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PROFESSOR:

Now, today I need to get started by finishing up what I did last time. Namely, talking about numerical methods. And I want to just carry out one example. And then I want to fill in one loose end. And then we'll talk about the unit overall. We were talking, last time, about numerical integration. I'm going to illustrate this just with the simplest example that I can. We're going to look at the integral from 1 to 2 of dx / x . Which we know perfectly well already is the log of x evaluated between 1 and 2, which is $\ln 2 - \ln 1$. Which is just $\ln 2$. Now, if you punch that into your calculator, you're going to get something like this. I hope I saved it here. Yeah. It's about 0.693147. That's more digits than we're going to get in our discussion here. Anyway, that's about how big this number is.

And the numerical integration methods will give you about as much accuracy as you can get on the function itself. And, of course, some functions we may have more trouble approximating. But the function $1 / x$, we know pretty well how to do, because we know how to divide. So since the function that we're integrating here is $1 / x$, it's going to be not too difficult to get some arithmetic. Nevertheless, I'm going to do this in the simplest possible case. Namely, just with two intervals.

Now, you really can't expect things to work so well with two intervals. That's a pretty ridiculous approximation to your function. When you have two intervals, that means you're looking at the graph of this hyperbola. And you have 1 here, and you have 2 here and you have $3/2$. And you're really only keeping track of the values at these three spots. So the idea that you can approximate the area just by knowing the values of three places is a little bit of a stretch of the imagination. But we're going to try it anyway. Now, the trapezoidal rule is the following formula. It's $\Delta x (1/2 \text{ the first value} + \text{the second value} + 1/2 \text{ the third value})$. In this case, the pattern is $1/2, 1, 1, 1, 1, 1, 1/2$. And in this case, $\Delta x = 1/2$ because this interval's of length 1. The $b - a$, right. Let's just point that out here. Here, $b = 2$. $a = 1$. $b - a = 1$. And the number n is 2. And so, Δx , which is $(b - a) / n$, is $1/2$.

So here's what we get. And let's just see what this number is. It's $1/2$ of the value at here. Well,

so let's just check what these values are. This value is 1, this value over here is $2/3$, and the last value is $1/2$. Because the function, of course, was $y = 1/x$. And those were the three values that we have. So y_0 , this one is y_0 , this one is y_1 , and this one is y_2 . Now, here we have $1/2 * 1 + 2/3 + 1/2 * 1/2$. Now, on an exam, I don't expect you to add up long messes of numbers like this. When you have two numbers, I expect you to add them up if they're reasonable, or subtract them. Just as we do when we take antiderivatives. Like, for example, I don't want you to leave the answer to an integration like this in this form. I want you to simplify it at least down to here. And I of course don't expect you to know the numerical approximation. But I certainly expect you to be able to do that. On the other hand, when the arithmetic gets a little bit long, you can relax a little bit. But I did carry this out on my calculator. Unless I'm mistaken, it's about 0.96. It's pretty far off.

So remember what it was. It's what you get when you get these straight lines. And there are these little extra pieces of junk there. Now, don't trust that too much, but the point is that it's far off. So now, let's take a look at Simpson's Rule. And I claim that Simpson's Rule is surprisingly accurate. In this case, really, even a little more than it deserves to be. The formula is $(\Delta x / 3) (y_0 + 4y_1 + y_2)$. So the pattern is 1, 4, 1, or 1, 4 and then it alternates 2's and 4's until 4, 1 at the very end. And if I just plug in the numbers now, what I get is $1/6$, because $\Delta x = 1/2$ again. And the value for y_0 was 1. And the value for y_1 was $2/3$. And the value for y_2 was $1/2$. So here's the estimate in this case. And this one I did carry out carefully. And it came out to 0.69444. Which is actually pretty impressive, if you think about it. Given what the logarithm is.

Now, what's going on with Simpson's Rule in general is this. If you-- Simpson's minus the exact answer, in absolute value, is approximately of the size of $(\Delta x)^4$. That's really the way it behaves. Which means that if Δx is about $1/10$, so if we had divided this up into 10 intervals, which we didn't, but if we'd divided it up into 10 intervals, then you could expect that Δx -- the error would be about 10^{-4} . In other words, four digits of accuracy here for this thing. But the exact analysis of this, a more careful analysis of this, is in your textbook. And I'm not going to do it. But I just want to point out that it is an effective method. It really does give you nice four-digit with manageable-- you could even really do it by hand. It's so convenient, the Simpson's Rule. Whereas the other rules aren't really that impressive as far as giving fairly accurate answers.

