Applications:
Bargaining Model of War
Why are there wars?

The next few weeks of class will propose many explanations.

This is not a settled question, but theories can still be useful.

- Today: bargaining failures because of indivisible resources, uncertainty, shifting power.
- Later: Domestic political incentives.
- Later: Differences between democracies and autocracies
- Later: Leader psychology
Why are there wars?

A simple bargaining model shows why war is more surprising than our intuition suggests.

This approach takes (possibly) Realist assumptions and applies game theory. We’ll identify three “rationalist” causes of war:

- Indivisible Goods
- Uncertainty about costs of war
- Shifting power

Basics of the model

Assume that states are engaged in zero-sum bargaining to divide territory.

Simplification:

- Actors: two states
- Interests: to get maximum territory
- Interaction: sequence of proposals followed by a lottery
- Institutions: rules about bargaining

This is not fully realistic, but that’s not the point.
Basics of the model

Utility/Payoff: Amount of satisfaction received from a specified outcome

Expected Utility: Average amount of utility from each possible outcome weighted by that outcome’s probability of occurring

Discount Factor: Present value of utility received in the next period (i.e., today’s value of future payoffs)
Basics of the model

The model treats war as a lottery.

Two (mathematically identical) interpretations of war outcomes

1. the probability a state gains the entire resource
2. the proportion of the resource the state gets

Equivalent in terms of expected utility
Basics of the model

War isn't free:

1. It costs money to fight. Troops die, airplanes destroyed, people get injured.

2. Also destroys the resource that states are battling over!
Basics of the model

Two states, A and B.

Bargaining over a resource worth one “unit” (this just makes the math easier).

- Power. Expected amount of resource an actor gets following war: \( p \).

- Costs of war: \( c_A, c_B > 0 \)

Bargaining protocol:

1. A makes a demand to B, \( x \), giving B \( 1 - x \).
2. B decides to accept or reject the demand.
Basics of the model

Resource (land), Power (cannons), and Costs (destruction)

Demand $x$ is a proposal from Blue on how to divide
Basics of the model

Consider a demand $x$. Expected utility to $B$ of accepting: $1 - x$

Expected utility to $B$ of rejecting: $(1 - p) \times 1 - c_B$

Should $B$ accept?

- Accept if $1 - x \geq 1 - p - c_B$
- Accept if $p + c_B \geq x$

$B$’s “decision-rule”: **Reject** if $p + c_B < x$
What should A demand in order to maximize its utility?

Consider A’s expected utility of war.

\[ p - c_A \]

Thus A prefers having an accepted demand over war that the demand gives them more than the expected utility of war:

\[ x \geq p - c_A \]
Basics of the model

A prefers having a demand $x$ accepted to war if $x \geq p - c_A$.

B prefers war to accepting a demand $x$ to war if $p + c_B < x$

Key question: Are there values of $x$ such that both prefer peace to war?
Answer: YES if the costs of war are positive.

$\therefore x \in p - c_A, p + c_B$

This “range” is called the bargaining range.
Basics of the model
Basics of the model

But this analysis suggests that bargaining should just follow the distribution of power, and this should be peaceful.

But we know this is not right, because wars happen.
The bargaining range is finite. In our model, it is \( x \in p - cA, p + cB \).

If the thing states are fighting over is:
1) worth more than the bargaining range
2) indivisible
then war results.

What are some examples in IR?

**Indivisible Goods**

Model set-up

<table>
<thead>
<tr>
<th>Indivisible Goods</th>
<th>Uncertainty</th>
<th>Shifting Power</th>
<th>Appendix: Fuller derivation for Shifting Power</th>
</tr>
</thead>
</table>

15
Indivisible Goods

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Indivisible Goods

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What are some examples in IR?
Uncertainty about costs of war
Uncertainty about costs of war

What if the costs of war \((c_A, c_B)\) of the other state not known.
Uncertainty about costs of war

Size of costs affects the size of the bargaining range
Uncertainty about costs of war

Bargaining range size affects payoffs

Low Costs

High Costs
Uncertainty about costs of war

Because of this payoff variation, states have incentives to lie (under-report) their costs of war

→ incentive to misrepresent size of your costs
Uncertainty about costs of war

Incomplete information plus incentives to misrepresent can foster war

if 🙆‍♂️ thinks 🙆‍♂️ has High Costs → 🙆‍♂️ makes ungenerous proposal
Uncertainty about costs of war

Incomplete information plus incentives to misrepresent can foster war

but if actually has Low Costs prefers war
Is credible communication possible?

Incomplete information can generate war because states cannot credibly communicate their costs of war.

- **Cheap talk:** No costs to bluffing about my costs
Is credible communication possible?

Costly Signaling: Communication can succeed if talk isn’t cheap. If signals carry costs, then they can credibly reveal information.

Examples

- **Sinking costs**: Moving an army into position to attack takes resources and is costly. This can reveal a state’s resolve.

- **Tying hands**: Leader promises citizens that she will not back down to a challenger. Backing down will now carry costs for her (re-election).
Is credible communication possible?

Sufficiently costly signals can separate different types of states.

Only highly resolved states will send these very costly signals.

Less costly signals cannot separate high resolve states from low resolve states.
Logic:
I Rising states cannot credibly commit to uphold bargains
I Declining states fear future unfavorable bargains
I For a severe enough power shift, a declining state does best by fighting when still at its strongest

Model set-up
Indivisible Goods
Uncertainty
Shifting Power
Appendix: Fuller derivation for Shifting Power

Shifting Power
Shifting Power

Logic:

- Rising states cannot *credibly commit* to uphold bargains
- Declining states fear future unfavorable bargains
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Multi-round bargaining

Lets use the same basic bargaining model as before but add:

- Multiple rounds of bargaining \((x_{1A}, x_{2A})\)
- Shifting power \((p_1, p_2)\)
Bargaining occurs in two rounds.

