

17.810/17.811 – Game Theory

Lecture 7: Dynamic Games of Incomplete Information

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Where We Are/Where We Are Headed

- We now complete our survey of the field with **dynamic games of incomplete information**.
 - This will require introducing our last equilibrium concept, **Perfect Bayesian Equilibrium**, which gives us a similar notion to subgame perfection in a setting where we may not have proper subgames
 - This equilibrium will build on two new concepts, **sequential rationality** and **weak consistency** of beliefs
- Then we will walk through some examples of **signaling games**.

These slides will focus on the following readings:

- Dynamic Games of Incomplete Information
 - Gibbons, 4.1, 4.2, and 4.3A

Dynamic Games of Incomplete Information

Motivation: Escalation Game

Perfect Bayesian Equilibria

Sequential Rationality

Weak Consistency of Beliefs

Signaling Games

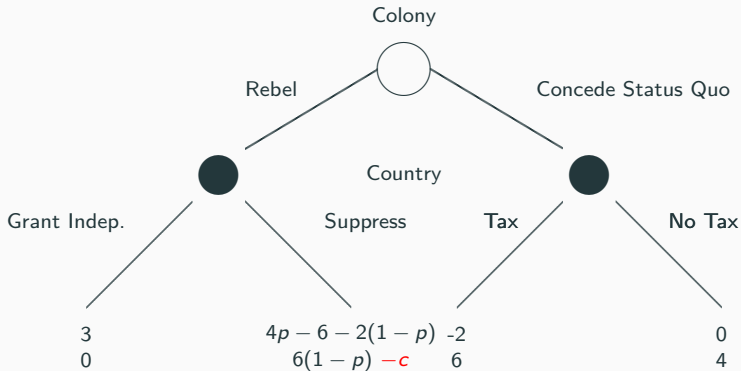
Example 1: Beer-Quiche Game

Example 2: Advice from Experts

Dynamic Games of Incomplete Information

- Earlier we learned that uncertainty about the preferences of other players fundamentally alters the strategic situation in static normal form games
- In dynamic multi-stage games, uncertainty leads to even more interesting strategic possibilities
 - However, we need to strengthen our equilibrium concept to rule out odd equilibria in these richer games Perfect Bayesian Equilibrium (PBE)
 - PBE strengthens SPE by extending the equilibrium concept to non-singleton information sets
- To motivate our analysis, let's reconsider the revolution game
 - Recall that the unique subgame perfect equilibrium involved a revolt by the colony and the grant of independence, $(R, (G, T))$

Modified Revolution Game



- Before, with $c = 6$, we had a unique SPE at $(R, (G, T))$
- Now suppose $c = 0$. The unique SPE is now $(C, (S, T))$
- How should *Colony* behave if it's not sure what game it's playing? ($c = 0$ vs. $c = 6$)

Dynamic Games of Incomplete Information

- We again model uncertainty about the game we're playing using the **Harsanyi maneuver**:
 - A game of **incomplete information** (uncertainty about payoffs)
→ a game of **imperfect information** (uncertainty about node of a game where Nature took the first move)
 - Nature randomly selects players' types from a known probability distribution. Not all players, however, observe the realization of Nature's draw.
 - To model a situation in which player i does not know player j 's preferences, we assume that Nature chooses player j 's payoffs (type) prior to agent i 's decision, and we model player i as facing an information set with multiple nodes because she does not observe the initial choice by Nature

An Example

- Consider a model of conflict between two nations.
- Country A first chooses whether or not to initiate a conflict.
 - If no conflict is initiated, the game ends.
 - On the other hand, if A initiates conflict then nation B decides whether to acquiesce or escalate.
- Suppose that the payoffs from this interaction can be given by either **Game I** or **Game II**, in which the escalation costs are different.

An Example

Table 1: Game I

If A does not initiate $(0, 0)$

If A initiates and B acquiesces, $(4, -4)$

If A initiates and B escalates $(-8, -8)$

Table 2: Game II

If A does not initiate $(0, 0)$

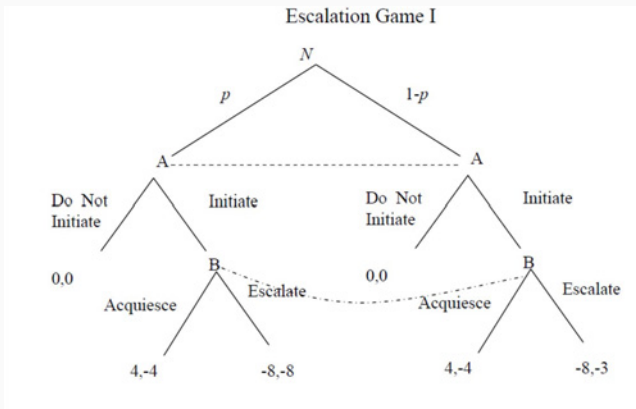
If A initiates and B acquiesces, $(4, -4)$

If A initiates and B escalates $(-8, -3)$

- Suppose that **Game I** is played with probability p and **Game II** is played with probability $1 - p$.
- Several distinct information structures are possible! We will consider three of them.

