The Second Fundamental Theorem of Calculus

We're going to start with a continuous function f and define a complicated function $G(x) = \int_a^x f(t)\,dt$. The variable x which is the input to function G is actually one of the limits of integration. The function f is being integrated with respect to a variable t, which ranges between a and x. The variable t is a dummy variable, and is the variable of integration. Don't get t and x mixed up, even if your textbook does.

Theorem: If f is continuous and $G(x) = \int_a^x f(t)\,dt$, then $G'(x) = f(x)$.

From the point of view of differential equations, $G(x)$ solves the differential equation

$$y' = f, \qquad y(a) = 0.$$

The second fundamental theorem of calculus tells us that we can always solve this equation (by using Riemann sums if necessary).

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