17.810/17.811: Problem Set 7

Read the following instructions carefully:

- All answers must be typed or clearly written up and stapled.
- Late submission of the write-up will not be accepted.
- You are encouraged to work in groups after a solo effort has been taken first, but you should write up your answers alone and tell us who you worked with
- For analytical (proofs) questions, you should include your detailed derivation for all intermediate steps (logical arguments).

Part I - Selection in Representative Democracy[1]

Set Up

There are two periods, 1, and 2, and a pair of possible actions $\{a, b\}$ in each period. Among the two possible actions, one is better for every voter than the other. However, no voter knows whether it is a or b. The prior belief for voters that a is the optimal choice is p, $p > \frac{1}{2}$.

The politician is more knowledgeable than voters, and we assume the politician knows what is best for the society and what is best for her. There are two types of politicians though, a good type and a bad type. There is a probability π that a politician is a good one, which means the politician inherently shares the same preference with the voters. That is, if nature decides the socially optimal policy is a, the personally optimal policy for the politician is a. On the other hand, there is a probability $1 - \pi$ that a political of bad type, which means the politician has an opposite preference compared with the voters. That is, if nature decides a is the socially optimal policy, b will be the personally optimal policy of the politician.

If a politician of good type is elected, she will always implement the socially optimal policy. If a politician of bad type is elected, she will always implement the policy that is personally optimal for himself.

Voters are interested in maximizing their expected utility. Voters get a payoff 1 for each term when the socially optimal policy is implemented, while they get a payoff 0 for each term when the socially optimal policy is not implemented.

The game is played in the following way:

1. Nature decides the socially optimal policy, with probability p that a is socially optimal.

2. Nature decides whether the current incumbent politician is of good type or bad type.

3. The politician learns about the socially optimal policy, and implements the policy according to her own type.

4. The voters observe the policy implemented, and decide whether to re-elect the incumbent politician or to remove the incumbent politician.

5. If the voters re-elect the incumbent politician, the incumbent politician implements a policy again, according to her type. However, if the voters remove the incumbent politician, a challenger will be elected, and nature decides whether the challenger is of good type or bad type. The elected challenger learns the socially optimally policy, and implements policy a or policy b according to her type.

6. The game ends after two periods.

[1]This problem is adapted from a paper authored by two Nobel Prize Laureates, i.e. Maskin, Eric, and Jean Tirole. "The Politician and the Judge: Accountability in Government." *American Economic Review* 94, no. 4 (2004): 1034-1054.

Note: (a) the socially optimal policy does not change across terms of the politicians; (b) a politician knows her type; (c) voters do not know the type of the politician, but knows that the prior probability that a politician is of good type is π; and (d) we model voters in the game as one representative voter.

Problem 1 What is the posterior probability at the end of the first term that a political who implements policy a is of good type?

Problem 2 What is the posterior probability at the end of the first term that a political who implements policy a is of bad type?

Problem 3 What is the posterior probability at the end of the first term that a political who implements policy b is of good type and bad type respectively?

Problem 4 What is the expected payoff a voter could get in the second term by removing a politician who implements policy a in the first term? What is the expected payoff a voter could get in the second term by keeping a politician who implements policy a in the first term? Suppose $0 < \pi < 1$, what is the best response for voters when they have observed the incumbent politician has implemented policy a in her first term?

Problem 5 What is the expected payoff a voter could get in the second term by removing a politician who implements policy b in the first term? What is the expected payoff a voter could get in the second term by keeping a politician who implements policy b in the first term? Suppose $0 < \pi < 1$, what is the best response for voters when they have observed the incumbent politician has implemented policy b in her first term?

Problem 6 What is the Bayesian Nash Equilibrium of the game?

Part II - Trade Restrictions

Two nations contemplate restrictive trade policies. Let $N = \{1, 2\}$ and suppose that each country has two possible types $\Theta_i = \{u, b\}$. A type u country wishes to limit its imports from the other country unilaterally, and a type b country wishes to pursue a bilateral policy of limiting trade only if the other country does so. The country types are independently drawn, and type u occurs with probability $p \in (0, 1)$. The strategy space for each country is $S = \{l, f\}$, where l denotes enacting an import limit and f denotes a free-trade policy. The payoffs for country i are

$$u_i(s_i, s_{-i}; \theta_i) = \begin{cases} 3 & s_i = l, s_{-i} = f, \theta_i = u \\ 2 & s_i = f, s_{-i} = f, \theta_i = u \\ 1 & s_i = l, s_{-i} = l, \theta_i = u \\ 0 & s_i = f, s_{-i} = l, \theta_i = u \\ 3 & s_i = f, s_{-i} = f, \theta_i = b \\ 2 & s_i = l, s_{-i} = f, \theta_i = b \\ 1 & s_i = l, s_{-i} = l, \theta_i = b \\ 0 & s_i = f, s_{-i} = l, \theta_i = b \end{cases}$$

A strategy in this game is a mapping $s_i(\theta_i) : \{u, b\} \to \{l, f\}$. A key feature of this game is that a u-type country always receives a higher payoff from l independently of the actions of the other country. If it is common knowledge that both countries are type u, the game is a Prisoner's Dilemma; each country has a dominant strategy to choose l. Alternatively, if it is common knowledge that both countries are type b, there are two pure strategy Nash Equilibria, (f, f) and (l, l).

In the following problems, we will work you through the process of the solving a symmetric Bayesian Nash Equilibrium. By symmetric, we mean that the two countries play the same strategy.

Problem 1

To compute Bayesian Nash Equilibrium, we begin with conjectures about equilibrium strategies and then check to see whether they satisfy the equilibrium requirements. What is the dominant strategy that u type country has? Can we eliminate some strategies that cannot be played in the Bayesian Nash Equilibrium?

Problem 2

Suppose further $p < \frac{1}{2}$, verify whether any of the remaining strategies is a symmetric Bayesian Nash Equilibrium. Show your work.

Problem 3

Suppose now $p > \frac{1}{2}$, how many symmetric Bayesian Nash Equilibria does this game have?