The last little remark to make is that the reason is that Simpson's Rule is matching a parabola.

And somehow the parabola follows this curve better. It's giving the exact answer. So I'll mention this. Simpson's Rule is derived using the exact answer for all degree 2 polynomials. In other words, parabolas. All parabolas. But even all the ones of lower degree. So straight lines would work, and constants would work as well. Whereas the other ones only work for, say, straight lines. The trapezoidal rule only works for straight lines. But there is a weird accident. It turns out that it also works for cubics. Once you get the formulas, it works for cubics. So it's also exact for cubics. And that's what explains the fourth order validity. The last thing that I want to point out is that this is extremely vague, what I said there. And you should be a little bit cautious about it. You need to watch out for $1/x$ for x near 0. All bets are off if the function is singular. And there's a lot of area under there. And it's also true that if the derivative messes up, you're in trouble too. You really need for the function to be nice and smooth in order for Simpson's Rule to work. This is wath out. That's a real wath out, but we'll try to-- Watch out. Watch out for whenever x near 0. Then this thing doesn't work. This thing really depends on bounds on derivatives. But I'm going to be relatively vague about that. I'm not attempting to give you an error analysis here.

OK, so if you were doing this on an exam, how do you remember this strange pattern of numbers? The one thing that I want to recommend to you is, as a way of remembering it, so the one mnemonic device, we'll call it a mnemonic device here, for remembering what it is that you're doing, is to remind yourself of what happens for the simplest possible case. Which is $f(x) = 1$. It seems very modest, but if it doesn't give you the exact answer for $f(x) = 1$, you've got the wrong weightings. And here, if you check out what happens in the first formula here, $y_0 / 2 + y_1 + \dots$, well, we'll go all the way to $y_{(n-1)} + y_n / 2$. If you check that formula out here, this is the trapezoidal rule. If you check it out for this case, then what you get is that this is equal to Δx times what? Well, all of these are 1's. And how many are there in the middle? There are $n - 1$ of them in the middle. So it's $1/2 + n - 1 + 1/2$. At the tail end. So all told it's $\Delta x n$. And I remind you that $\Delta x = b - a / n$. So, Δx , this thing, is equal to $b - a$. And that's just as it should be. What we just calculated is an approximation to this integral here. Which is just the area of the rectangle of base $b - a$ and height 1. Which of course is $b - a$.

So this is the check that you got your weighted average correct here. You've put the correct weightings on everything. And you can do this same thing with Simpson's Rule. And match up those quantities. There was a question in the room at some point. No, OK.

So now, the next thing I want to do for you is the loose end which I left hanging. Namely, I want to compute that mysterious constant square root of $\pi / 2$. This is really one of the most famous computations in calculus. And it's a very, very clever trick. I certainly don't expect you to come up with this trick. I certainly wouldn't have myself. But it's an important thing to calculate. And it's just very useful. So I'm going to tell you about it. And it's just on the subject that we're dealing with in this unit; namely, slicing. Or adding up.

So the first step, which is just something that we already did, was that we found the volume under this curve. This bell-shaped curve, e^{-r^2} . But rotated around an axis. Rotated around this axis. Around this way. So we figured that out. And that was a relatively short computation. I'm just going to remind you, it goes by shells. We integrate the whole range from 0 to infinity. And we have $2\pi r e^{-r^2} dr$. So this again is the circumference of the shell. This is the height of the shell, and this is the thickness of the shell. Circumference, height, thickness. So we're just taking a little piece here and sweeping it around. And then adding up. And then this antiderivative is $-\pi e^{-r^2}$, evaluated at 0 and infinity. And we worked this out last time. This is π . It's $\pi(1 - 0)$. Which is π .

So the conclusion is that $V = \pi$. We already know that. Now, the problem that we want to deal with now is the problem not of a volume, but an area. And this looks quite different. And of course the answer is going to be different. But let's do it. So this is this question mark here. And I'm going to do the one from minus infinity to infinity. And I'll relate it to what we talked about earlier in this unit, in just a couple of minutes when I show you the procedure that we're going to follow. So here's the quantity and now, what this is interpreted as is the area under this bell curve. This time, Q is really an area.

