = X_1^{\,2}\, X_2 = 2 \log (X_1) + \log (X_2)$

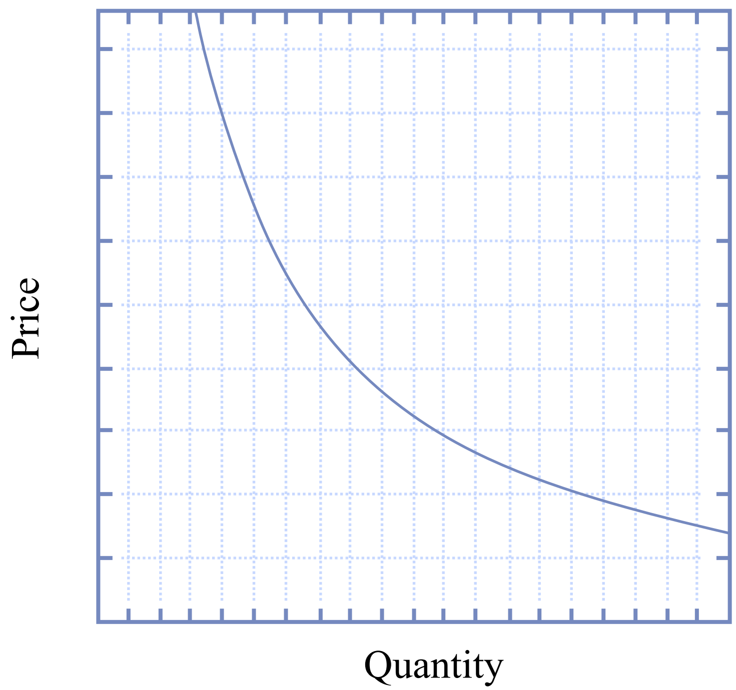
Value Functions

Isovalue Contours



Demand Function

(Maximize value given a budget constraint)



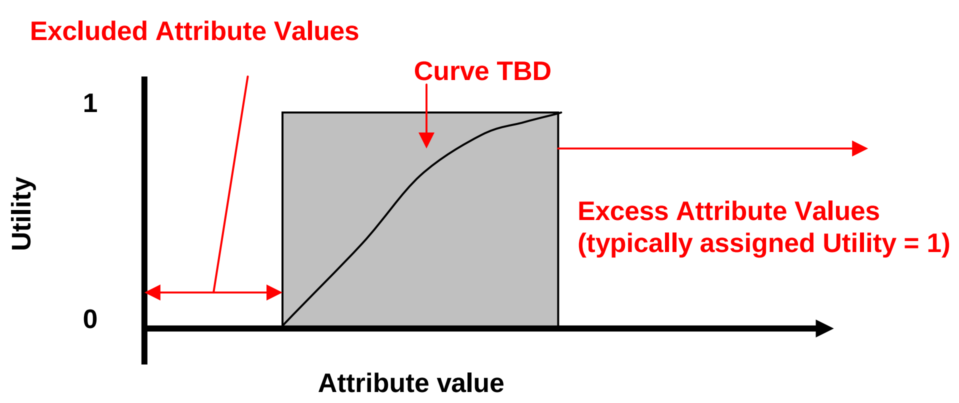
Adapted from: de Neufville, Richard. *Applied Systems Analysis: Engineering Planning and Technology Management.* New York: McGraw-Hill Co., 1990.

The Utility Function

- Utility Function - U(*X*)
- Definition:
  - U(*X*) is a special V(*X*),
  - Defined in an <u>uncertain</u> environment
- It has a special advantage
  - Units of U(*X*) DO measure relative preference
  - CAN be used in meaningful calculations
  - Is measured on a Cardinal Scale

Single Attribute Utility

- Postulate a dimensionless metric that is a function of attribute X: $U_i = U_i(X)$
- Set U_i = 0 at the least desirable, *but still acceptable* value of X
- Set U_i = 1 at the highest (most desirable) value of X
- U_i, the "single attribute utility," can be used to express the relative desirability of values of X



Cardinal Scales

- Ratio
  - Zero value implies an absence of phenomenon e.g., Distance, Time note: F'(x) = a F(x) defines an equivalent measure (e.g., meters and feet)
- Ordered Metric
  - Zero is relative, arbitrary
    - Example: Temperature
  - Need to define two points:
    - 0 degrees C - freezing point of pure water
    - 100 degrees C - boiling point of pure water at standard temperature and pressure
    - 0 degrees F - freezing point of salt water
    - 100 degrees F - What?
  - Equivalent measures under a positive linear transformation
    - Note: f'(x) = a f(x) + b (e.g. F = (9/5) C + 32)

Utility Axioms

- Value function axioms
  - Completeness
  - Existence of preference
  - Transitivity
  - Monotonicity
- Preferences in probability (more likely good)
- Substitutions (expected value)

More Info

Utility Axiom Consequences

- Utility exists on an ordered metric scale
  - The utilities have meaning only compared with other utilities
  - The units of utility have constant relative meaning but no absolute meaning
  - A linear transformation of a utility function (e.g. changing from °F to K) does not change the decisions that will be made
- To measure, sufficient to
  - Scale 2 points arbitrarily
  - obtain relative position of others by probability weighting -- Similar to triangulation in surveying
- How do we measure utility?
  - Since it is empirical -- Measure
  - Since it is personal – Measure Individuals
- Solution: some form of an interview
  - oral
  - computer based

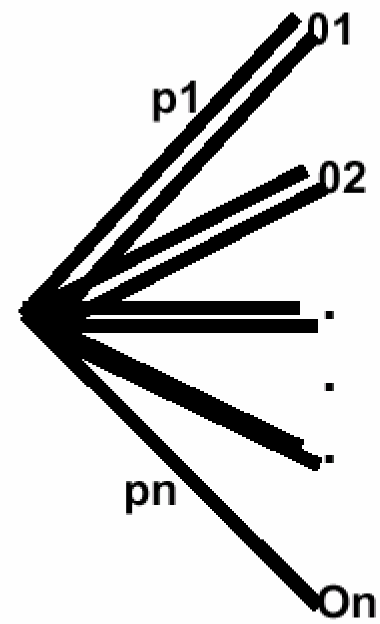
Measuring Utility

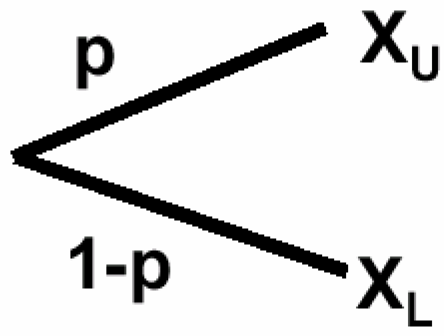
- Psychometric considerations
  - Nature of interview
  - Context
  - Scale of response
  - Method obtained (bracketing)
  - Consistency and replicability (computer programs)
- Step-by-Step Procedure
  - Defining X
  - Setting context
  - Assessment
  - Interpretation
  - Numerical approximation

More details

Probs & Outcomes: Lotteries

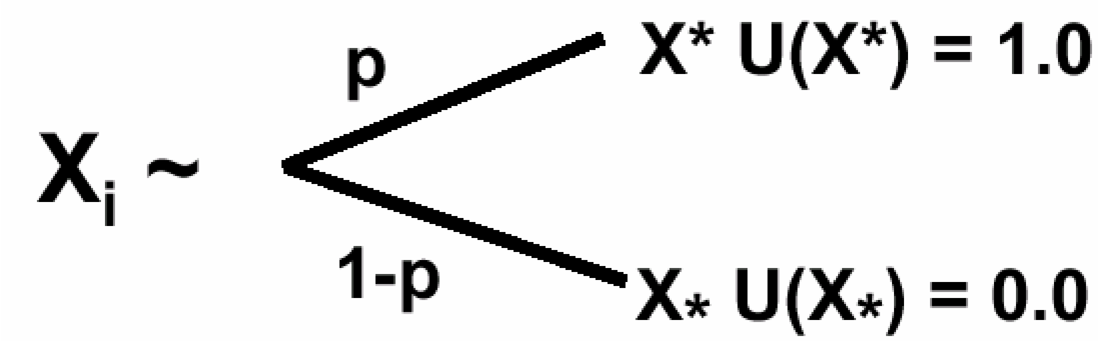
- Lottery
  - A risky situation with outcomes 0_j at probability p_j
    - Written as $(0_1, p_1; 0_2, p_2; ...)$
- Binary Lottery
  - A lottery with only two branches, entirely defined by X_U, p_U, X_L
    - $p(X_L) = 1 - P_U$
    - Written as $(X_U , P_U ; X_L)$
- Elementary Lottery
  - A lottery where one outcome equals zero, that is, the status quo
    - Written as (X,p)





Utility Measurement (CE)

- Conventional Method
- Certainty Equivalent (CE) - Balance X_i and a lottery
  - Define X^* - best possible alternative on the range
  - Define X_* - worst possible alternative on the range
- Assign convenient values - $U(X^*)=1$; $U(X_*) = 0$
- Conduct data collection/interview to find X_i and p
- Note: $U(X_i) = p$



- For each X_i, vary p until indifferent

Source: de Neufville, Richard. Applied Systems Analysis: Engineering Planning and Technology Management. New York: McGraw-Hill Co., 1990.

Utility Measurement (LEP)

- New method
- Avoid certainty equivalents to avoid "Certainty Effect"
- Consider a "Lottery Equivalent"
  - Rather than comparing a lottery with a certainty
  - Reference to a lottery is not a certainty

- Vary "P_e" until indifference between two lotteries. This is the Lottery Equivalence

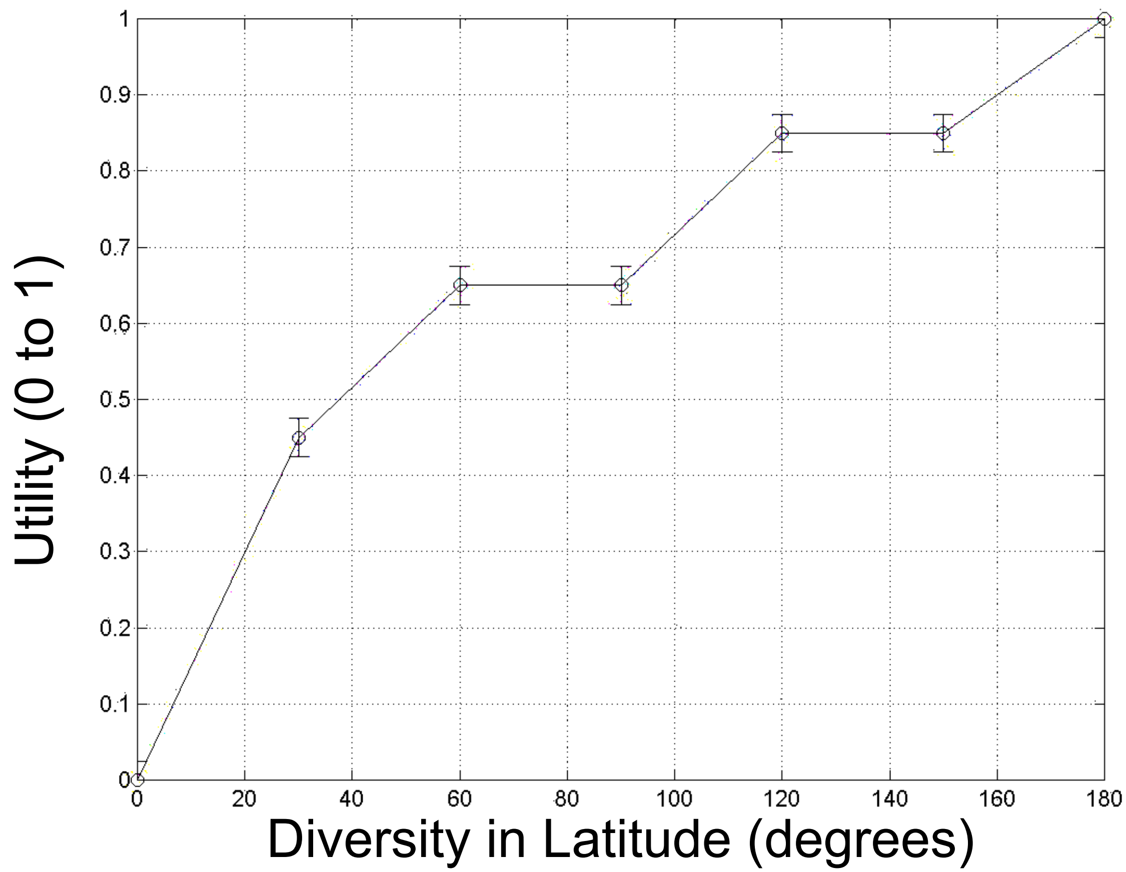
More details

Source: de Neufville, Richard. Applied Systems Analysis: Engineering Planning and Technology Management. New York: McGraw-Hill Co., 1990.

Single Attribute Utility

- Mapping of attributes to perceived-value under uncertainty
- Utility is an ordered metric scale (e.g. °F)
- Not required to have "analytic" form

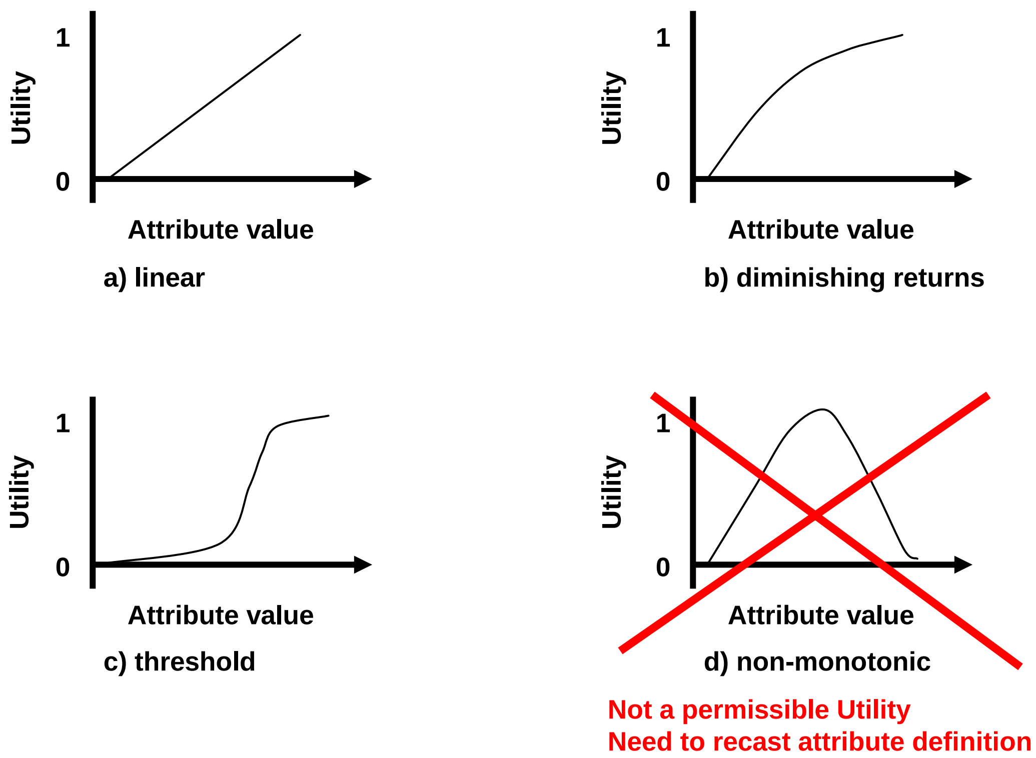
More SAU Examples



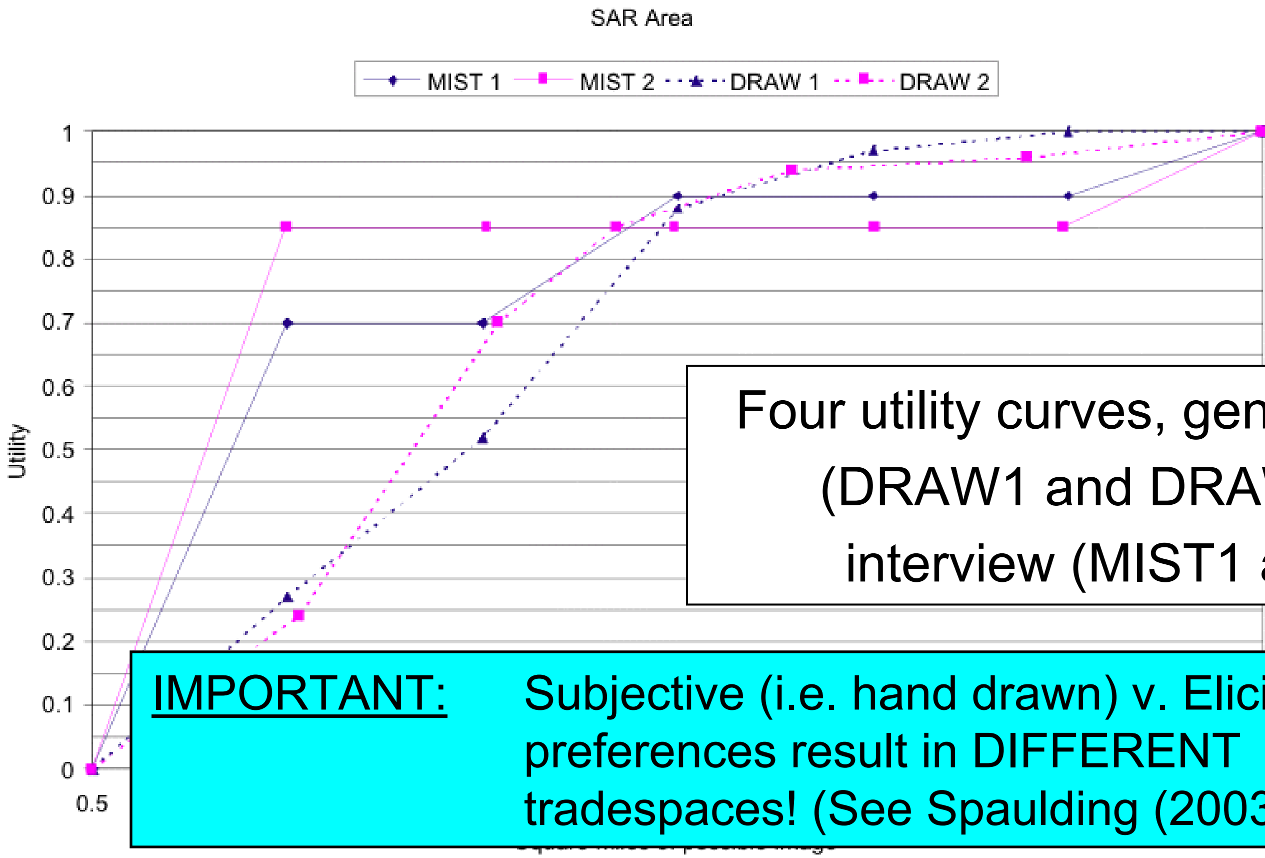
Source: X-TOS Final Design Report, MIT 16.89 Spring 2002