Escalation Game I

Suppose first that neither country observes Nature's move, as in the following figure:



Equilibrium

- Information is imperfect but symmetric in that both players find themselves in the same situation
- We only need to compute country B 's expected utility of escalation and modify the game accordingly
- B 's expected utility of escalation is:

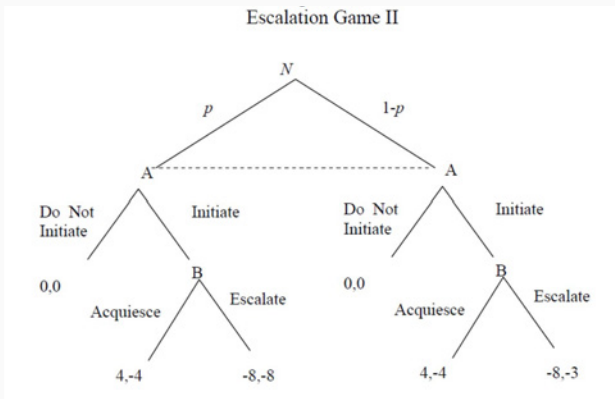
$$-8p - 3(1 - p) = -3 - 5p$$

So it prefers to escalate when $-3 - 5p \geq -4$, or $p \leq \frac{1}{5}$

- Thus, if $p \leq \frac{1}{5}$, the outcome is $\{Do\ Not\ Initiate, Escalate\}$ otherwise it is $\{Initiate, Acquiesce\}$

Escalation Game II

Now suppose that only B observes Nature's move, as in the following:



Escalation Game II

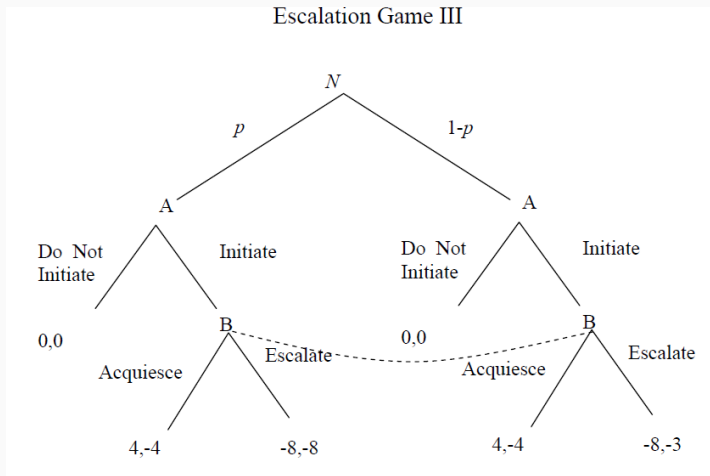
- This information structure means that A is uncertain of B 's type.
- B acquiesces in Game I (left) and escalates in Game II (right)
- Thus A 's expected utility from initiating is:

$$4p - 8(1 - p) = -8 + 12p$$

- A prefers initiating if $-8 + 12p \geq 0 \rightarrow$ if $p \geq \frac{2}{3}$

Escalation Game III

Finally, suppose that only A observes Nature's move:



Escalation Game III

- Now the first mover has more information than the second mover $\rightarrow A$ must consider what information her choices convey to B
- To illustrate, consider a natural way of playing the game where A initiates in **Game I**, but not in **Game II**
- If A plays these strategies, B infers from A 's initiation that they are playing **Game I** and acquiesces. If B responds in this way (whenever she sees initiation), A has an incentive to initiate in **Game II** to get 4 instead of 0.

Strategic Use of Information

We focus on the strategic use of information in dynamic settings. Incomplete information raises a number of important issues.

- *Strategic Use of Information*: Do any of the players have a strategic advantage based on how information is allocated? In many games, informed players have important advantages. But sometimes the uninformed player is advantaged – ignorance can be bliss!
- *Learning*: Can the uninformed players get more information from observing the actions of the informed players? How do these possibilities affect the strategies of the informed players?
- *Signaling*: Can the informed players credibly communicate information about the game to the uninformed players? Can informed players mislead uninformed players?

Dynamic Games of Incomplete Information

Motivation: Escalation Game

Perfect Bayesian Equilibria

Sequential Rationality

Weak Consistency of Beliefs

Signaling Games

Example 1: Beer-Quiche Game

Example 2: Advice from Experts

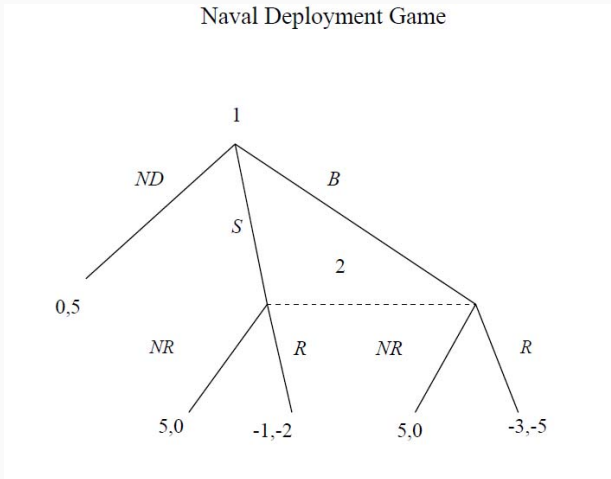
Perfect Bayesian Equilibria

- Although subgame perfection rules out some unreasonable Nash equilibria, many extensive form games with imperfectly observed actions require a stronger equilibrium concept. Consider the following example.
- Player 1 chooses whether to secretly deploy military capability to attack Player 2's island. She chooses between a small fleet of ships (S), a big fleet of ships (B), or not to deploy any ships (ND).
- Player 2 only observes whether there was a deployment. Lookouts, relying only on telescopes, can see the ships coming but cannot determine how many are coming.

Perfect Bayesian Equilibria

- If there is no deployment Player 2 keeps the island and the payoffs are $(0, 5)$.
- If there is a deployment, Player 2 decides whether to respond to the attack (R).
- If there is no response (NR) Player 1 wins the island.
- If there is a response, Player 2 gets to keep the island but the casualties for Player 2 are much higher under B than under S . The casualties for Player 1 are also higher under B than under S .

Naval Deployment I



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Perfect Bayesian Equilibria

- There are three Nash equilibria to this game.
 - (ND, R) where Player 1 does not deploy, but if she did Player 2 would respond.
 - (B, NR) where Player 1 deploys a big line of ships, and Player 2 does not respond.
 - (S, NR) where Player 1 deploys a small line of ships, and Player 2 does not respond.
- There is something odd about the first Nash equilibrium.
 - Regardless of whether B or S is played, Player 2 is better off playing NR . Shouldn't Player 1 recognize this and attack?
 - In the past we used subgame perfection to eliminate similar problems, but subgame perfection fails in this case.
 - Why? Because this game has **no proper subgames**, so all Nash equilibria are trivially subgame perfect.

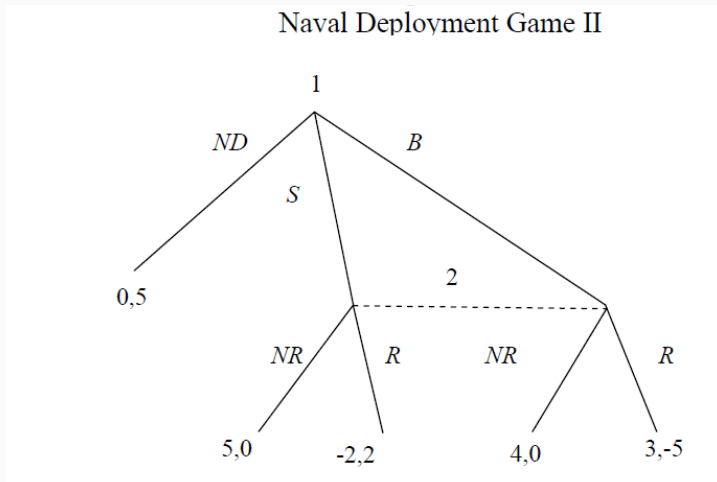
Perfect Bayesian Equilibria

- This profile is unreasonable because Player 1 should anticipate a rational response from Player 2 at Player 2's information set. The threat of R is not credible.
- The goal is to incorporate this type of sequential rationality into an equilibrium concept.
- So we require that agents form beliefs about the history reached at each information set and select best responses given these beliefs.
- These equilibria are called Perfect Bayesian Equilibria (or PBE).
- The problem of having no proper subgames can be “solved” by introducing beliefs.

Perfect Bayesian Equilibria

- Returning to the example:
 - No belief about the history of play at Player 2's information set justifies the selection of R as a best response: whether S or B was played, Player 2 is better off choosing NR .
- The previous example is somewhat trivial, so now consider a more interesting example.
 - Suppose that Player 1 wins the island only if she selects B .
 - Moreover, Player 2 prefers to defend the island if Player 1 has selected S .

Naval Deployment II



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- In this version, whether R or NR is sequentially rational depends on what beliefs Player 2 assigns to the two possible histories in her information set.
 - If she believes that S was played then R is sequentially rational.
 - Conversely if she believes that B was played then NR is sequentially rational.
- What should she believe?
 - Clearly, her beliefs are based on expectations about what Player 1 does.
 - But Player 1's choice depends on what she expects Player 2 to believe.