Now, what's going to turn out to happen, is this. This is the trick. We're going to compute V in a different way. And you'll see it laid out in just a second. We will compute V by slices. We're going to slice it like a piece of bread here. We're going to solve for that same thing here. And then, amazingly, what's going to happen is that we will discover that $V = Q^2$. That's going to be what's going to come out. And that's the end of the computation that we want. Because actually we already know what V is. We don't want to read this equation forward. We want to read it the other way. We want to say $Q^2 = V$, which we already know is π . And so Q is equal to the square root of π .

I haven't shown this yet, this is the weird part. And I'm going to put it in a little box so that we know that this is what we need to check. We need to check this fact here. We haven't done

that yet. Now, let me connect this with what we did a few days ago. With what I called one of the important functions of mathematics besides the ones you already know. And so the function that we were faced with, and that we discussed, was this one. And then, we were interested in the value at infinity. We were interested in this. Which, if you draw a picture of it, and you draw the same bell curve, that's the area under half. of it. That's the area starting from 0 and going to infinity. That's the area under half. So this chunk is F of infinity.

And now I hope that this part of the connection is not meant to be fancy. The idea here is that $Q = 2 F(\text{infinity})$. This number here. And so $F(\text{infinity})$ is equal to the square root of π over 2, if we believe what we said on the last panel. And that was the thing that I drew a picture of on the board. Namely, the graph of F looked like this. And there was this asymptote, which was the limit $F(x)$ tends to square root of π over 2, as x goes to infinity. That was that limiting value. Which is F of infinity. So this is the asymptote. And now I've explained the connection between what we claimed before, which was quite mysterious, and what we're actually going to be able to check now. Concretely, by making this computation.

So how in the world can you get something like this. What's in that orange box there, that $V = Q^2$. Again, the technique is to use slices here. And I'm going to have to draw you a 3-D picture to visualize the slice. Let's do that. I'm going to draw three axes now, because we're now going to be in three-dimensional space, and I want you to imagine the x -axis as coming out of the blackboard, the y -axis is horizontal, and there's a new axis, which I'll call the z -axis, which is going up. So what's happening here is that I'm thinking of this-- This is, if you like, some kind of side view. And this is a view where I've tilted things a little bit up to the top. Now, the distribution, or you could think of this target in the plane, where the most likely places to hit were in the middle and it died off. As we went down. Now, I want to draw a picture of this graph. I'm going to draw a picture of $e^{-(r^2)}$. And it's basically a hump. So I'm going to take the first-- the slice along $y = 0$. The $y = 0$ slice. And I claim that that goes up like this. And then comes back down. Let me shade this in, so that you can see what kind of a slice this is. This is supposed to be along this vertical plane here. Which is coming out of the blackboard and coming towards you. And that's a slice.

Now, I'm going to draw one more slice so that you can see what's happening. I'm going to draw a slice at another place. Along here. This will be $y = b$. Some other level. And now I'm going to show you what happens. What happens is that the hump dies down a little bit. So the bump is just a little bit lower. And it's going to look a little bit the same way. But it's just going to

be a bit smaller. So there's another slice here. Like that. And I want to give a name to these slices. I'm going to call this $A(b)$. That is, the area of the b slice. Under the surface. OK? Yes, question.

STUDENT: [INAUDIBLE]

PROFESSOR: Yeah, the solid. Yeah. We're trying to figure out this volume here, which is the one we started out with, by slices. So first I have to think of-- I'm going to visualize-- So here I didn't even visualize. I took a cross section and I thought about how to spin it around without actually doing that in three-dimensional space. But now I'm going to take a different kind of slice. I'm going to take that same bump, which is a three-dimensional object. I'm going to lay it down on a plane. Which looks like this. And then it's a bump here. It's a hump. And now I'm going to try to slice it by various planes.

STUDENT: [INAUDIBLE]

PROFESSOR: So one way of defining the bump, as you just suggested, is you take this curve and you rotate it around this z -axis. So in other words, you make this the axis of rotation, you spin it around. That's correct. So that shows you that the peaks as you go down here are going to descend the same way. But I don't want to draw those lines. I want to imagine what the parallel slices are. Because I don't want to get cross slices. I want all slices parallel to the same thing.