Determining Single Attribute Utility

- Interviewing – axiomatically based, but time-consuming
- Sketching – not accurate, but easy



Noise is inevitable



Four utility curves, generated by sketch (DRAW1 and DRAW2) and LEP interview (MIST1 and MIST 2)

<u>IMPORTANT:</u> Subjective (i.e. hand drawn) v. Elicited preferences result in DIFFERENT tradespaces! (See Spaulding (2003))

Both random noise and effect of generation method are evident

<u>Source:</u> Spaulding, Timothy J. "Tools for Evolutionary Acquisition: A Study of Multi-Attribute Tradespace Exploration (MATE) Applied to the Space Based Radar (SBR)." SM, Massachusetts Institute of Technology, 2003.

MAUT Motivation

- Desire to capture preferences over multiple attributes
- Curse of Dimensionality
  - Procedure for 1-dimensional utility function can, in theory, be applied to an n-dimensional utility function
  - But, consider the number of points to be assessed if we divide a range of N dimensions into quarters

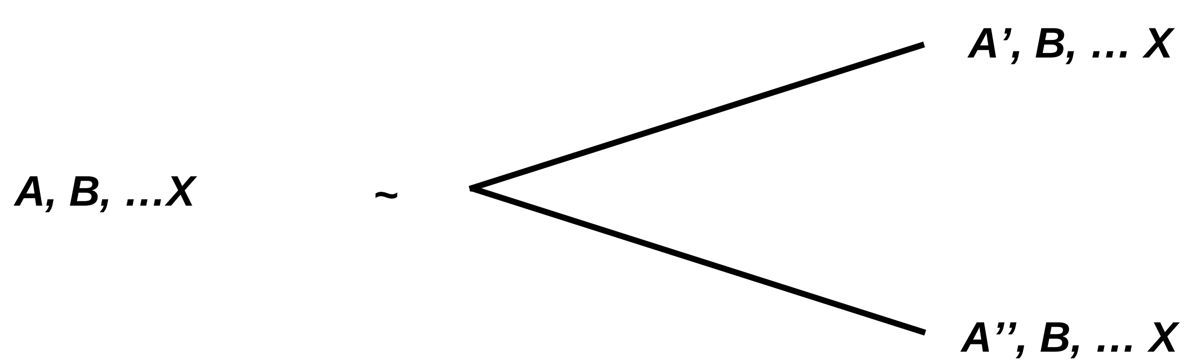
Dimensions	Number of points
1	5 – 2 = 3
2	(5)(5) – 2 = 23
3	(5)(5)(5) – 2 = 123

MAU: Axiomatic Basis (1)

- Preferential Independence - An ordinal condition
  - The order of preference between any 2 pairs of outcomes is constant, regardless of level of other outcomes
  - If $(X_1', X_2') > (X_1'', X_2'')$ for any $(X_3', \ldots X_N')$
  - Example
    - I prefer (1 cup coffee, black) to (2 cups coffee, w/sugar), regardless of wealth
  - Consequence
    - Can compare dimensions two at a time, independently of others

MAU: Axiomatic Basis (2)

- Utility Independence – a cardinal condition
  - The relative intensity of value for different amounts of one type of outcome is independent of level of all other outcomes.



True for all consistent sets of *B, …X*

MAU: Axiomatic Basis (3)

– Example
  - When hungry, I prefer 1 plate of food for sure to a 50:50 gamble on 2 plates or none, regardless of noise
– Consequence
  - Can assess U(X_i) once, and use it for all cross sections of U($\underline{\boldsymbol{X}}$), subject to a positive linear transformation
  - "Shape" of U(X_i) constant
    – $U'(\underline{\boldsymbol{X}}) = a\,U(\underline{\boldsymbol{X}}) + b$

Derivation of Keeney-Raiffa MAUF*

*Source: Keeney, Ralph L., and Howard Raiffa. Decisions with Multiple Objectives--Preferences and Value Tradeoffs. 2nd ed. Cambridge: Cambridge University Press, 1993.

Multi-Attribute Utility Fcns

- Combine multiple single-attribute utilities U_i into a single metric U
- May not be possible
- May be simple
  - Weighted sum $$U = \sum_{i=1}^{n} k_i U_i$$
  - Multiplicative function $$U = \prod_{i=1}^{n} U_i$$
  - Inverse multiplicative function $$1 - U = \prod_{i=1}^{n} (1 - U_i)$$
- Generalized form - Keeney-Raiffa function

Axiomatic

$$KU + 1 = \prod_{i=1}^{n} (Kk_i U_i + 1)$$

MAU Analysis Process

Method:

1. Define attributes
2. Define attribute ranges (worst→best case)
3. Compose utility questionnaire (context)
4. Conduct utility interview with Customer/User
   - Find single attribute utilities, $U(X_i)$
   - Find “corner weights”, k_i
5. Calculate utility function
6. Conduct validation interview
7. Conduct sensitivity analysis

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Attributes as Decision Metrics

- A decision maker-perceived metric that measures how well a *decision maker-defined* objective is met
- Set of attributes must be:
  - Complete
  - Operational
  - Decomposable
  - Non-redundant
  - Minimal
  - Perceived Independent*
- "Rule of 7": Human mind limited to roughly 7 (7 ± 2) simultaneous concepts**
- In the limit ranges converge to a point, the attributes become requirements

Attribute Characteristics

- Definition
- Units
- Range (least-most acceptable)

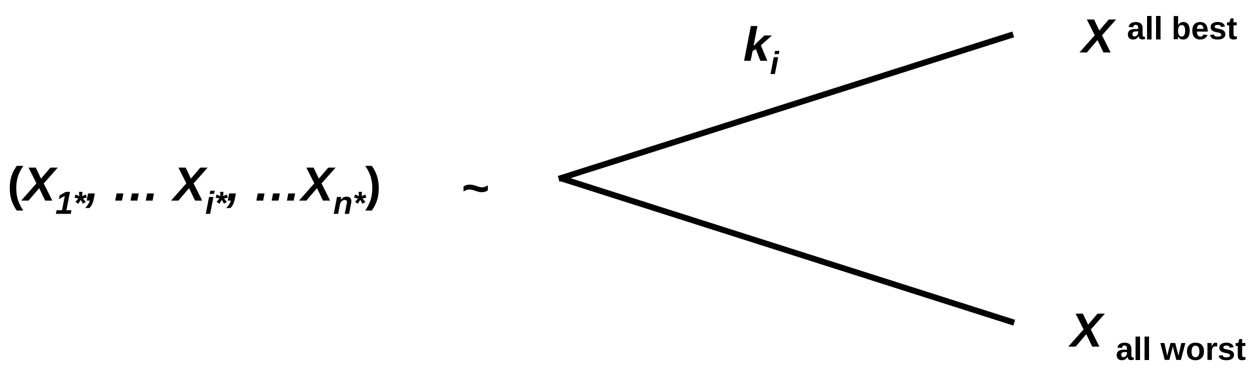
* Not strictly necessary, but reduces interview time and complexity.

**Miller, George A. "The Magical Number Seven, Plus or Minus Two: Some Limits on Our Capacity for Processing Information." *The Psychological Review* 63 (1956): 81-97.

Procedure for U($\underline{X}$)

- Establish the range of each dimension
  - X_{i*} to X_i^*
- Set $\underline{\boldsymbol{X}}_* = (X_{1*}, \ldots, X_{n*})$ $\quad U(\underline{\boldsymbol{X}}_*) = 0$

  $\underline{\boldsymbol{X}}^* = \text{(all the best)}$ $\quad U(\underline{\boldsymbol{X}}^*) = 1$
- Establish the relative value of each dimension:



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Multi-Attribute Utility Function

Multi-Attribute Utility Function*

$$KU(\underline{X}) + 1 = \prod_{i=1}^{N} \left(K k_i U(X_i) + 1 \right)$$

Normalization constant

Multi-attribute utility function

Relative “weight”

Single attribute utility

*Keeney, Raiffa, 1976.

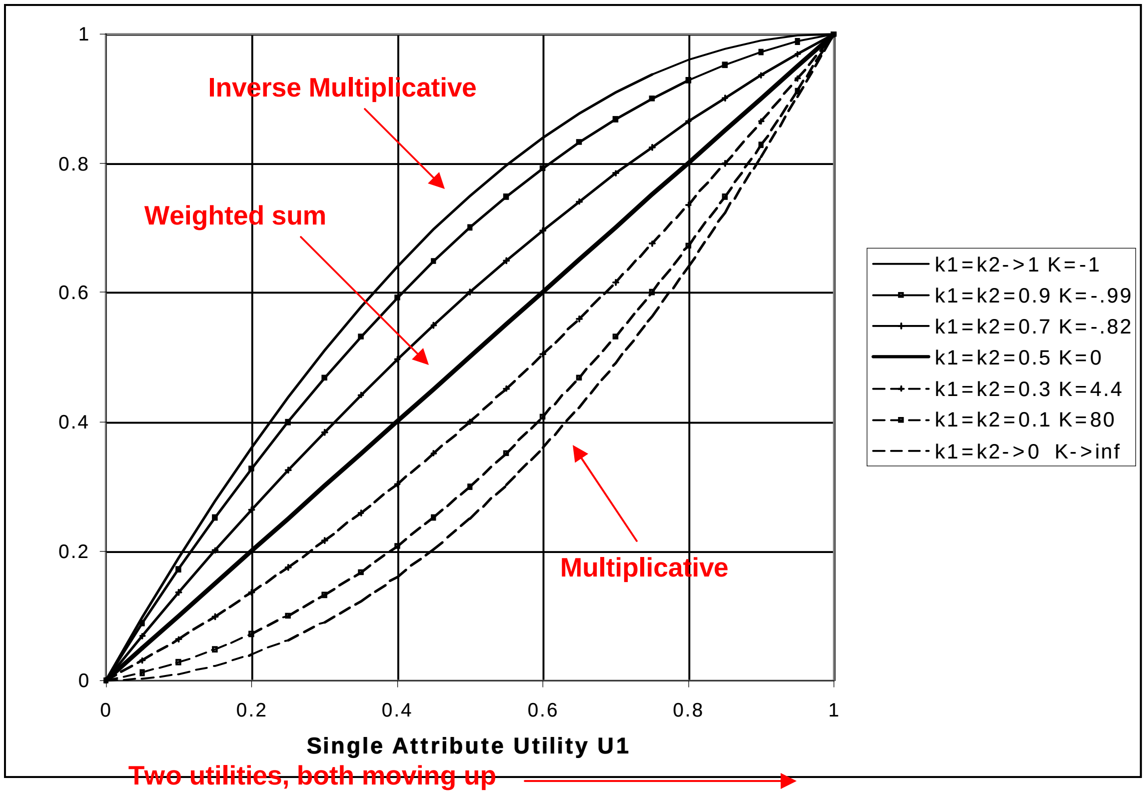
Solving for K

$$K + 1 = \prod_{i=1}^{N} \left(K k_i + 1 \right)$$

- Set $U(\underline{\boldsymbol{X}}) = U(\underline{\boldsymbol{X}}^*) = 1$, then $U(\boldsymbol{X_i}) = U(\boldsymbol{X_i}^*) = 1$
- Guidelines:
  - If the sum of all $k_i < 1$ — $K > 0$
  - If the sum of all $k_i > 1$ — $-1 < K < 0$
  - If the sum of all $k_i = 1$ — $K = 0$

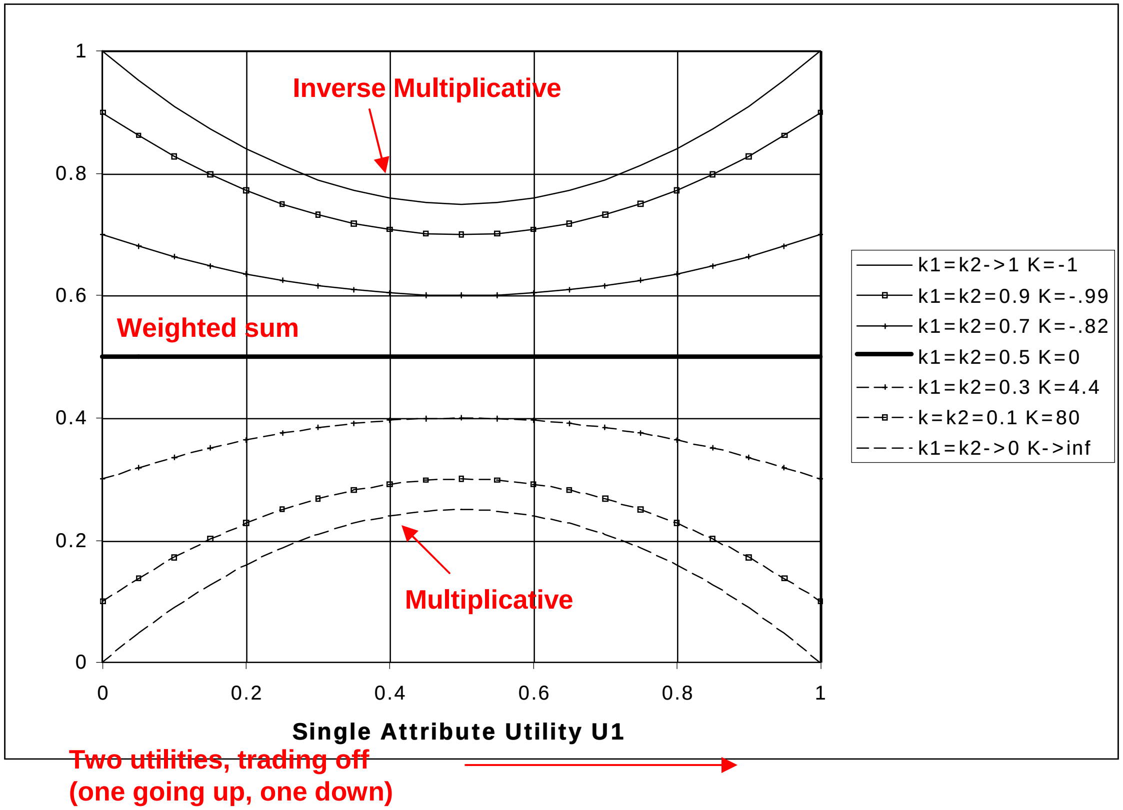
    $U(\boldsymbol{X}) = \Sigma\, k_i\, U_i(\boldsymbol{X_i})$