This brings us to our first requirement for PBE:

Definition (Requirement 1)

At each information set, the player with the move must have a **belief** about which node in the information set has been reached by the play of the game. For a nonsingleton information set, a belief is a probability distribution over the nodes in the information set; for a singleton information set, the player's belief puts probability 1 on the single decision node.

Beliefs

- In the example above, a belief on Player 2's information set is a probability distribution over $\{S, B\}$.
- A **belief profile** describes a complete list of beliefs for all information sets.
- Because only one player makes a decision at each information set, there is no ambiguity about whose beliefs are relevant on each portion of the belief profile: the player making the choice at that time.

Dynamic Games of Incomplete Information

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Example 2: Advice from Experts

Sequential Rationality

Our second requirement states that all strategies must be optimal at each information set **given a specific belief profile**.

Definition (Requirement 2)

Given their beliefs, the players' strategies must be **sequentially rational**. That is, at each information set the action taken by the player with the move (and the player's subsequent strategy) must be optimal given the player's belief at that information set and the other players' subsequent strategies (where a "subsequent strategy" is a complete plan of action covering every contingency that might arise after the given information set has been reached).

Returning to the example above, if the beliefs assign a probability close to 1 on S then R is sequentially rational at the information set. Similarly, if Player 2 believes B is sufficiently likely then NR is the sequentially rational response.

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Weak Consistency of Beliefs

We now place some reasonable restrictions on players' beliefs. But first, a reminder:

Definition

For a given equilibrium in a given extensive-form game, an information set is **on the equilibrium path** if it will be reached with positive probability if the game is played according to the equilibrium strategies, and is **off the equilibrium path** if it is certain not to be reached if the game is played according to the equilibrium strategies.

Weak Consistency of Beliefs

Weak consistency consists of two further requirements:

Definition (Requirement 3)

At information sets **on the equilibrium path**, beliefs are determined by Bayes' rule and the players' equilibrium strategies.

Definition (Requirement 4)

At information sets **off the equilibrium path**, beliefs are determined by Bayes' rule and the players' equilibrium strategies **where possible**.

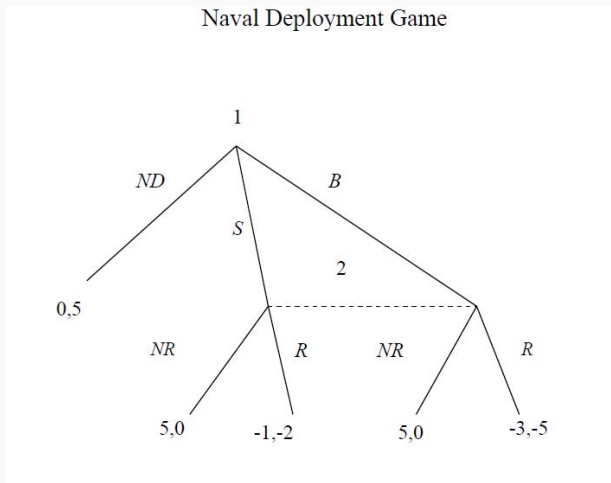
Perfect Bayesian Equilibrium

Definition (Perfect Bayesian Equilibrium)

A **perfect Bayesian equilibrium** consists of strategies and beliefs satisfying Requirements 1 through 4.

PBE for Naval Deployment I

Returning to Naval Deployment I, we now characterize PBE.

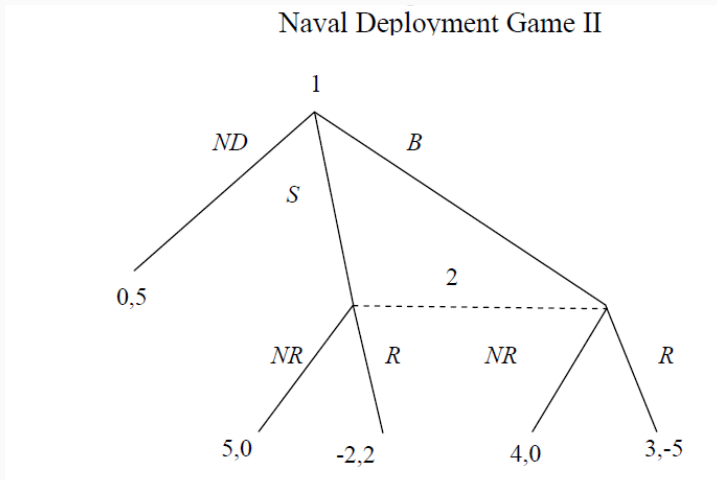


- The Nash equilibrium (ND, R) is not a PBE.
 - For any beliefs about whether Player 1 choose S or B , NR is the unique sequentially rational response.
 - Given that Player 2 chooses NR , Player 1's optimal choice is to play either S or B .