STUDENT: [INAUDIBLE]

PROFESSOR: OK. This is not particularly easy to visualize. Now, here's the formula for volume by slices. The formula for volume by slices is that you add up the areas of the slices. That's how you do it. You take each slice. You add the cross-sectional area, and then you take a little thickness, dy , and then you add all of them up. Because this is extending over the whole plane, we're going to have to go all the way from minus infinity to plus infinity. And this is the formula for volumes by slicing. And now our goal, in order to do this calculation, we're going to just fix y is equal to some b . We're just going to fix one of these slices. And we're going to calculate $A(b)$. That's what we need to do in order to make this procedure succeed.

This is the only place where this method works. But it's an important one. In order to make it work, I'm going to have to again draw the plot from a different point of view. I'm going to do the top view. So I want to look down on this x - y plane here. This is the x -direction, and here's the y -direction. And then again I want to draw my slice. My slice is here. At $y = b$. So we're just

right on top of it. And it's coming up at some kind of bump. Here, with a little higher in the middle and going down on the sides. Now, the formula for the height is this. If I take a distance r here, the formula for the height of the bump is $e^{(-r^2)}$. I'll store that over here. $e^{(-r^2)}$ is the height at this place. If this distance to the origin is r . That's true all the way around. And in terms of b and x , we can figure out that by this right triangle. This height is b , and this distance is x . So $r^2 = b^2 + x^2$. Question.

STUDENT: [INAUDIBLE]

PROFESSOR: The question is, is that the x - y plane. So the answer is that over here I cleverly used the letter r . I avoided using y 's and z 's or anything. And over here, this is the distance r . And you like, this is z , going up. That's the way to think of it. So that all of the letters are consistent. So I just avoided giving it a name. That's good, that's exactly the point. Alright. So now, I claim I have a formula for r^2 . And so I can write this down. This $e^{(-b^2 + x^2)}$. But now I'm going to use the rule of exponents. Which is that this is the same as $e^{(-b^2)}$ times $e^{(-x^2)}$. And that's going to be the main way in which we use the particular function that we're dealing with here. That's really the main step, amazingly. So now I get to compute what $A(b)$ is.

$A(b)$ is the area under a curve. So it's going to be, let me write it over here, $A(b)$ is the area under this curve here. Which is some constant times-- so if you imagine, call this thing the name c . Under some curve, $ce^{(-x^2)}$. Where the c is equal to $e^{(-b^2)}$. That's what our slice is. In fact, it looks like one of those. It looks like one of those bumps. Here's its formula again. It's the integral from minus infinity to infinity of $e^{(-b^2)} e^{(-x^2)} dx$. We just recopied what I had up there. And this is the height at each value of x , with b fixed. And now, so we have a lot of steps here. But each of them is very elementary. The first one was just that law of exponents. That we could split the two into products. Now I'm going to make that splitting even further. This is a constant. It's not varying with x . So I'm going to factor it out of the integral. This is $e^{(-b^2)}$ times the integral from minus infinity to infinity of $e^{(-x^2)} dx$. So this might look frightening, but actually it's just the property of an integral. All integrals have this kind of property. You can always factor out a constant.

And now here comes the remarkable thing. This is $e^{(-b^2)}$ times a number which is now familiar to us. What is this number? This is what we're looking for. This is our unknown, Q . So I've computed $A(b)$, and now I'm ready to finish the problem off. $A(b) = e^{(-b^2)} Q$. Q is that strange number which we don't know yet. What it is. So now I'm going to compute the whole volume. The whole volume, remember, it's over there, it's minus infinity to infinity, $A(y) dy$. And

now I'm just going to plug in the formula that we've found for A. Now I'm doing this for each b, so I'm doing it varying over all b's. So I have the integral from minus infinity to infinity. And here I have $e^{(-y^2)}$. I've replaced b by y. And now I have Q. And I have dy. I just recopied what I had over there into the formula for slicing. And now, I'm going to do this trick of factoring out the constant a second time.

This is a constant. It doesn't depend on y. It's the same for all y, it just will factor out. So this is the same as Q times the integral from minus infinity to infinity, $e^{(-y^2)} dy$. And now, lo and behold, this expression here. Of course, notice how I defined Q. Let's go back carefully to where Q is defined. Here's Q. This t is a dummy variable. It doesn't matter what I call it. I can call it x, I can call it u, I can call it v. In this case, I've given it two different names. At this stage, I called it x. And at this stage I'm calling it y. But it's the same variable. And so this little chunk is Q and altogether I have two of them, for Q^2 being the total. And that's the end of the argument. It's a real miracle.

