17.810/17.811: Problem Set 6

Read the following instructions carefully:

- All answers must be typed or clearly written up and stapled.
- Late submission of the write-up will not be accepted.
- You are encouraged to work in groups after a solo effort has been taken first, but you should write up your answers alone and tell us who you worked with
- For analytical (proofs) questions, you should include your detailed derivation for all intermediate steps (logical arguments).

Part I - Majority Rule Bargaining with Asymmetric Proposal Power

Set Up

This problem is an extension of the Majority Rule Bargaining under a closed rule as discussed during the lecture.

- There are N (odd) members in the Congress, and any proposal would require $n = \frac{N+1}{2}$ votes to pass.
- There are two parties in the Congress. Among these N members, $N - m$ of them belong to Party A, and m of them belong to Party B. Without loss of generality, we assume $N - m \geq n$, which means Party A is the majority party.
- Instead of the random recognition rule as discussed in the lecture, now members from Party A has a probability p to be recognized, while members from Party B has a probability q to be recognized. As Party A is the majority party, we assume $p > \frac{1}{N}$, and $q < \frac{1}{N}$, such that each member of Party A has a slightly higher probability to be recognized. This is because meetings are chaired by members from the majority party. As a regularity assumption, the sum of the probability of being recognized equals 1, or formally $(N - m)p + mq = 1$.
- We assume symmetry in the sense that each legislator from the same party plays the same strategy and has the same continuation value. The members of the two parties have continuation values v_A and v_B respectively. To make the problem more accessible, we only consider the case where $v_A > v_B$.
- A member of Party A votes for any proposal that provides her at least δv_A, while a member of Party B votes for any proposal that provides her at least δv_B. As you may have observed, the discount rate is assumed to be the same for every member of the Congress.
- All other rules follow in the Majority Rule Bargaining as discussed in the lecture, for example the total value to divide is 1 dollar.

We also explore only stationary equilibrium in this problem. A stationary equilibrium is one in which:

1. A proposer proposes the same division every time she is recognized regardless of the history of the game.
2. Voters vote only on the basis of the current proposal and expectations about future proposals. Because of assumption 1, future proposals have the same distribution of outcomes in each period.

Problem 1

If a member from the Party A has been recognized, is maximizing her proposer's share and at the same time securing a majority consent (n votes), would she prioritize to allocate resources to her colleagues from her same party, or would she prioritize to allocate resources to members of Party B? Discuss.

Hint: $v_A > v_B$

Problem 2

If a member from the Party A has been recognized, how would she make her proposal? Write out the equation for z_A, the proposer's share if the proposer is from Party A, as a function of n, m, δ, v_A, v_B.

Problem 3

If a member from the Party B has been recognized, how would she make her proposal? Write out the equation for z_B, the proposer's share if the proposer is from Party B, as a function of n, m, δ, v_A, v_B.

Problem 4

The continuation value v_A for a member from Party A is the expectation of her share at the start of each game. Write out the equation for v_A as a function of $p, q, N, n, m, \delta, v_A, v_B, z_A, z_B$.

Hint: It equals the sum of (1) the product of the probability that she herself is recognized and her share as a proposer, (2) the product of the probability that a member of Party A other than her is recognized, her probability of being in the winning coalition and the portion she would get as a member in the winning coalition from Party A, (3) the product of the probability that a member from party B is recognized, her probability of being in the winning coalition and the portion she would get as a member in the winning coalition from Party B.

Problem 5

The continuation value v_B for a member from Party B is the expectation of her share at the start of each game. Write out the equation for v_B as a function of $p, q, N, n, m, \delta, v_A, v_B, z_A, z_B$.

Problem 6

Let $N = 3$, and $m = 1$, solve for z_A, z_B, v_A, and v_B.

Hint: $2p + q = 1$

Part II - Bayesian Game

Set Up

This problem will walk you through the steps to solve Bayesian Nash Equilibrium with an action space containing discrete actions.

Bob is a politician working in the Congress. He is reported to have been involved in a scandal by an anonymous twitter post. A journalist, Alice, is trying to interview Bob for his response to the twitter post.

