17.810/17.811: Problem Set 1

Read the following instructions carefully:

- All answers must be typed or clearly written up.
- Late submission of the write-up will not be accepted.
- You are encouraged to work in groups after a solo effort has been taken first, but you should write up your answers alone and tell us who you worked with.
- For analytical (proofs) questions, you should include your detailed derivation for all intermediate steps (logical arguments).

Set Up

Let's walk you through some basic concepts regarding preferences, choice and utility. Recalling from the lecture notes, we have the following definitions and propositions. These can be useful in answering the questions below. Following the notations in the lecture notes, we have

Definition 1. *Given a choice structure $(\mathcal{B}, C(\cdot))$, we say that the rational preference relation R rationalizes $C(\cdot)$ relative to $\mathcal{B}$ if:*

$$C(B) = C^*(B, R) \ \forall \ B \in \mathcal{B}$$

Recalling that:

$$C^*(B, R) = \{x \in B : xRy \text{ for every } y \in B\}$$

Definition 2. *The choice structure $(\mathcal{B}, C(\cdot))$ satisfies the weak axiom of revealed preference (WARP) if the following property holds:*

If for some $B \in \mathcal{B}$ with $x, y \in B$ we have $x \in C(B)$, then for any $B' \in \mathcal{B}$ with $x, y \in B'$ and $y \in C(B')$, we must also have $x \in C(B')$.

Proposition 1. *Suppose that $\succsim$ is a rational preference relation. Then the choice structure generated by $\succsim$ satisfies the weak axiom of revealed preference.*

Proposition 2. *A preference relation $\succsim$ can be represented by a utility function if it is rational and X is finite.*

Problem Set Questions

Problem 1

A common electoral rule in multi-candidate contests is the *runoff rule* or *two-round system*. One of the most prominent elections using this system is the French presidential election. According to Wikipedia, the rule is also used to elect the presidents of many other countries such as Afghanistan, Argentina, Austria, ..., Turkey, Ukraine, Uruguay, and Zimbabwe (40 in total).

A simplified version of the system elects the winning candidate via the following procedure:

1. Each voter cast 1 vote for their most preferred candidate in the first round.
2. If one candidate wins more than 50% of the vote, she becomes the winner. Otherwise, the two candidates who win most votes enter the second round.
3. Between the two remaining candidates, whoever wins more votes in the second round wins the election.

Show that this version of *runoff rule* may violate *WARP*. Further explain why in this case the underlying preference relation is not rational.

Problem 2

John is planning to get a bowl of soup at a restaurant called *Game Theory Diner* in Cambridge. There are two options on the menu, Clam Chowder (x) and Brown Windsor soup (y). He decides to order Clam Chowder (x) after carefully thinking through the available options. Slightly different from what we asserted in the lecture notes, in this problem and following ones, we unrealistically (and probably incorrectly) assume that chosen elements are members of $C(A)$, while other elements are not members of $C(A)$. Show that John's choice is rationalizable. If the choice is not rationalizable, explain why (*Hint:* See Definition 1 above).

Problem 3

However, the waitress tells John that actually the Clam Chowder is not New England Clam Chowder (x), but Manhattan Clam Chowder (z). John then decides to order Brown Windsor soup (y). However, after consulting with the manager, the waitress tells John that actually both New England Clam Chowder (x) and Manhattan Clam Chowder (z) are available, but John keeps his order of Brown Windsor soup (y). Could John's choice be rationalized by a rational preference? Explain.

Problem 4

Next, suppose that John does not keep his order of Brown Windsor soup (y) after knowing that both New England Clam Chowder (x) and Manhattan Clam Chowder (z) are available. Could John's choice be rationalized by a rational preference? Explain.

Problem 5

Finally, suppose that John does not keep his order of Brown Windsor soup (y) after knowing that both New England Clam Chowder (x) and Manhattan Clam Chowder (z) are available. And we further know that John would choose New England Clam Chowder (x), if only New England Clam Chowder (x) and Manhattan Clam Chowder (z) are available. Is John's choice rationalizable? Can John's preference be represented by a utility function?