17.810/17.811: Problem Set 5

Read the following instructions carefully:

- All answers must be typed or clearly written up and stapled.
- Late submission of the write-up will not be accepted.
- You are encouraged to work in groups after a solo effort has been taken first, but you should write up your answers alone and tell us who you worked with
- For analytical (proofs) questions, you should include your detailed derivation for all intermediate steps (logical arguments).

Part I - Bargaining

Two politicians are bargaining on the allocation of $1 million US Dollars. Each politician wants more money to be allocated into her own district. Politicians are friendly to each other, and they always accept a plan if they are indifferent between alternative plans. The bargain occurs according to the following rule:

- Politician 1 proposes an allocation plan.
- Politician 2 decides whether to accept the allocation plan or not.
- If Politician 2 accepts, the game ends and the money will be allocated accordingly. If Politician 2 rejects, she will have to make a counter-offer to Politician 1. However, the money available will be reduced to $\$\delta$ million US Dollars. ($0 < \delta < 1$)
- Then Politician 1 decides whether to accept the counter-offer or not.
- If Politician 1 accepts the counter offer, the plan will be carried out accordingly. If Politician 1 declines, the money will expire and nobody gets any money.

How much money will Politician 1 and Politician 2 get respectively if they are playing pure strategies in the Subgame Perfect Equilibrium? Show your steps and your reasoning.

Part II - Investigation on Foreign Campaign Contributions

Set Up

Two candidates, A and B are running for the position of Prime Minister in the Commonwealth of Game Theory. They both have received secret but illegal money from foreign entities in the Republic of Political Philosophy to fund their campaigns.

The two candidates understand that they both receive illegal foreign campaign contribution, but have no solid evidence for their opponent's illegal behavior. They play a sequential but infinite game, in which they take turn to decide whether to hire a team to investigate on their opponent's illegal receipt of foreign campaign contributions. There is no discount factor.

On each stage, if the candidate decides to investigate the illegal behavior of her opponent, she has a probability p to find solid evidence. The probability of discovery p is the same for both candidates. The probability p is independent of whether any other investigation finds evidence. If solid evidence regarding one candidate's receipt of illegal foreign campaign contributions, the game ends and the candidate will be charged and deprived of the right to run for any public office, which generates a huge negative payoff, say -100.

Each candidate cares only about his NOT being discovered solid evidence of receiving illegal foreign money. That is, candidates do not care about their opponent's probability of being discovered.

Problem 1

Show that neither candidate hiring the investigation team on all histories is a Subgame Perfect Equilibrium of the game.

Problem 2

Show that both candidates always hiring the investigation team is a Subgame Perfect Equilibrium of the game.

Part III - Individual Rationality

Set Up

The stage game as depicted in the following table is played infinitely by Player 1 and Player 2 simultaneously in each stage. Both players have a common discount factor δ.

		Player 2	
		L	R
Player 1	U	9,9	1,10
	D	10,1	7,7

Problem 1

What outcomes in the stage-game are consistent with a Nash equilibrium?

Problem 2

Let v_1 and v_2 be the repeated game payoffs to player 1 and player 2 respectively. Draw the set of feasible payoffs from the repeated game.

Problem 3

Find the set of individually rational feasible set of payoffs. And draw the set on the plot you have produced for Problem 2.

Problem 4

Find Subgame Perfect Equilibrium in which the players obtain $(9, 9)$ each period. Specify the equilibrium strategy of each player, and respective restrictions on δ.