Behavior of MAU's



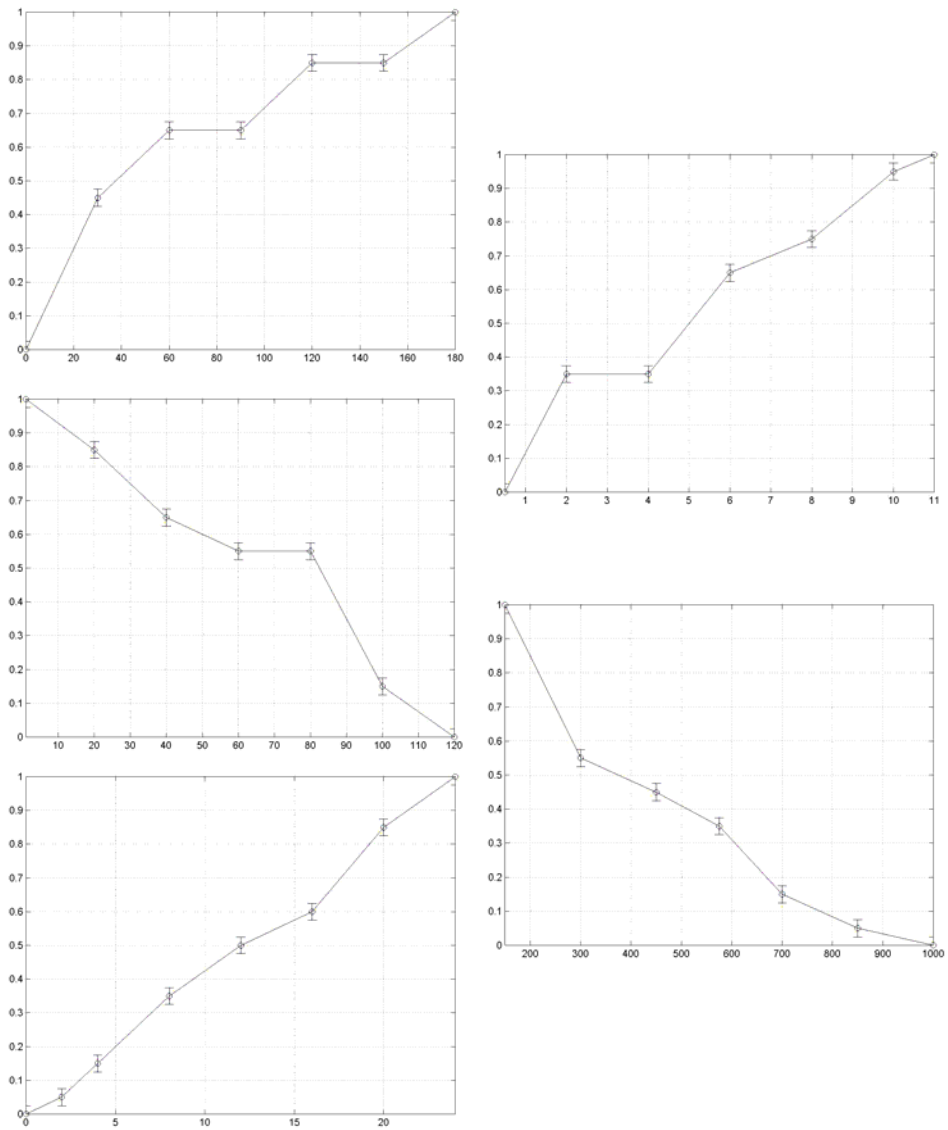
A family of possible interactions

Behavior of MAU's



A family of possible trade-off behaviors

SSPARC Multi-Attribute Utility Function*

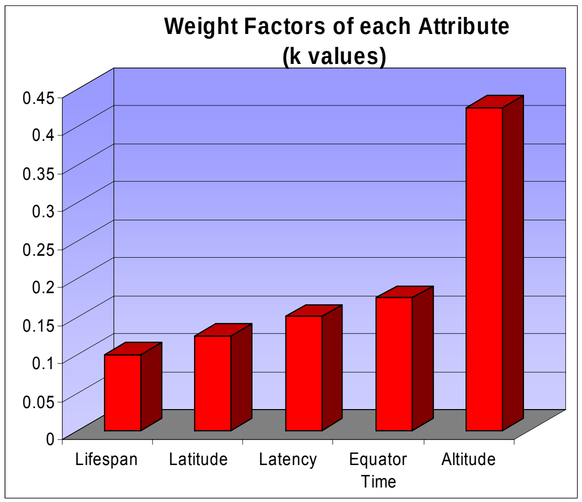


Single attribute utility

$$KU(\underline{X})+1=\prod_{i=1}^{N}\left(Kk_iU(X_i)+1\right)$$

Normalization constant

Relative "weight"



*Keeney & Raiffa, 1976.

Prospect Theory

- Questions rational utility theory
- Proposed by psychologists Kahneman and Tversky in 1979
- Incorporates how people *actually* make decisions
  - Comparisons
  - Biases
  - Heuristics

$$\max\{E[V(X)]_i\} = \max\left\{\left(\sum_j \pi(P(X_j))V(X_j)\right)_i\right\}$$

Source: *Kahneman, D. and Tversky, A. (1979). "Prospect Theory: An Analysis of Decisions under Risk." Econometrica **47**: 263-291.*

Prospect Value Function

$$V(f) = \sum_{i=-m}^{n} \pi_i \, v(x_i) \qquad v(x) = \begin{cases} x^{\alpha} & , x \geq 0 \\ -\lambda(-x)^{\beta} & , x < 0 \end{cases}$$

Value function — Subjective probability — Value of outcome x_i — Loss aversion constant

- Empirical research has shown $\lambda \approx 2$. Meaning people value losses about twice as much as gains. (Loss aversion)
- Parameters α and β are defined such that people are *risk seeking* in the loss domain and *risk averse* in the gain domain.
- (Note that *x* is defined *relative to some reference level*.)

PT: Subjective Probability

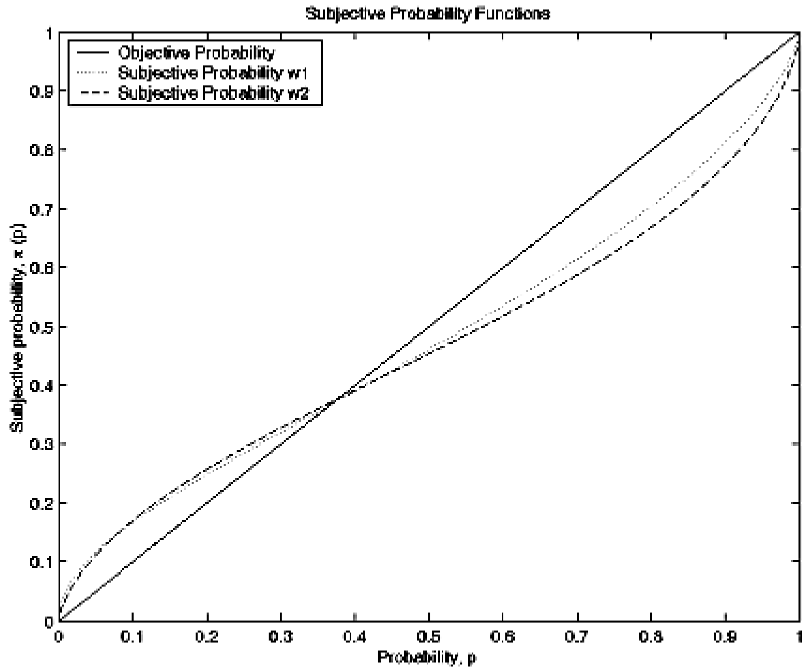
- The probability weighting function, $\pi_i = w(p_i)$, has been estimated to be either:

$$w_1(p) = \exp(-\beta(-\ln p)^{\alpha}) \quad ,(1)$$

- with empirical evidence showing $\alpha = 0.7$ and $\beta = 1$†, or

$$w_2(p) = \frac{p^{\gamma}}{(p^{\gamma} + (1-p)^{\lambda})^{1/\gamma}} \quad ,(2)$$

- with empirical evidence showing $\gamma = 0.61 - 0.69$*.



- Overweight low probabilities: i.e. 0.01% ~ 1%
- Underweight high probabilities: i.e. "certainty bias"

Sources: † *Kahneman, D. and Tversky, A. (1992). Advances in Prospect Theory: Cumulative Representation of Uncertainty. Choices, Values, and Frames. D. Kahneman and A. Tversky. New York, Cambridge University Press: 44-65.*

* *Fox, C. R. and Tversky, A. (1998). A Belief-based Account of Decision under UncertaintyIbid.: 118-142.*

PT: Decision-making Heuristics

- *"Availability"*
  - the tendency of people to weight the probability of an event by the ease with which some relevant information comes to mind; other information, although relevant, is ignored simply because it does not come to mind so quickly.
- *"Representativeness"*
  - the tendency to ignore good probabilistic information on the basis of information that is irrelevant in fact, but that is believed by the decision maker to be representative of relevant information.
- *"Anchoring"*
  - the tendency of many people, even after learning that they have based probability estimates on worthless information, to continue to be influenced by the earlier assessments.

Source: *Kahneman, D., Slovic, P., et al., Eds. (1982). Judgment Under Uncertainty: Heuristics and Biases. New York, Cambridge University Press.*

Utility v. Prospect Theory

- Prescriptive v. Descriptive
- Desire for rational decision-making?
  - Agreement with axioms
  - Better off in “long run”
- Prospect Theory gives *insight* into *actual* decision-making processes to help to *avoid* making *bad decisions*
- Conclusion: use Utility Theory

Multiple Stakeholders

- There is no optimum solution to a problem where multiple stakeholders have conflicting needs
- In practice, need to negotiate
- Knowledge of tradespace greatly aids negotiation
  - Finding the win-win changes (moving toward the pareto front
  - Finding the real trades (on the pareto front)
  - Negotiation advantage for stakeholders who understand tradespace

Summary

Utility theory translates system attributes into user preferences

- Not a perfect system
  - Difficult, to confirm all conditions
  - Noise and biases
- Probably the best available (like democracy…)
- Needs to be used as a tool for tradespace exploration
- Should not be used mechanistically (e.g. as an objective function for MDO)

Users/stakeholders should stay in the loop as tradespace exploration proceeds

Extra Slides

Further Details…

Requirements for Single Attribute Utility

- For a utility function to exist:
  - The user must have a preference for a given value of the attribute over other values of the attribute
  - The preferences must be transitive
  - The preferences must be monotonic
- Single attribute utilities are “ordered metric scales”
  - The utilities have meaning only compared with other utilities
  - The units of utility have constant relative meaning but no absolute meaning
  - A linear transformation of a utility function (e.g. changing from °F to K) does not change the decisions that will be made

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Lottery Equivalent Probability (LEP)

- Set up a questionnaire with hypothetical questions
- User must choose between “lotteries” (uncertain outcomes) e.g. 40% chance of x(1) vs 60% chance of x(2)
- Teases out preference in a (relatively) unbiased way
- Requires:
  - The attributes must have meaning in the presence of uncertainty, i.e. the statement “there is a 10% probability that the system will have attribute x_i” must be meaningful
  - The user must have preference under uncertainty, i.e. must prefer a higher probability of a desirable result over a lower probability
  - The user’s preference must be linear with probability, *at least within the bounds of the problem stated in the LEP interview*

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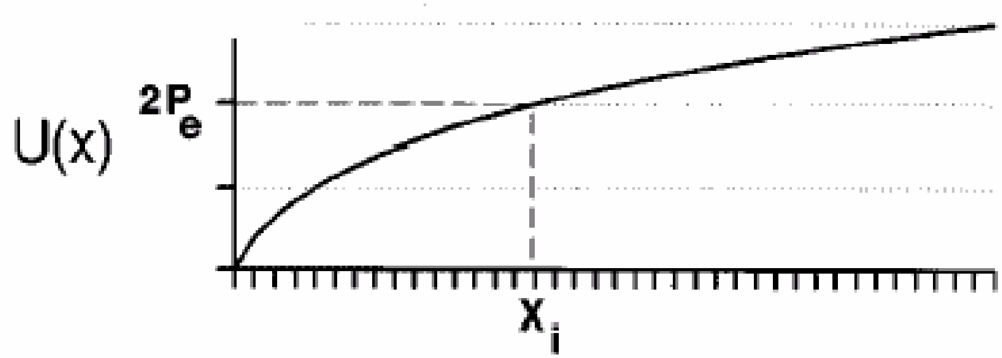
Utility Analysis