STUDENT: [INAUDIBLE]

PROFESSOR: Great question. The question is, wait a minute. As y changes, doesn't x change. And so then this wouldn't be a constant. So that's the way in which we've used the letters x and y in this whole course. When you get to 18.02, you'll almost never do that. Always y and x will be different variables. And they won't have to depend on each other. Now, let me show you where on this picture the x and the y are. We've got a whole x-y plane, and here I'm fixing $y = b$, y isn't varying. Whereas x is changing. So, in other words, I don't have a relationship between x and y, unless I fix it. In this case I've decided that y is going to be constant. For all x. Over here, I made a computation. And I have a Q, which is just a single number. No matter which b I took, it didn't matter which. No matter which y equals b. Of course, I changed the name to b so it wouldn't be so jarring to you. But in fact this b was y all along. It's just that the x varied completely independently of the y. I could fix the y and vary the x, I could fix the x and vary the y. So it's a different use of the letters. From what you're used to. It happens that y is not a function of x. In this case. Yes.

STUDENT: [INAUDIBLE]

PROFESSOR: Yes.

STUDENT: [INAUDIBLE]

PROFESSOR: The question is, because I'm rotating around the z-axis, doesn't x change exactly as much as y does. What happens is that x and y are symmetric variables. They can be treated equally. But if I decide to take slices with respect to y being fixed and x varying, then of course they're now separated, and I have a separate role for the x and a separate role for the y. Or if I'd sliced it the other way, I would have gotten the same answer. I just would have reversed the roles of x and y. So what's happening is that x and y are on equal footing with each other in this picture, and I could've sliced the other way. I would have gotten the same answer. That's more or less the answer to your question. OK.

Now I have given you a review sheet, and I want to run through, briefly, what's going to be on the exam. And this list of exam questions is what's going to be on the exam. There are, sorry this is not displayed correctly. So, exam questions, but now I'm just going to show you what they are. There are five questions on the exam. They are completely parallel to what you got last year. So you should look at that test. It's worth looking at. And you'll see in the descriptions on this sheet that what I'm describing is what's on that test. So what's going to happen is - and this is also posted on the Web - is that you'll be expected to calculate some definite integrals using the fundamental theorem of calculus. Do a numerical approximation. There'll be a Riemann, a trapezoidal rule and a Simpson's Rule. Calculate areas and volumes. And then some other cumulative sum. Either an average value or probability or perhaps work. And sketch a function which is given in this form as an integral. So those are the questions, and you'll see by the example of last year's exam exactly the style. They're really going to be very similar. Yes, question.

STUDENT: [INAUDIBLE]

PROFESSOR: OK, good question. So the question is, for Riemann sums, what's the difference between upper and lower, and right and left? So here we have a Riemann sum. And I'm going to give you a picture which is, maybe this function $y = 1/x$, which was the one that we were discussing earlier. If you take the function $y = 1/x$ and you break it up into pieces here, however it doesn't matter how many pieces, let's just say there are four of them. Then the lower Riemann sum is the staircase which fits underneath. So this one is a picture of the lower sum. It's always less. And in the case of a decreasing function, it's going to be, so since if you like, since $1/x$ decreases, the lower sum equals the right sum. You can see that visually on this picture. The values you're going to select are going to be the right ends of the rectangles. The upper sum is the left one. Now, if the function wiggles up and down, then you have to pick

whichever side is appropriate. Or maybe it'll be a point in the middle, if the maximum is achieved in the middle. Yeah, another question.

STUDENT: [INAUDIBLE]

PROFESSOR: Correct. If the function is increasing, then the lower sum is the left sum. So it just exactly reverses what's here. So this is decreasing, lower sum is right-hand sum. Increasing, lower sum is left-hand sum.

STUDENT: [INAUDIBLE]

PROFESSOR: Yes.

STUDENT: [INAUDIBLE]

PROFESSOR: Good question. Suppose you're faced with a function like this in this last problem. Which, generally, these are the trickiest problems. And the question is, how are you ever going to be able to decide on an asymptote, even whether there is an asymptote. And the answer is, you're not. It's going to be pretty tricky to get keep track of what's happening as it goes to infinity. We had an example on the homework where it was oscillating and it's very unclear what's going on. You have to do a very long analysis for that. So in fact, just don't worry about that now. At the very end of the class, we'll talk a little bit about these asymptotes. And really, the first issue is whether they exist or not. And that's even something. That's a serious question which we'll address at the very end of this course.

