Why L'Hôpital's Rule Works

Suppose we're considering a limit:

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

that is indeterminate; i.e. $f(a) = g(a) = 0$.

We do the same thing we did in the previous example:

$$
\begin{aligned}
\lim_{x \to a} \frac{f(x)}{g(x)} &= \lim_{x \to a} \frac{f(x)/(x-a)}{g(x)/(x-a)} \\
&= \frac{\lim_{x \to a}(f(x)/(x-a))}{\lim_{x \to a}(g(x)/(x-a))} \\
&= \frac{\lim_{x \to a} \frac{f(x)-f(a)}{x-a}}{\lim_{x \to a} \frac{g(x)-g(a)}{x-a}} \qquad (f(a) = g(a) = 0) \\
&= \frac{f'(a)}{g'(a)}.
\end{aligned}
$$

We can use this formula to calculate limits of indeterminate expressions provided that $g'(a) \neq 0$.

Question: Is there a more intuitive way to understand this rule?

Answer: There are other ways to understand the rule, but none that are much more intuitive than this. It helps to understand these other ways as well. For example, we'll soon see how we can derive l'Hôpital's rule by looking at the linearizations of f and g at a.

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