- Analysis $(X^*, P_e; X_*) \sim (X_i, P; X_*)$

$$
\begin{aligned}
\Rightarrow\; P_e U(X^*) + (1-P_e) U(X_*) &= P\, U(X_i) + (1-P)\, U(X_*) \\
P_e\, [U(X^*) - U(X_*)] &= P\, [U(X_i) - U(X_*)] \\
P_e &= P\, U(X_i)
\end{aligned}
$$

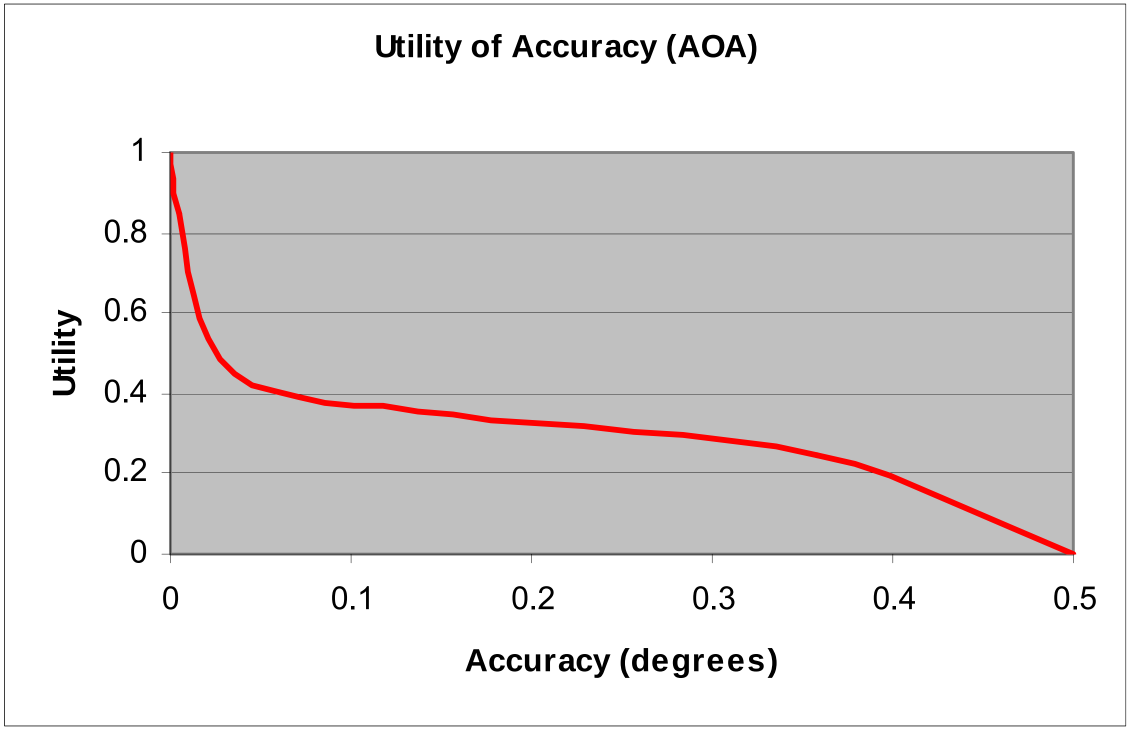
$\Rightarrow$ $U(Xi) = P_e/P$; or $U(Xi) = 2\,P_e$ when $P = 0.5$

- Graph



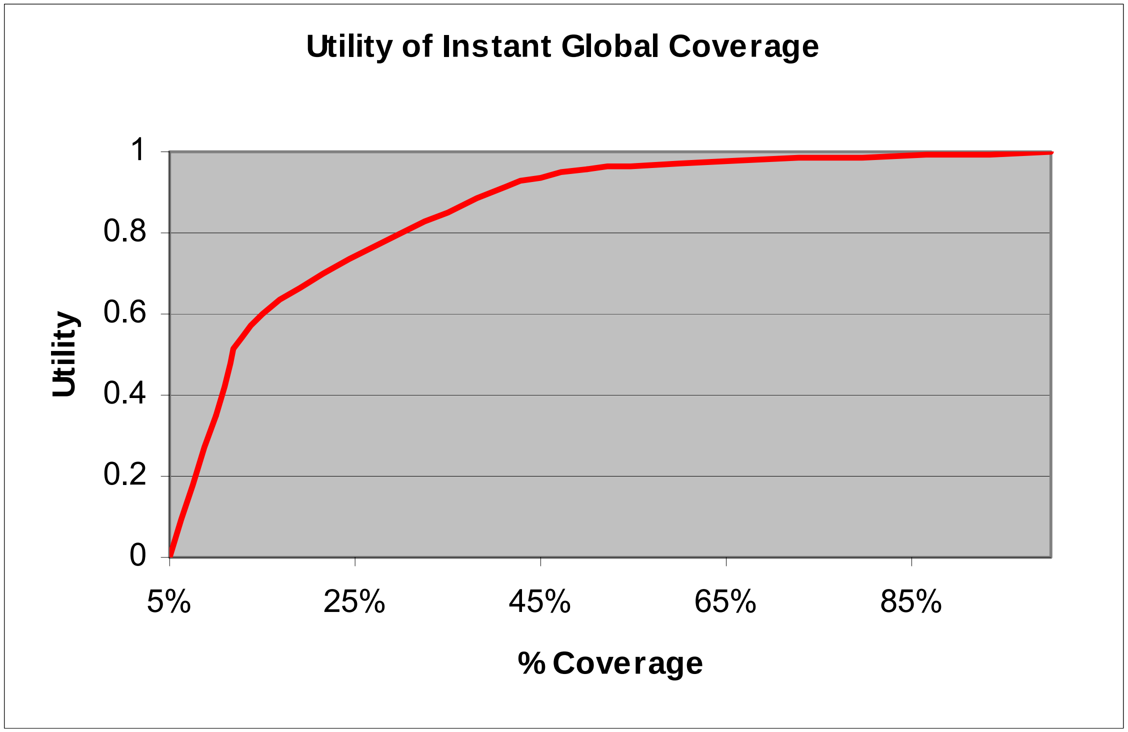
- Big Advantage - Avoids Large Errors (+/- 25% of "Certainty Equivalent" Method)

B-TOS: Accuracy (AOA)



- **AOA accuracy is the range of certainty of the angle of arrival measurement.**

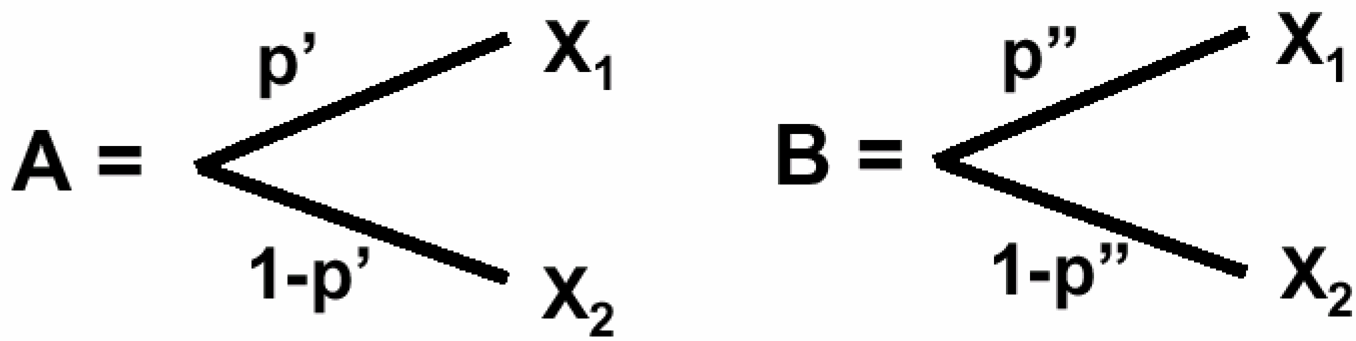
B-TOS: Instant. Global Coverage



- Instantaneous global coverage is the percentage of the globe over which measurements are taken within the time resolution of the system.