STUDENT: [INAUDIBLE]

PROFESSOR: That's right. It's not going to be anything that complicated. Other questions? We we still have a five whole minutes, and I have an example to give, if nobody has a question. Yeah.

STUDENT: [INAUDIBLE]

PROFESSOR: The question, uh, will I tell you which one of what to use?

STUDENT: [INAUDIBLE] **PROFESSOR:** When I tell you the numeric approximation is, you'll see on the exam. The practice exam that you have. I will ask you for all three. I will ask you for the Riemann sum, the trapezoidal rule, and the Simpson's rule. I'm guaranteeing you they'll all three be on the exam. I'm guaranteeing that every single thing which is on that piece of paper is on the exam. And you'll see it on the exam that you've got. It's exactly parallel to what's

there.

STUDENT: [INAUDIBLE]

PROFESSOR: So with areas and volume, the question is will I tell you which method to use. So let's discuss that.

So with areas and volumes, there's basically-- So this is always true with areas. And it's true with volumes of revolution. By the way you should read this sheet. Not everything that's on here have I said. But you should read it. Because it's all relevant. So with volumes of revolution, you always work your way back to some 2-D diagram. So there's some 2-D diagram which is always-- two-dimensional diagram, which is always connected with these problems. I mean, something this hard is really just too hard to do on an exam, right? I mean, I'm not going to ask you something this complicated on the exam. Because this involves a three-dimensional visualization. But once you're down to 2-D, you're supposed to be able to handle it. Now, what's the main issue after you've got your 2-D diagram? The main issue is, do you want to integrate with respect to dx or dy ? And the answer is that it will depend. And if there's one that's going to cause you incredible difficulty, and I feel that you're not able to dodge it, then I might give you a hint and say you'd better use shells, or you'd better use disks or washers or something like that. But if I feel that you're grown up enough to figure out which one it is, because one of them is so ridiculous you say forget it, immediately, after thinking about it. Then I won't tell you which one. Because I figure, in other words, I don't want you to waste your time. But I'm willing to waste a minute or two of your time on a wild goose chase.

So let me give you an example of this. Suppose you're looking at the curve y between 0 and $x - x^3$. So this is some kind of lump. Like this. It goes from 0 to 1, because the right-hand side is 0 at 0 and 1 here. It's some kind of thing. And there are these two possibilities. One of them is to do shells. And then, so this is supposed to be rotated around the y -axis. In this case. And the same would apply, actually, to the area problem. So I'm doing a slightly more complicated problem. But you could ask for the area underneath this, and so forth. OK. So we can integrate this dx , or we can integrate this dy . This indicates that I'm deciding that this is going to be of thickness dx , and I'm integrating dx . So that's a choice that I'm making. Now, the minute I made that choice I know that these are shells. Because they sweep around this way and that makes them shells. Cylindrical shells. And if I do that, the setup is this. It's $2\pi x (x - x^3) dx$. Now, I claim that when you get to this point, you already know you've won. Because this is an easy integral to calculate. So you're done here. You're happy.

Now, if you happened to say, oh gee, I hate to do this. I want to do something clever, you could try to do it with cutting this way. Let's do this. And this would be the dy thickness. And then when you sweep this around, you get what we call a washer. Which is really just the difference of two disks. So the shape here is this thing swung around this axis. And it looks like this. So it's going to be the difference of radii. So what's the formula for this? It's some integral of pi times the right end, which I'll call x_2 , and here the left end, which I'll call x_1 . So this is $\pi (x_2^2 - x_1^2) dy$. Now, already at this stage, you think to yourself this is more complicated than the other method. So you've already abandoned it. But I'm just going to go one step further into this one to see what it is that's happening. If you try to figure out what these values x_1 and x_2 are, that corresponds to solving for x_1 and x_2 in terms of y . So that's the following equation. x_1 and x_2 solve the equation that-- the curve, $x - x^3$ is equal y .

Now, look at this equation. That's the equation $x^3 - x + y$, I guess. Let's see. Yeah, that's right, is equal to 0. This is a cubic equation. Although there is a formula for this. You've never been taught the formula for this equation. So therefore, you will never, ever be able to get a formula for x_2 and x_1 as a function of y . And you'll never be able to compute this one. This is more than just a dead end, it's like crash, burn, and, you know, self-destruct. So there may be such a thing, so do the other way. Good luck, folks.