More Exam 5 Practice Problems

Here are some further practice problems with solutions for Exam 5. Many of these problems are more difficult than problems on the exam.

I. Areas of regions bounded by polar curves. In each of the following, find the area of the region bounded by the polar curves.

I.1 The region bounded by $r = ae^{b\theta}$ for $0 \leq \theta \leq \pi/2$.

I.2 The region bounded by $r^2 = 2a^2\cos(2\theta)$ for $-\pi/4 \leq \theta \leq \pi/4$.

I.3 The region bounded by $r = \cos(\theta)/(p\sin^2(\theta))$ for $\pi/4 \leq \theta \leq \pi/3$.

II. Inverse substitution. For each of the following, use inverse substitution to evaluate the antiderivative.

II.1 Use a hyperbolic substitution to find,

$$\int \sqrt{x^2+6x}dx.$$

II.2 Use a hyperbolic substitution to find,

$$\int \frac{1}{\sqrt{2x^2+4x+10}}dx.$$

II.3 Use a trigonometric substitution to find,

$$\int \frac{1}{(16-4x^2)^{3/2}}dx.$$

II.4 Use a hyperbolic substitution to find,

$$\int \sqrt{x^2+4x}dx.$$

II.5 Use a trigonometric substitution to find,

$$\int \frac{\sqrt{8+2x-x^2}}{(1-x)^2}dx.$$

Hint. Because $\cot^2(\theta)$ equals $\csc^2(\theta) - 1$, the antiderivative of $\cot^2(\theta)$ equals $-\cot(\theta) - \theta + C$.

III. Partial fractions. Use polynomial division, factoring and partial fractions to compute the following integrals.

III.1

$$\int \frac{x^5}{x^2+1} dx.$$

III.2

$$\int \frac{t^2+2}{t^2+3t-4} dt.$$

III.3

$$\int \frac{2y^2+2}{y^3-y^2} dy.$$

III.4

$$\int \frac{2x+3}{x^3+x} dx.$$

III.5

$$\int \frac{2x+3}{x(x^2+2)} dx.$$

IV. Integration by parts. Use integration by parts (possibly combined with other methods) to compute each of the following.

IV.1

$$\int x \sin(x) dx.$$

IV.2

$$\int x^2 \sin(x) dx.$$

IV.3

$$\int x[\ln(x)^2] dx.$$

IV.4

$$\int \sin^{-1}(x) dx.$$

IV.5

$$\int \theta \sec^2(\theta) d\theta.$$

Solutions.

Solution to I.1

$$a^2(e^{\pi b}-1)/(4b).$$

Solution to I.2

$$a^2.$$

Solution to I.3

$$(9-\sqrt{3})/(27p^2).$$

Solution to II.1

$$1/2[(x+3)\sqrt{x^2+6x}-9\ln(x+3+\sqrt{x^2+6x})]+C.$$

Solution to II.2

$$(\sqrt{2}/2)\ln(x+1+\sqrt{x^2+2x+5})+C.$$

Solution to II.3

$$x/(32\sqrt{4-x^2})+C.$$

Solution to II.4

$$(1/2)[(x+1)\sqrt{x^2+4x}-4\cosh^{-1}((x+2)/2)]+C.$$

Solution to II.5

$$\sqrt{8+2x-x^2}/(1-x)-\sin^{-1}((x-1)/3)+C.$$

Solution to III.1

$$(x^4/4)-(x^2/2)+(1/2)\ln(x^2+1)+C.$$

Solution to III.2

$$t-(18/5)\ln(|t+4|)+(3/5)\ln(|t-1|)+C.$$

Solution to III.3

$$(2/y)-2\ln(|y|)+4\ln(|y-1|)+C.$$

Solution to III.4

$$(-3/x)+\ln(|x+1|/|x|)+C.$$

Solution to III.5

$$2\tan^{-1}(|x|/\sqrt{2})+(3/4)\ln(x^2/(x^2+2))+C.$$

Solution to IV.1

$$-x\cos(x)+\sin(x)+C.$$

Solution to IV.2

$$-x^2\cos(x) + 2x\sin(x) + 2\cos(x) + C.$$

Solution to IV.3

$$(x^2/4)(2[\ln(x)]^2 - 2\ln(x) + 1) + C.$$

Solution to IV.4

$$x\sin^{-1}(x) + \sqrt{1-x^2} + C.$$

Solution to IV.5

$$x\tan(x) + \ln(|\cos(x)|) + C.$$