U(X): Basic Axioms (1)

- Probability
  - Probabilities exist - can be quantified
  - More is better



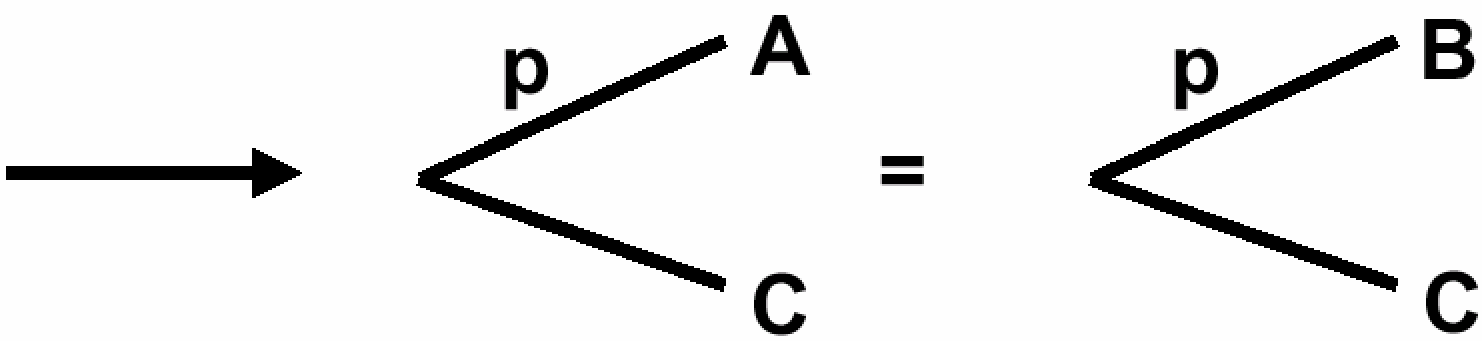
If $X_1 > X_2$; A > B if p’ > p”

is preferred to

U(X): Basic Axioms (2)

- Preferences
  - Linear in Probability

(substitution/independence) - Equals can be substituted if a subject is indifferent between A and B



Not a good assumption for small p (high consequences) !

Interview Issues

- Put person at ease
  - this individual is expert on his values
  - their opinions are valued
  - there are no wrong answers
  - THIS IS NOT A TEST!!
- Scenario relevant to
  - person
  - issues to be evaluated
- Technique for obtaining equivalents:
  - BRACKETING
  - Basic element for measurement: LOTTERIES

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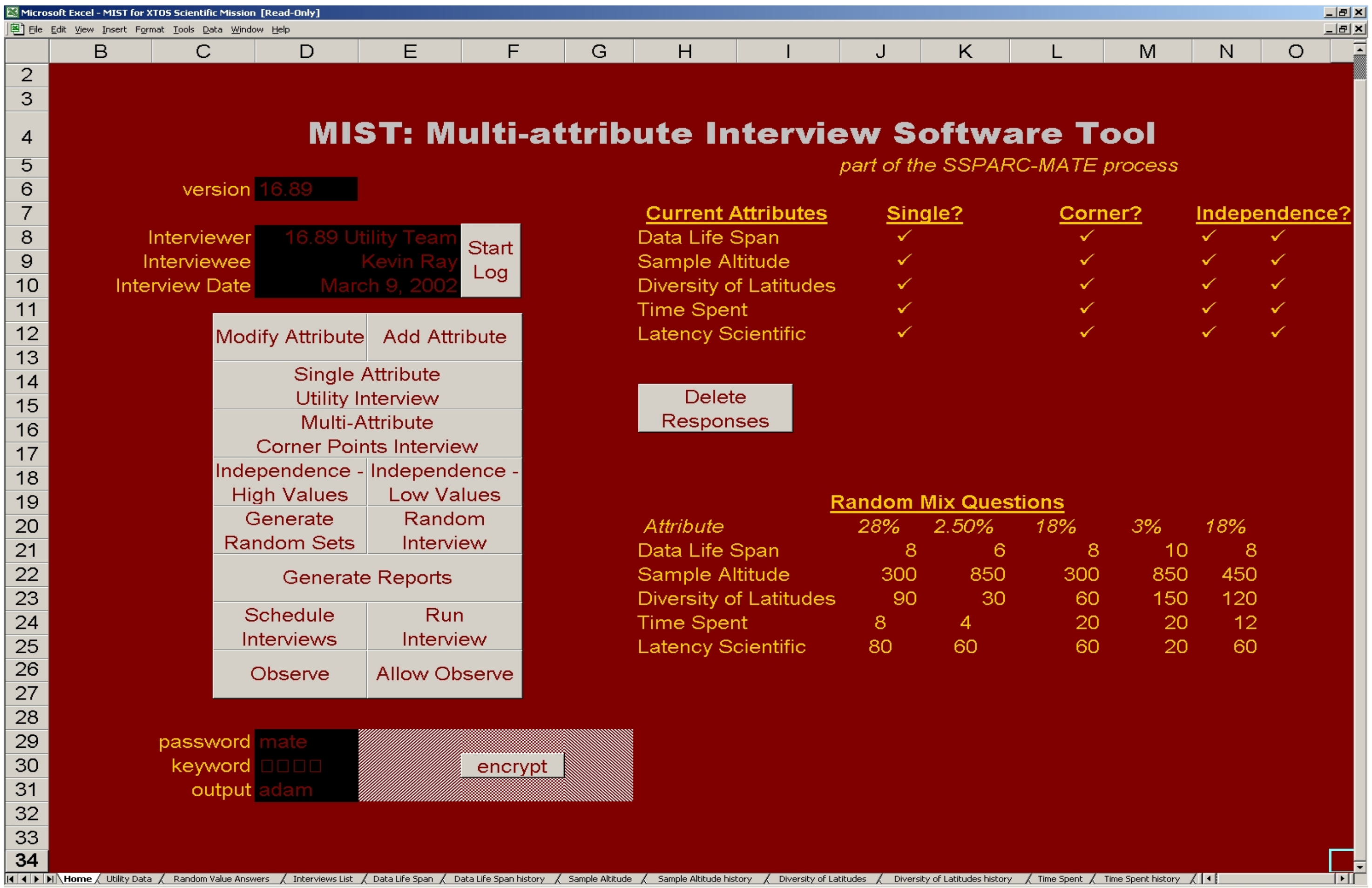
Utility Interview Software (MIST)

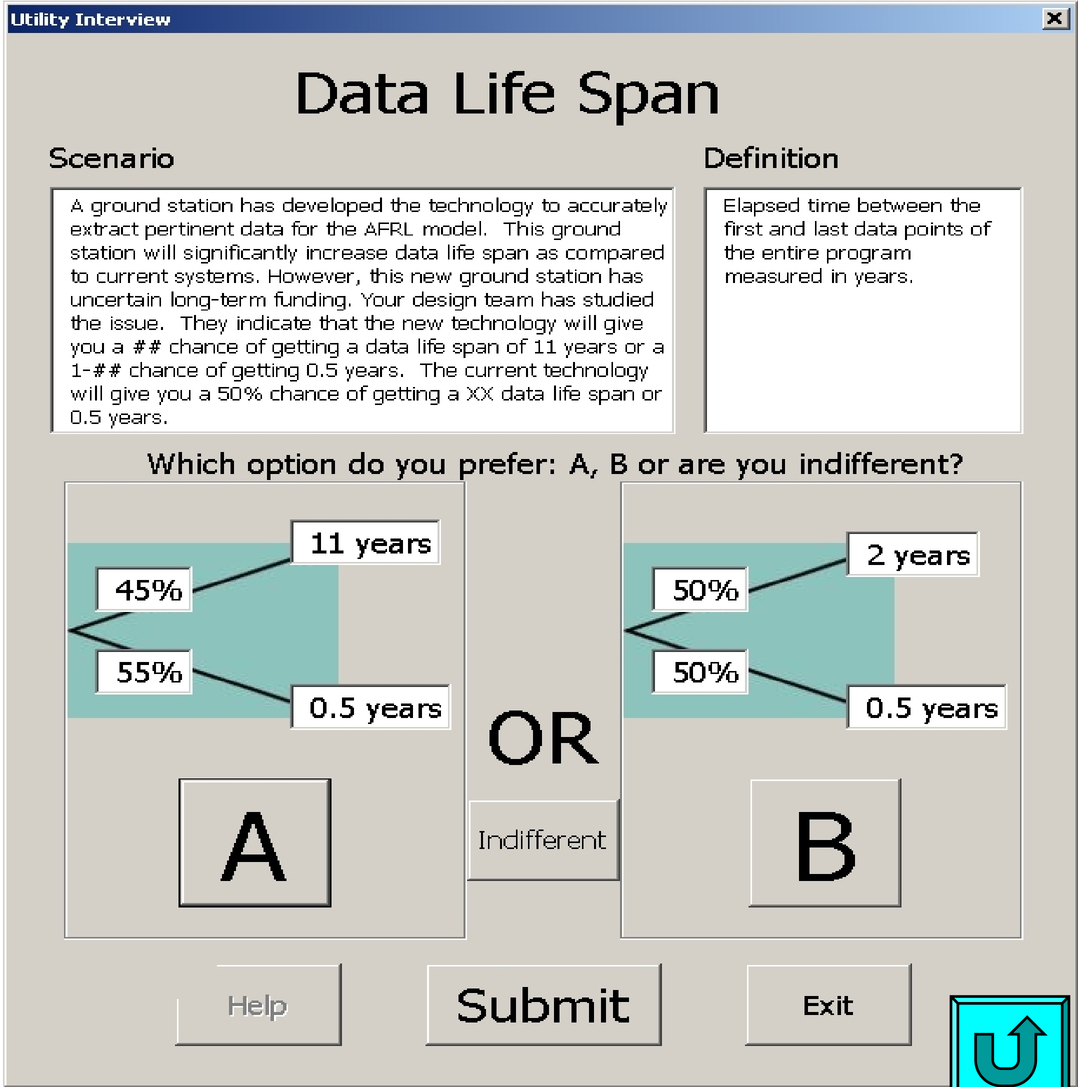


Interviews require interaction with decision makers to determine utility functions

*MIST created by Satwik Seshasai, MIT 2002

Approach to Determining Utility

- Attributes framed by "scenarios"—meant to take each attribute in isolation
- MIST uses "lottery equivalent probability" to create a curve
- User first rates each attribute individually, then balances each against the others



Conditions for MAU

- Weighted sum requires U_i to be fully independent and

$$\sum_{i=1}^{n} k_i = 1$$

- Multiplicative function accounts for simple interaction (e.g. need *all* U_i to approach 1 before U approaches one)
- Inverse multiplicative function accounts for opposite interaction (all U_i need to approach 0 before U approaches 0)
- Keeney-Raiffa accounts for single interaction - depends on K
  – requires that the value of any U_i perform no more than a linear transform on the values of other U_i - which doesn't change the decisions!