In the first period player A makes a demand, $x_{1A}$, to player B. This leaves player B with $1 - x_{1A}$. 

Proposal

You take

I get

$x_{1A}$

$1 - x_{1A}$
If the first period demand is accepted, then in the second period there is another bargaining stage, where player A makes a demand $x_{2A}$. 
War in period 1 or 2?

If the first period demand is rejected then war occurs.

Player A gets $p_1$ and player B gets $1 - p_1$.
But both players pay costs $c_A = c_B$.

If the offer is rejected in the second period, both players pay the cost and A gets $p_2$ and B gets $1 - p_2$. 
Backwards Induction

Assume that states “look ahead” into the future. If Player B expects a large unfavorable power shift (and thus small payoff) in period 2:

Then B starts war even with the most generous possible first round offer:

Player A can not offer (demand little) enough to make player B accept the demand in light of what they expect to get in period 2.
Shifting Power

Reviewing the logic:

- Rising states cannot *credibly commit* to uphold bargains
- Declining states fear future unfavorable bargains
- For a severe enough power shift, a declining state does best by fighting when still at its strongest

Lots of things like this. “Time inconsistency problems” in

- saving money
- working out
- party platform vs. policy
- dating and marriage?
War is inefficient, yet we observe war in the world. Why?

- Indivisible Goods
- Uncertainty about costs of war
- Shifting power
Shifting Power: more detailed derivation
Shifting Power: more detailed derivation

Logic:

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Backwards Induction

Assume that states “look ahead” into the future, and condition their current decisions on what they think will happen in the future.

Last decision by actor B. In Period 2 player B accepts demand $x_{2A}$ (rather than start war)

$$\underbrace{1 - p_2 - c_B}_{\text{Expected Payoff from Fighting}} \leq \underbrace{1 - x_{2A}}_{\text{Payoff from Proposal}}.$$

Thus if $x_{2A} \leq p_2 + c_B$ then actor B accepts the demand.
In period 2 player A’s expected utility from war is $p_2 - c_A$. Hence A will prefer to make a demand $x_{2A} = p_2 + c_B$ if

$$\underbrace{p_2 + c_B}_\text{Payoff from Proposal} > \underbrace{p_2 - c_A}_\text{Expected Payoff from Fighting}.$$

This holds because we assume $c_B, c_A \geq 0$.

They can’t make more than this demand because then it will be rejected.

So, the optimal demand in the second period is

$$x_{2A}^* = p_2 + c_B.$$
Optimal demand in the second period is $x_{2A}^* = p_2 + c_B$. This leaves $B$ with $1 - x_{2A}^*$, equal to its expected payoff for war.
Now consider period 1. B’s utility from rejecting the proposal in period 1 is

\[
(1 - p_1) - c_B + \delta(1 - p_1),
\]

where \(\delta\) represents the discount rate for the second period, or the likelihood that the second period is played.

We will assume that \(\delta = 1\) for simplicity.
Player B’s utility from accepting a demand $x_{1A}$ is equal to

$$1 - x_{1A} + \delta(1 - p_2 - c_B).$$
Payoff:

Round 1

Proposal
You take I get

Round 2

Proposal
You take I get

WAR
Thus player B will reject the demand $x_{1A}$ if

$$\frac{(1 - p_1) - c_B + \delta(1 - p_1)}{1 - x_{1A} + \delta(1 - p_2 - c_B)} > \frac{1 - x_{1A} + \delta(1 - p_2 - c_B)}{p_1 + \delta p_1 - \delta p_2 + c_B - \delta c_B}$$

Hence they will be indifferent if $x_{1A} = p_1 + \delta p_1 - \delta p_2 + c_B - \delta c_B$.

$x_{1A} = 2p_1 - p_2$ for $\delta = 1$
Now consider A’s expected utility in period 1. If they have the proposal rejected their expected utility is

$$ p_1 - c_A + \delta p_1. $$

Round 1

If they have some demand $x_{1A}$ accepted then they get

$$ x_{1A} + \delta(p_2 + c_B). $$

Round 1
A will want their first period demand accepted if

\[
\begin{align*}
\text{Expected Payoff from Accepting} & \geq \text{Expected Payoff from War} \\
\left( x_{1A} + \delta(p_2 + c_B) \right) & \geq (p_1 - c_A + \delta p_1) \\
x_{1A} & \geq p_1 + \delta p_1 - \delta p_2 - c_A - \delta c_B \\
x_{1A} & \geq 2p_1 - p_2 - c_A - c_B; \text{ for } \delta = 1
\end{align*}
\]

Now note that the right hand side of this is almost identical to what will make B indifferent, except that it is slightly smaller.

Hence A will make demand:

- \( x_{1A}^* = p_1 + \delta p_1 - \delta p_2 + c_B - \delta c_B \)
- \( x_{1A}^* = 2p_1 - p_2; \text{ for } \delta = 1 \)
- \( x_{2A}^* = p_2 + c_B \)
First period demands will always be rejected (preventive war) when $x^*_1A$ is less than 0. (when $p_2 > 2p_1$)

→ When player A can not offer (demand little) enough to make player B accept the demand in light of what they expect to get in period 2.

If Player B expects a large unfavorable power shift (and thus small payoff) in period 2:

Then B starts war even with the most generous possible first round offer: