## Antiderivative of $\tan x \sec ^{2} x$

Compute $\int \tan x \sec ^{2} x d x$ in two different ways:
a) By substituting $u=\tan x$.
b) By substituting $v=\sec x$.
c) Compare the two results.

## Solution

a) Compute $\int \tan x \sec ^{2} x d x$ by substituting $u=\tan x$.

If $u=\tan x$ then $d u=\sec ^{2} x d x$ and:

$$
\begin{aligned}
\int \tan x \sec ^{2} x d x & =\int u d u \\
& =\frac{1}{2} u^{2}+c \\
& =\frac{1}{2} \tan ^{2} x+c
\end{aligned}
$$

b) Compute $\int \tan x \sec ^{2} x d x$ by substituting $v=\sec x$.

If $v=\sec x$ then $d v=\sec x \tan x d x$ and:

$$
\begin{aligned}
\int \tan x \sec ^{2} x d x & =\int \sec x(\tan x \sec x d x) \\
& =\int v d v \\
& =\frac{1}{2} v^{2}+C \\
& =\frac{1}{2} \sec ^{2} x+C .
\end{aligned}
$$

c) Compare the two results.

At first glance you may think you made a mistake; it is not true that $\tan ^{2} x=$ $\sec ^{2} x$. However, you can see from the graph in Figure 1 that your two answers may only differ by a constant.


Figure 1: Graphs of $\tan ^{2} x$ (blue) and $\sec ^{2} x$ (red).

In fact, that is the case:

$$
\begin{aligned}
\tan ^{2} x & =\frac{\sin ^{2} x}{\cos ^{2} x} \\
& =\frac{1-\cos ^{2} x}{\cos ^{2} x} \\
& =\sec ^{2} x-1
\end{aligned}
$$

We conclude that $\frac{1}{2} \tan ^{2} x=\frac{1}{2} \sec ^{2} x-\frac{1}{2}$ and so the two results are equivalent up to an added constant. Both answers are correct.

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