CHRISTINE BREINER: Welcome back to recitation. We're going to practice using some of the tools you developed recently on taking derivatives of exponential functions and taking derivatives of logarithmic functions. So I have three particular examples that I want us to look at. And I'd like us to find derivatives of the following functions.

The first one is f of x is equal to x to the pi plus pi to the x. The second function is g of x is equal to natural log of cosine of x. And the third one is-- that's an h not a natural log-- h of x is equal to natural log of e to the x squared. So you have three functions you want to take the derivative of with respect to x. I'm going to give you a moment to to work on those and figure those out using the the tools you now have. And then we'll come back and I will work them out for you as well.

OK, so let's start off with the derivative of the first one. OK, now, the reason in particular that I did this one-- it might have seemed simple to you, but the reason I did this one is because of a common mistake that people make. So the derivative of x to the pi is nice and simple because that is our rule we know for powers of x. So we can write this as that derivative is, pi times x to the pi minus one.

OK, but the whole point of this problem for me, is to make sure that you recognize that pi to the x is not a power of x rule that needs to be applied. It's actually an exponential function right, with base pi. So if you wrote the derivative of this term was x times pi to the x minus one, you would not be alone in the world.

But that is not the correct answer, all the same. Because this is not a power of x, this is x is the power. So this is an exponential function. So the derivative of this, we need the rule that we have for derivatives of exponential functions. So that's natural log of pi times pi to the x. That's the derivative of pi to the x. So that's the answer to number one.

OK. Number two, I did for another reason. I think it's an interesting function once you find out what the derivative is. So, this is going to require us to do the chain rule. Because we have a function of a function. But you have seen many times now, when you have natural log of a function, its derivative is going to be 1 over the inside function times then the derivative of the inside function.

So again, what we do is we take the derivative of natural log. Which is 1 over cosine x. So we take the derivative of the natural log function, evaluate it at cosine x. And then we take the derivative of the inside function, which is the derivative of cosine x. So you get negative sine x.

So you get this whole thing is negative sine over cosine. So this is negative tangent x. So the reason I, in particular, like this one is that we see, "Oh, if I wanted to find a function whose derivative was tangent x, a candidate would be the negative of the natural log of cosine of x." That in fact gives us a function whose derivative is tangent x.

So it's interesting, now we see that there are trigonometric functions that I can take a derivative of something that's not just trigonometric and get something that's trigonometric. So that's kind of a nice thing there.

And then the last one, example three, I'll work out to the right. There's a fast way and there's a slow way to do this. So I will do the slow way first. And then I'll show you why it's good to kind of pull back from a problem sometimes, see how you can make it a lot simpler for yourself, and then solve the problem.

So, I'll even write down this is the slow way. OK, the slow way would be, well I have a composition of functions here. I have natural log of something and then I have e to the something else. Right? And then that function actually, is not just e to the x. So I have some things I have to, I have to use the chain rule here.

OK, so let's use the chain rule. So I'll work from the outside in. So the derivative of the natural log function, the derivative of the natural log of x is 1 over x. So I take the derivative of the natural log function, I evaluate it here. So the first part gives me 1 over e to the x squared. And then I have to take the derivative of the next inside function, which the next one inside after natural log, is e to the x squared. And the derivative of that, I'm going to do another chain rule. I get e to the x squared times the derivative of this x squared, which is 2x.

OK, so again, this part is the derivative of natural log evaluated at e to the x squared. This part is the derivative of e the x squared. This one comes just from the derivative of e to the x is e to the x. And so I evaluate it at x squared. And then this is the derivative of the x squared part.

So I end up with a product of three functions, because I have a composition of three functions. So I have to do the chain rule with three different pieces basically. So, but this simplifies, right? e to the x squared divided by e to the x squared is 1. So I get 2x. OK, so what's the fast way? That's our answer: 2x.

But what's the fast way? Well, the fast way is to recognize that the natural log of e to the x squared-- let me erase the y here-- e to the x squared is equal to x squared. OK? Why is that? That's because natural log function is the inverse of the exponential function with base e. Right? This is something you've talked about before.

So this means that if I take natural log of e to anything here, I'm going to get that thing right there. Whatever that function is. So natural log of e the x squared is x squared. OK? If you don't like to talk about it that way, if you don't like inverse functions, you can use one of the rules of logarithms, which says that this expression is equal to x squared times natural log of e. That's another way to think about this problem. And then you should remember that natural log of e is equal to 1.

So at some point you have to know a little bit about logs and exponentials. But the thing to recognize is, that h of x is just a fancy way of writing x squared. And so the derivative of x squared is 2x. So sometimes it's better to see what can be done to make the problem a little easier. But that is where we will stop with these.