Question: Can we use the original formula?

Earlier, we found that:

$$f(x) = \frac{e^{-3x}}{\sqrt{1+x}} \approx 1 - \frac{7}{2}x.$$

Could we use a different method to get a linear approximation of the function $f(x)$?

Yes. We could calculate f' and use the formula for linear approximation to find:

$$f(x) \approx f(0) + f'(0)x.$$

This must also be a linear approximation to $\dfrac{e^{-3x}}{\sqrt{1+x}}$.

We can easily find that $f(0) = 1$. Computing $f'(x)$ by the product rule is an annoying, somewhat long computation. Because of what we've just done we know that $f'(0)$ must equal $-\frac{7}{2}$. We used linear approximation as a shortcut to avoid computing $f'(0)$ directly.

When we study quadratic approximation we'll quickly see that combining approximations for complicated functions is far superior to differentiating them twice.

Question: If we find the linear approximation by differentiating, do we have to throw away an x^2 term?

Answer: No. But remember that when x is close to 0 throwing away an x^2 term has very little influence on our final value. Throwing away the x^2 was an easy way to simplify our expression; it's not something we should be trying to avoid here. (Linear approximation just captures the linear features of the function; we are not concerning ourselves with higher order terms here.)

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