- Now if Player 1 chooses B then weakly consistent beliefs assign probability 1 to Player 2 moving at history B .
 - Thus, one PBE is $\{(B, NR), \Pr(B \mid \neg ND) = 1\}$ where $\Pr(B \mid \neg ND)$ is the posterior probability of B given that Player 1 did not choose ND (i.e., conditional on reaching the information set).
 - Similarly there exists a PBE where $\{(S, NR)$ and $\Pr(B \mid \neg ND) = 0\}$.

PBE for Naval Deployment II

Now consider PBE for Naval Deployment II.



PBE for Naval Deployment II

- If Player 2 believes that $\Pr(B \mid \neg ND) = 1$, NR is the best response.
- On the other hand if Player 2 believes that $\Pr(B \mid \neg ND) = 0$ then R is the best response.
- One candidate PBE is (ND, R) and $\Pr(B \mid \neg ND) = 0$.
 - Does it satisfy **weak consistency**? Yes. Weak consistency does not impose any constraints on beliefs when Player 1 plays ND , so the belief $\Pr(B \mid \neg ND) = 0$ is weakly consistent relative to the strategy ND .
 - Does it satisfy **sequential rationality**? No. Player 1 prefers to play B rather than ND when she conjectures that Player 2 plays R .

PBE for Naval Deployment II

- Now suppose there is a pure strategy PBE in which ND is not played.
 - If B is played and beliefs are weakly consistent, the only sequentially rational strategy by Player 2 involves NR .
 - But if Player 1 conjectures that Player 2 is playing NR she wants to play S .
 - So B cannot be part of a pure strategy PBE profile.
 - On the other hand, if Player 1 chooses S , weakly consistent beliefs assign probability 1 to Player 2 moving at history S .
 - Thus, the only sequentially rational action involves playing R .
 - But if Player 1 conjectures that Player 2 is playing R then she wants to play B .
 - Thus, there are no pure strategy PBE in which S is played.
 - Consequently, there are no pure strategy PBE.
- There are, however, mixed strategy PBE.

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Signaling Games

- An important class of games of imperfect information involves interaction between a more informed agent, the **sender**, and a less informed agent, the **receiver**.
- When the sender moves first the game is called a **signaling game**.
- These games take their name from the possibility that the sender's action conveys information about her type to the receiver.
- We focus on the simplest possible signaling game to demonstrate the incentives agents face and to emphasize the various types of equilibria that might exist.

Signaling Games

- Two players: sender and receiver
- First, nature choose a type t_i from the type space $T_i \subseteq \mathbb{R}^n$ with prior probability $p(t_i)$
- Sender observes her type and then chooses a message or action $m \in M$
- Receiver observes m but not the sender's type and then chooses some action $a \in A = \{a_1, a_2, \dots, a_k\}$.
- The payoff to the sender is

$$u_s(t_i, m_j, a_k)$$

- The payoff to the receiver is

$$u_r(t_i, m_j, a_k).$$

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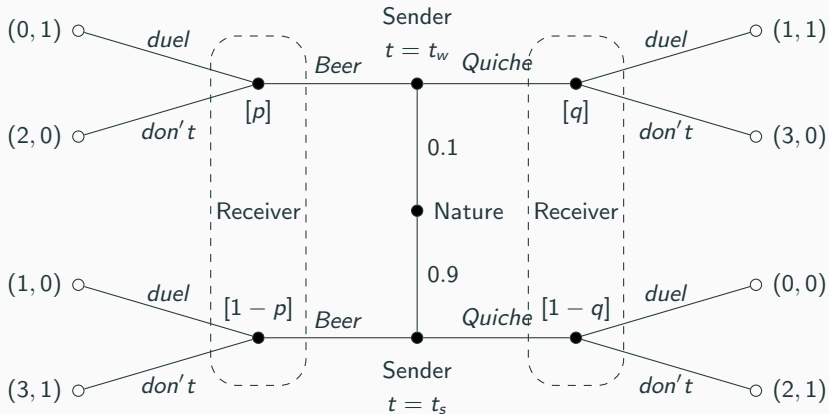
Weak Consistency of Beliefs

Signaling Games

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Example of a Signaling Game: Beer-Quiche Game



Solution Concepts

- Recall that Nash equilibrium requires (1) players best respond to conjectures (beliefs) (2) conjectures are correct in equilibrium.
- In extensive form games subgame perfection requires that actions and conjectures were credible.
- We can extend these ideas to incomplete information games.
 - Bayesian Nash: Nash equilibrium with expected utility, given strategies
 - Perfect Bayesian Nash eq. (PBE): Requires actions and beliefs to be consistent at all action sets.
 - PBE restricts actions and beliefs at any information set reached with positive probability.
 - Beliefs at information sets never reached in equilibrium are free parameters

Perfect Bayesian Equilibrium (PBE)

A PBE is two things:

- **Strategy**: for each information set a contingent plan of action (possibly mixed).
- **Beliefs**: a probability distribution over each information set, i.e., for each node I could be at given the history, what is the probability I am at a particular node?

such that:

- Strategies are **sequentially rational**: optimal given beliefs.
- Beliefs are **consistent** with the strategy profile and obtained from Bayes' rule where ever possible.

