

17.810/17.811: Problem Set 4

Read the following instructions carefully:

- All answers must be typed or clearly written up and stapled.
- Late submission of the write-up will not be accepted.
- You are encouraged to work in groups after a solo effort has been taken first, but you should write up your answers alone and tell us who you worked with
- For analytical (proofs) questions, you should include your detailed derivation for all intermediate steps (logical arguments).

Part I - Incumbent Entrant Game

Set Up

In the upcoming congressional election in the Game Theory District of the State of Political Science, there is one incumbent candidate (I) and one potential challenger (C).

- In the election, the challenger first decides and announces whether she enters the election. After observing whether the challenger enters the election, the incumbent candidate decides the amount of her campaign spending. The incumbent candidate has two options, high spending and low spending.
- The benefit of the seat for the incumbent, the enjoyment of serving the people as you may interpret, is 5. The cost of high campaign spending is 3 for the incumbent, and the cost of low campaign spending is 1 for the incumbent. The payoff of not being able to stay in power is normalized to zero for the incumbent.
- The benefit of the seat for the challenger is 4, while the cost of the challenger to enter the election is 2. If the challenger stays out of the election, her payoff is normalized to zero.
- If the challenger enters the election and incumbent chooses low spending, the incumbent will win the election with 50% of chance, and hence the probability that the challenger will win the election is 50% as well. On the other hand, if the challenger enters the election and the incumbent chooses high spending, she wins the election for sure.
- If the challenger does not enter the election, the incumbent wins automatically, but as she has already pre-paid her spendings to purchase advertisements, the spendings could not be refunded.

Problem 1

Write the game in an extended form.

Problem 2

Write the game in a normal form.

Problem 3

Find all pure strategy Nash Equilibria, and all pure strategy Subgame Perfect Nash Equilibria.

Part II - Control of Politicians

Set Up

Delegates to the Constitutional Convention of the United States of Political Science is debating on whether frequent elections can hold the President more accountable to the people. One delegate says yes because if the President does not work hard enough but slacks, she will be voted out of the position in the election. This problem will help you evaluate this argument¹.

An elected President has two options, (a) to work very hard for the people, and (b) to slack. Working hard incurs a cost of 1 for the President, while slacking incurs no cost. On the other hand, a hardworking President gives the people a positive payoff 5, while a slacking President gives people a smaller positive payoff of 3.

The President gets a direct utility of 3 for each term being in the office. At the end of each term, the President always chooses to run for re-election if the law allows. If the incumbent President loses her re-election, a challenger will win the election, and the people will get a pay-off of 4, the average of the pay-off brought in by a hardworking President (5) and the pay-off brought in by a slacking President (3). However, the losing president gets a zero pay-off.

Problem

Suppose the law limits the number of terms that a President could serve to 2, write the game in extensive form and solve the pure strategy Subgame Perfect Nash Equilibrium. Discuss the implication of the solution to your extensive-form game.

Part III - Finitely Repeated Game

Problem 1

Suppose that two players play with each other for two periods. In the first period, they play the game at left below, and in the second period they play the game at right. There is no discounting between periods. Players observe the action their opponent took in the first period before choosing their second period actions.

¹In political science literature, such debates can be summarized as “sanction versus selection”. People debate on whether voters sanction unqualified politicians and stimulate them to do well in elections, or select good politicians who are self-motivated (or with similar preference with voters) in elections. An important work on this is Fearon, James D. “Electoral accountability and the control of politicians: selecting good types versus sanctioning poor performance.” *Democracy, Accountability, and Representation* 55 (1999): 61. His model is slightly different, as he assumes there are only two periods, and thus his model produces slightly different predictions. You may refer to the article if you are interested.

		Player 2	
		L	R
Player 1	U	4,4	-10,2
	D	6,-1	0,0
		Stage 1	

		Player 2	
		L	R
Player 1	U	10,4	0,0
	D	0,0	4,10
		Stage 2	

Find a Subgame Perfect Equilibrium in which Player 1 receives a payoff of 14, and specify the respective behavioral strategy.

Problem 2 (Extra Credit)

Suppose that two players play with each other for two periods. In the first period, they play the game at left below, and in the second period they play the game at right. There is no discounting between periods. Players observe the action their opponent took in the first period before choosing their second period actions.

		Player 2	
		L	R
Player 1	U	4,4	-10, x
	D	y , -1	0,0
		Stage 1	

		Player 2	
		L	R
Player 1	U	10,4	0,0
	D	0,0	4,10
		Stage 2	

Find all possible values of x and y , such that this game has a pure strategy Subgame Perfect Equilibrium where player 1 receives a payoff of 14.

(Hint: The possible values of x and y are intervals in the form of $x \leq a$ and $y \leq b$. You only need to find the values of a and b .)

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