17.810/17.811: Problem Set 3

Read the following instructions carefully:

- All answers must be typed or clearly written up and stapled.
- Late submission of the write-up will not be accepted.
- You are encouraged to work in groups after a solo effort has been taken first, but you should write up your answers alone and tell us who you worked with
- For analytical (proofs) questions, you should include your detailed derivation for all intermediate steps (logical arguments).

Note: In this problem set, "Find all pure strategy Nash Equilibria" does not imply the existence of at least one pure strategy Nash Equilibrium. If you can not find one, you can say that there is no pure strategy Nash Equilibrium, but you have to state your reasons clearly for the statement.

Part I - Mixed Strategies

Problem 1

Find all Nash Equilibria in pure and mixed strategies for the following game.

Player 2

		L	R
Player 1	T	1,1	2,2
	B	0,5	4,3

Problem 2

Find all Nash Equilibria in pure and mixed strategies for the following game.

Player 2

		L	M	R
Player 1	T	3,3	1,1	0,0
	B	0,0	3,1	4,4

Part II - Logic of Collective Action

Set Up

Suppose there is a public good which could only be provided if at least one person (out of N people) gives up c dollars ($0 < c < 1$) to pay for it. Consider the following game: each person simultaneously decides whether or not to contribute c. If at least one person contributes c, each contributor gets $1 - c$ and each noncontributor gets 1. If no one contributes, everyone gets a payoff of zero.

Problem 1

Find all pure strategy Nash Equilibria of this game.

Problem 2

Find the unique symmetric mixed strategy equilibrium of this game. A symmetric mixed strategy equilibrium is an equilibrium in which everyone uses the same mixed strategy.

Problem 3

What is the probability that at least one person contributes in this unique symmetric mixed strategy equilibrium you find in the previous part?

Problem 4

Show that when $N > 1$, as N increases, the probability that at least one person contributes the Public Good decreases. And find limit of the probability as $N \to +\infty$. (**Hint:** the limit could be a function of the cost of provision c).

Part III - Voter Turnout [1]

Set Up

There are three voters in the game, indexed by $1, 2, 3$. Voter 1 and Voter 2 are Democrats, while Voter 3 is a Republican. Each voter decides independently whether she turns out to voter or not. The cost of turning out to vote is c, $c < \frac{1}{2}$. Each voter only cares about whether her own party wins, and will get a pay-off 1 if her party wins, and get zero if her party loses. The election adopts simple plurality rule. The party who gets the largest number of votes wins the election. If there is a tie, a coin is tossed, and each party will win the election with the probability $\frac{1}{2}$.

Problem 1

Fill in the blanks of the following three-player pay-off matrices.

Voter 2

Voter 1		Votes for Dem	Votes for Rep	Stay at Home
	Votes for Dem	$(1-c, 1-c, -c)$		
	Votes for Rep	$(1-c, 1-c, -c)$		
	Stay at Home	$(1, 1-c, -c)$		

Voter 3 Votes for Dem Party

Voter 2

Voter 1		Votes for Dem	Votes for Rep	Stay at Home
	Votes for Dem			
	Votes for Rep			
	Stay at Home			

Voter 3 Votes for Rep Party

[1] This problem is based on Palfrey, Thomas R., and Howard Rosenthal. "A strategic calculus of voting." *Public Choice* 41, no. 1 (1983): 7-53. You may find it helpful to read the paper. However, the problem is self-contained, and requires no more knowledge beyond what has been covered in class.

Voter 2

Voter 1		Votes for Dem	Votes for Rep	Stay at Home
	Votes for Dem			
	Votes for Rep			
	Stay at Home			

Voter 3 Stays at Home

Problem 2

Find all pure-strategy Nash Equilibria.

Problem 3 (Extra Credit)

Find one mixed-strategy Nash Equilibrium in which all Democrats turn out to vote with the same probability p_1, and the Republicans turn out to vote with the same probability p_2.