Back to Beer Quiche

In this game:

- Types are strong, weak
- Messages are beer, quiche
- Actions are duel, don't

What are the equilibria?

Solving Dynamic Games of Incomplete Information

There are three kinds of equilibria to dynamic games of incomplete information:

- ① Pooling equilibrium: A PBE to a dynamic game of incomplete information is **pooling** if all types of senders send the same message.
- ② Separating equilibrium: A PBE to a dynamic game of incomplete information is **separating** if all types of senders send different messages.
- ③ Semi-separating equilibrium: A PBE to a dynamic game of incomplete information is **semi-separating** if some types send the same message while other types send different messages (or types send the same messages with different probabilities).

Solving Dynamic Games of Incomplete Information

To solve games we conjecture strategy profiles of these three varieties and see if they can be sequentially rational.

Pooling Equilibrium: Beer, Beer

Let's check to see if the following is a PBE:

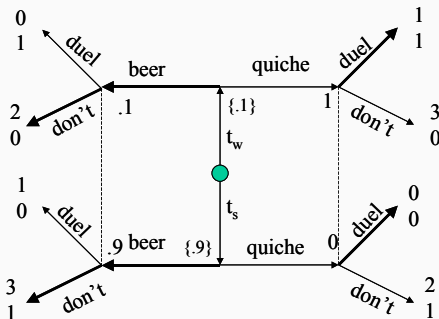
Strategies: **Player 1:** both types choose **beer**, **Player 2:** **don't duel** at information set **beer**, **duel** at information set **quiche**

Beliefs: **Player 2**, after seeing **beer**, believes probability of strong type is .9 and weak type is .1; after seeing **quiche**, believes Player 1 is weak with certainty (probability 1).

(This is the proper characterization of a PBE: (1) a complete contingent plan of actions for every player-type at every information set, including off the equilibrium path; (2) beliefs at every information set, which satisfy consistency on the equilibrium path and are free parameters otherwise.)

Pooling Equilibrium: Beer, Beer

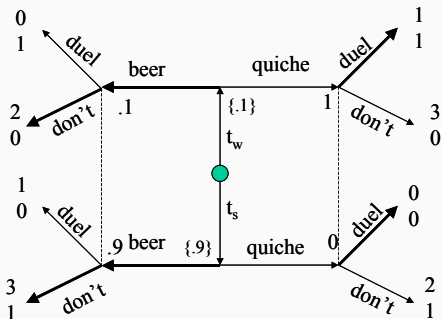
First let's check **beliefs**. If both types of P1 pick beer, P1's action is uninformative about type; the consistent belief is the prior. If neither type of P1 picks quiche, this is off the equilibrium path and we don't require consistent beliefs.



Pooling Equilibrium: Beer, Beer

Now let's check that this equilibrium is **sequentially rational** for P2.

$$EU_2(\text{duel}|\text{beer}) = (.1)(1) + (.9)(0) = .1$$
$$< EU_2(\text{don't}|\text{beer}) = (.1)(0) + (.9)(1) = .9$$



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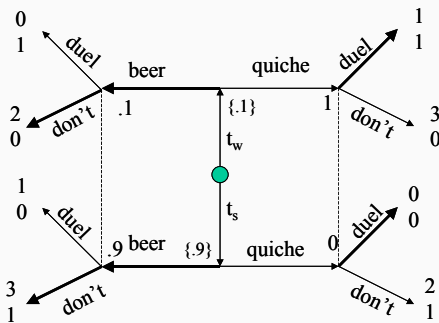
$$EU_2(\text{duel}|\text{quiche}) = 1 > EU_2(\text{don't}|\text{quiche}) = 0$$

Pooling Equilibrium: Beer, Beer

What about sequential rationality for P1?

Weak type: $EU_1(\text{beer}) = 2 > EU_1(\text{quiche}) = 1$

Strong type: $EU_1(\text{beer}) = 3 > EU_1(\text{quiche}) = 0$



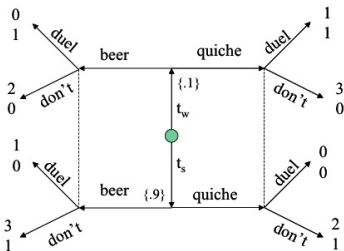
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Pooling Equilibrium: Quiche, Quiche

Exercise: Check that there is another pooling equilibrium where both types of Player 1 play **quiche**. What are the beliefs for Player 1 and strategies for Player 2 that justify this equilibrium?

