Hyperbolic Sine

In this problem we study the hyperbolic sine function:

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

reviewing techniques from several parts of the course.

a) Sketch the graph of $y = \sinh x$ by finding its critical points, points of inflection, symmetries, and limits as $x \to \infty$ and $-\infty$.

b) Give a suitable definition for $\sinh^{-1} x$ (the inverse hyperbolic sine) and sketch its graph, indicating the domain of definition.

c) Find $\dfrac{d}{dx} \sinh^{-1} x$.

d) Use your work to evaluate $\displaystyle\int \frac{dx}{\sqrt{a^2 + x^2}}$.

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