These conditions may not be met, and are hard to prove rigorously

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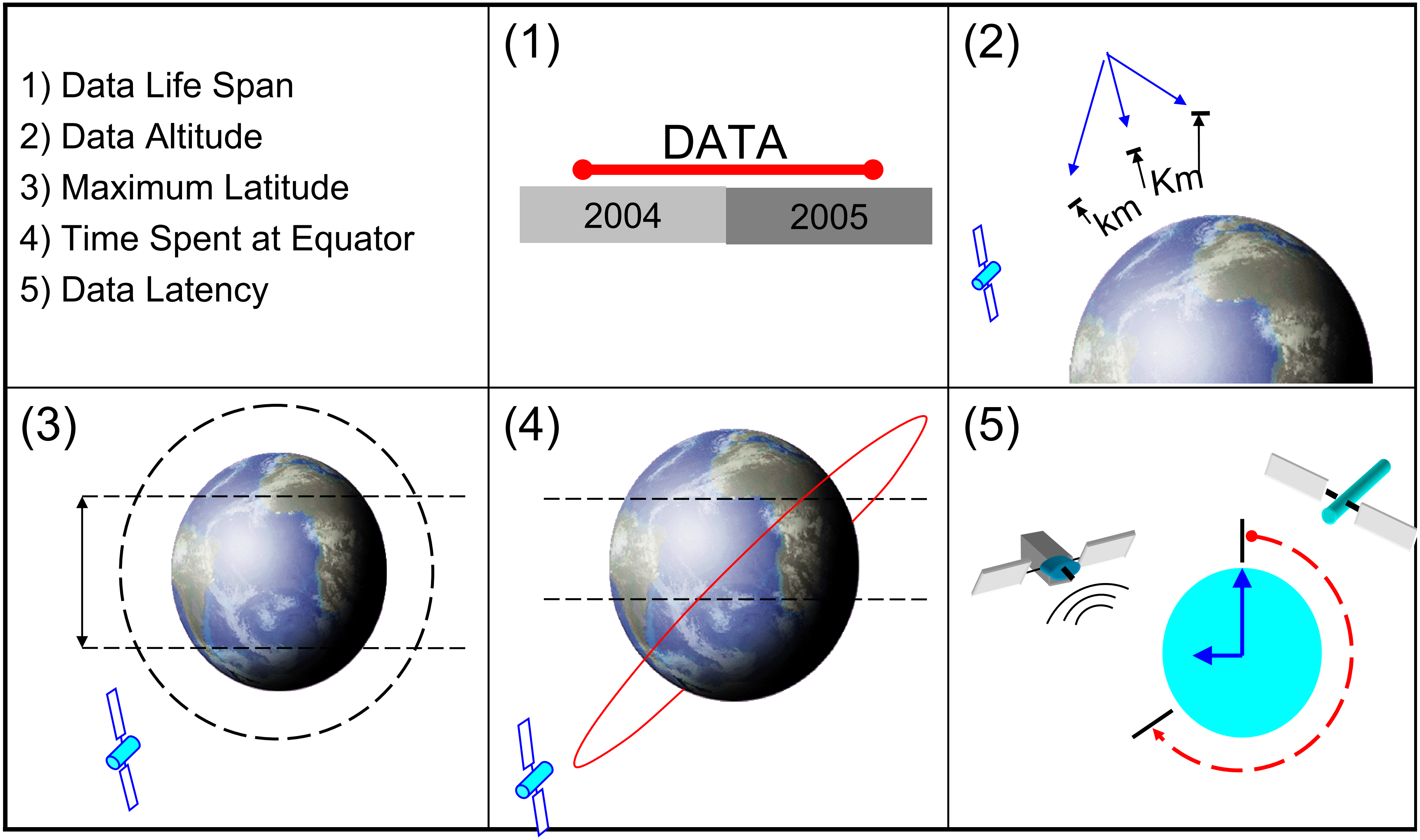
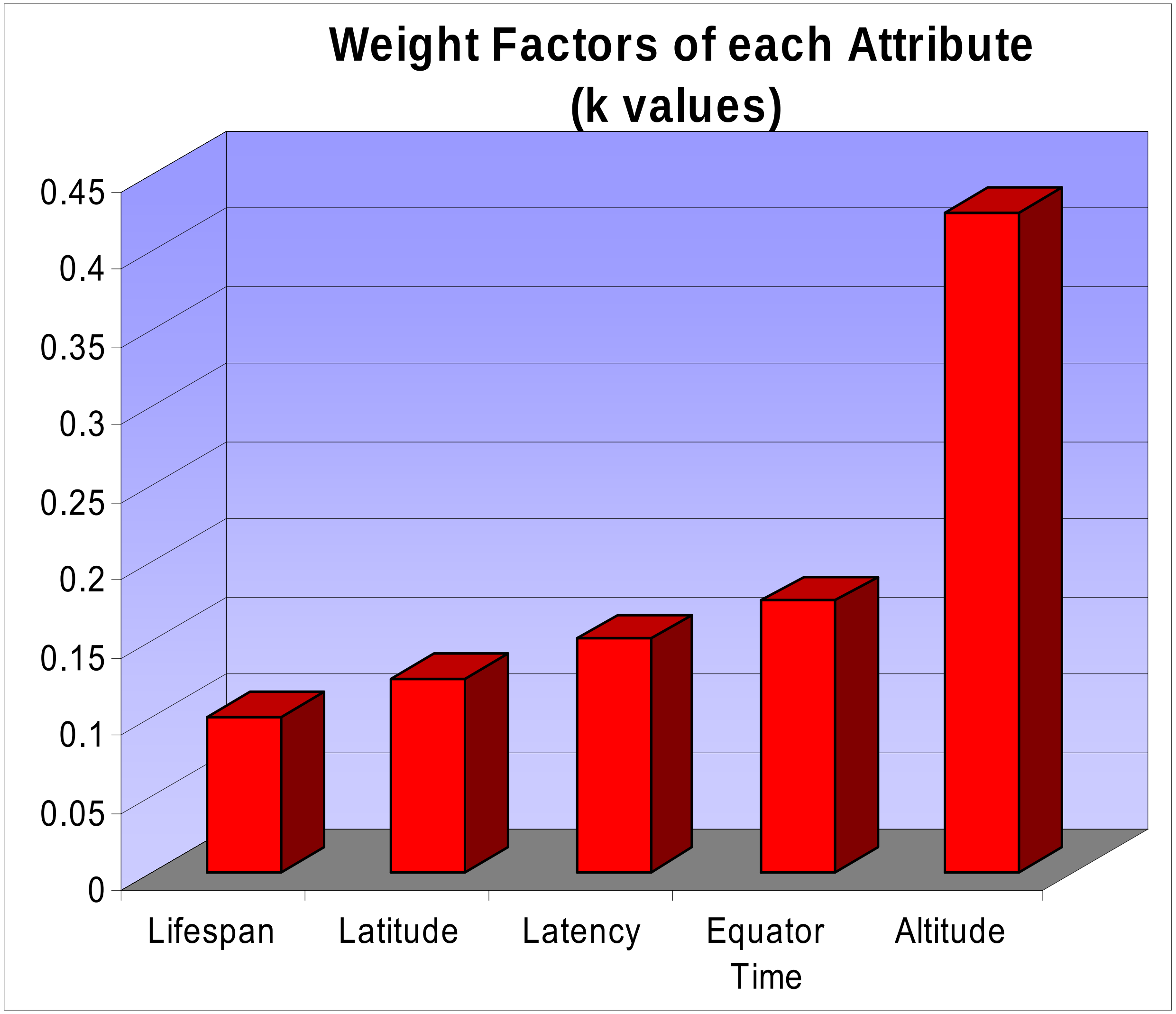
X-TOS User Attributes

1) Data Life Span
2) Data Altitude
3) Maximum Latitude
4) Time Spent at Equator
5) Data Latency



Single Attributes Aggregated

- Depicts the relative importance of each attribute to the decision maker
- Resolution of ±0.025
- User interviewed for ~2 hours



16.892/ESD.353 Space Systems Architecture and Design

MATE

Utility Theory and Assessment

Adam M. Ross

October 4, 2004