Bob has two offices, respectively at DC and in his own constituency, say State of Game Theory (GT). Alice is trying to catch Bob in his office, and will get payoff 1 if she successfully catches Bob in his office (and then interviews him). Bob is consulting his lawyer on whether he should take the interview. The lawyer of Bob is looking into the case, and will tell Bob whether he is innocent in the scandal, i.e. sending a signal as nature to Bob.

If Bob is innocent, he will prefer to meet up with the journalist to respond to the scandal directly. If Bob is not innocent, he will prefer to avoid meeting with the journalist. Further, Bob enjoys staying at DC simply because the musical Hamilton is on during that week in DC, and he has got the tickets. Alice also prefers interviewing in DC to interviewing in GT because the headquarter of her news agency is based in DC.

We can formalize the game in the following payoff matrix. In the game, nature generates $\theta \in \{0, 1\}$, and the value is known by Bob. Alice believes that $\theta = 0$ with probability $\frac{1}{2}$, and $\theta = 1$ with probability $\frac{1}{2}$. Everything else is common knowledge.

		Bob	
		GT	DC
Alice	GT	1,θ	0,2$(1-\theta)$
	DC	0,$1-\theta$	2,2θ

Problem 1

Write out the two payoff matrices corresponding to the two types of Bob. Specify the best response Bob has for each strategy played by Alice and his respective payoff, conditional on the type of Bob.

Hint: $B_B(GT|\theta = 0) = DC$ and the payoff for Bob will be 2.

Problem 2

We now focus on the payoffs that Alice will get. Alice believes that there is a probability of $\frac{1}{2}$ that $\theta = 0$, and a probability of $\frac{1}{2}$ that $\theta = 1$. Fill in the payoff matrix below for Alice. In each cell of the matrix, specify the payoff Alice will get.

		Bob			
		GT,GT	GT,DC	DC,GT	DC,DC
Alice	GT		$\frac{1}{2} \times 1 + \frac{1}{2} \times 0 = \frac{1}{2}$		
	DC				

Problem 3

Solve all pure strategy Bayesian Nash Equilibria in the game.

Part III - Lemon Market (Extra Credit)

This is a simplified version of the Nobel Prize winning idea[1]. Lemon here is a jargon which means second hand cars of poor quality.

Suppose we have a second-hand car market, in which 101 (potential) sellers are trying to sell their cars, and a representative buyer is trying to buy a car from the market. The value of a second hand car is fully determined by its inherent quality. For the 101 sellers, the value of their cars are respectively \$1,000, \$1,010, \$1,020, ..., \$2,000. A seller fully knows the value of the car he is selling deterministically. On the other hand, the representative buyer cannot tell whether a specific car is good or bad, but only knows that the 101 cars on the market are worth \$1,000, \$1,010, \$1,020, ..., \$2,000 respectively. That is, the buyer only knows the distribution of the value of cars rather than how much a particular car is worth.

A seller decides whether to stay in the market, and she stays in the market if the expected price a representative buyer is willing to pay is higher than the value of her car. On the other hand, the representative buyer is maximizing her expected surplus, i.e. the expected value of the car she gets minus the price she pays.

Show that in the Bayesian Nash Equilibrium of the game, the buyer purchases the car of the worst quality (lowest value).

[1] Akerlof, George A. "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism." *The Quarterly Journal of Economics 84, no. 3 (1970): 488-500.* Fun facts: According to Wikipedia, both the American Economic Review and the Review of Economic Studies rejected the paper for "triviality", while the reviewers for Journal of Political Economy rejected it as incorrect. Only on the fourth attempt did the paper get published in Quarterly Journal of Economics. Today, the paper is one of the most-cited papers in modern economic theory and most downloaded economic journal paper of all time in RePEC. There is also an interesting article written by graduate students interviewing their established professors whether their most known articles have ever been rejected that you may want to read Gans, Joshua S., and George B. Shepherd. "How are the mighty fallen: Rejected classic articles by leading economists." *The Journal of Economic Perspectives 8, no. 1 (1994): 165-179.*