What about a Separating Equilibrium?

Consider a strategy for P1 that has the weak type choose quiche and the strong type choose beer. Can this be part of a PBE? **No.**

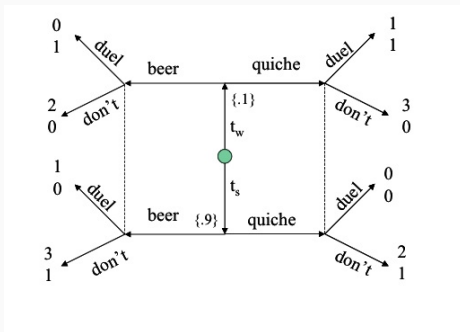


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- Consistent beliefs dictate that P2 thinks P1 is strong with probability 1 when observing beer, weak when quiche
- P2 will thus play **don't** when beer, **duel** when quiche
- But then the weak type of P1 will want to deviate to beer (2 vs. 1)

What about a Separating Equilibrium?

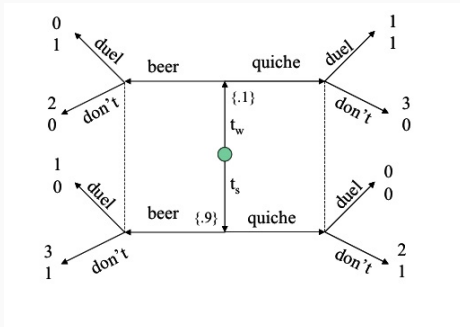
Exercise: Also check that there is no separating PBE the other way: weak type chooses beer, strong type chooses quiche.



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What about a Semi-Separating Equilibrium?

In many games of this type, you will get a **semi-separating equilibrium**: where both types of P1 play a mixed strategy, but (for example) the strong type is **likelier** to play beer than the weak type, and there is **partial information transmission** to P2.

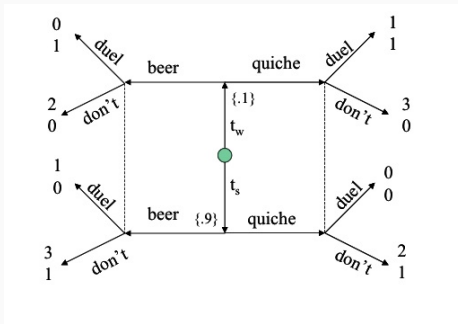


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How do we solve for equilibria of this type?

What about a Semi-Separating Equilibrium?

Now P2's beliefs over P1's type are not immediately obvious; we have to use **Bayes' Rule**.



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Suppose strong type plays **quiche** with probability Q and weak type plays **quiche** with probability q . What are the compatible beliefs for Player 2?

What about a Semi-Separating Equilibrium?

Suppose strong type plays **quiche** with probability Q and weak type plays **quiche** with probability q . What are the compatible beliefs for Player 2?

$$\begin{aligned}\mu_2(\text{strong}|\text{quiche}) &= \frac{Pr(\text{quiche}|\text{strong})Pr(\text{strong})}{Pr(\text{quiche}|\text{strong})Pr(\text{strong}) + Pr(\text{quiche}|\text{weak})Pr(\text{weak})} \\ &= \frac{(Q)(.9)}{(Q)(.9) + (q)(.1)}\end{aligned}$$

$$\mu_2(\text{weak}|\text{quiche}) = \frac{(q)(.1)}{(Q)(.9) + (q)(.1)}$$

$$\mu_2(\text{strong}|\text{beer}) = \frac{(1 - Q)(.9)}{(1 - Q)(.9) + (1 - q)(.1)}$$

$$\mu_2(\text{weak}|\text{beer}) = \frac{(1 - q)(.1)}{(1 - Q)(.9) + (1 - q)(.1)}$$

What about a Semi-Separating Equilibrium?

Now what pure strategies for P2 are best-responses given P1's strategy and P2's beliefs?

Seeing **quiche**, P2's payoff from **duel** is:

$$\mu_2(\text{strong}|\text{quiche})(0) + \mu_2(\text{weak}|\text{quiche})(1) = \frac{(q)(.1)}{(Q)(.9) + (q)(.1)}$$

and payoff from **don't** is:

$$\mu_2(\text{strong}|\text{quiche})(1) + \mu_2(\text{weak}|\text{quiche})(0) = \frac{(Q)(.9)}{(Q)(.9) + (q)(.1)}$$

So P2 will **duel** when $.1q > .9Q$.

To complete the analysis, (1) check P2's best-responses for the information set **beer**, and (2) check that P1 has no incentive to deviate given P2's strategies and beliefs. (More [here](#).)

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Motivation: Escalation Game

Perfect Bayesian Equilibria

Sequential Rationality

Weak Consistency of Beliefs

Signaling Games

Example 1: Beer-Quiche Game

Example 2: Advice from Experts

Advice from Experts

- Often in politics, decision-makers do not have the time or human capital to assess the appropriateness of particular policies.
- This is especially true when there are many different policies to choose from and each policy is right for some situation, but no policy is right for all situations.
- Experts often have specific political orientations:
 - The median member of APSA is to the left of the average democrat.
 - The median economist is more conservative than the median voter.
 - The typical bureaucrat is biased toward his institution.
 - The typical environmental scientist is a conservationist.

Strategic Information Transmission from Experts

- Consider a situation where a Legislature (L) and an expert (E) have different preferences
- L needs to make a policy choice, but the right policy depends on the state of the economy
- E fully observes some information about the state of the economy (world), $t \in [0, 1]$, that is relevant for L 's policy choice, but L only knows the distribution of t

Strategic Information Transmission from Experts

The game:

- Nature draws t from a uniform distribution on the interval $[0, 1]$
- The expert (E) sends a message r to L , which may depend on t
- L takes an action y that sets a policy

Payoffs:

$$u_E(y, t) = -(y - (t + b))^2 \quad (1)$$

$$u_L(y, t) = -(y - t)^2. \quad (2)$$

Thus note that L wants to exactly match t , while E wants t plus some bias b .

Strategic Information Transmission from Experts

Looking for PBE:

- Can we get full information transmission (a separating equilibrium)?
 - One such strategy is E reports the truth: $r(t) = t$.
 - If E played this strategy, L would know the state of the world was t , and choose $y = t$.
 - But if L is just going to do whatever E says, E has incentive to deviate to $t + b$.
- Can we get pooling, i.e. $r(t) = c$ for all t ?
 - We need off-path beliefs, but if they equal the prior, then yes.
 - L chooses y such that:

$$\max_y \int_0^1 -(y - t)^2 1 dt \rightarrow y^* = \frac{1}{2}.$$

Strategic Information Transmission from Experts

What about partial information transmission?

- Let's look for a strategy for E such that:

$$r(t) = \begin{cases} \text{low} & \text{if } 0 \leq t < t^* \\ \text{high} & \text{if } t^* \leq t \leq 1 \end{cases}$$

- If L receives low signal, they believe t is distributed uniform on $[0, t^*] \rightarrow y^* = t^*/2$.
- If L receives high signal, they believe t is $U[t^*, 1]$ and $y^* = \frac{t^*+1}{2}$.
- Thus E can only induce $y = \frac{t^*}{2}$ or $y = \frac{t^*+1}{2}$.
- These strategies are optimal when **incentive compatibility** of messages holds.

This incentive-compatibility condition can be stated as follows:

$$\begin{aligned}u_E(t^*/2|t < t^*) &\geq u_E\left(\frac{t^* + 1}{2}|t < t^*\right) \\u_E\left(\frac{t^* + 1}{2}|t \geq t^*\right) &\geq u_E(t^*/2|t \geq t^*)\end{aligned}$$

Namely, experts who see a higher t want L to choose the higher of the two policies, and experts who see a lower t want L to choose the lower of the two policies \implies despite expert's bias, expert and decisionmaker are **close enough** in their preferences.

Strategic Information Transmission from Experts

- How close is close enough?
- Observe: if incentive compatibility holds, E is indifferent when $t = t^*$. Thus solve:

$$-(t^*/2 - (t^* + b))^2 = -\left(\frac{1+t^*}{2} - (t^* + b)\right)^2$$
$$t^* = \frac{1}{2} - 2b$$

- For the equilibrium to exist, we need $t^* > 0$, so $b < \frac{1}{4}$.
- So only if an expert has sufficiently small bias can she help the policy maker choose better policies.

Slackers and Zealots (Gailmard and Patty 2007)

- A more complicated, dynamic model is: when should bureaucracies give experts incentives to develop expertise?
- Bureaucrats have higher-paying outside options, i.e. in industry
- Therefore the incentives to stay in the bureaucracy are **promotion** and associated **discretion**
- To whom is promotion/discretion most appealing over monetary rewards? **Zealots**: those who care about policy and want something sufficiently different from the status quo/alternative
- The more experts stay in government, the more expertise they develop and thus the greater their **informational advantage** over the policymaker
- So bureaucracies have two choices: **regime of clerkship** and **politicized competence**. There is no way to get **neutral competence**.
- Which regime we want to choose depends again on the **ally principle** (ideological proximity between decisionmaker and expert), as well as how important it is to have the right information.

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