PROFESSOR: Hi. Welcome back to recitation. We've been talking about applications of integration, including finding the areas between curves. So I have here a nice little region that I like. I think it's kind of cute. So it's the region between y equals sine x and y equals cosine x. And, you know, those two curves cross each other over and over again. They wrap around each other. So I'm just interested in the region between the two consecutive points where they cross. So here, you know, so at pi over 4 we have sine x equals-- sine pi over 4 equals cosine pi over 4. And at 5 pi over 4 they're also equal again, down in the third quadrant there.

So, the question is to compute the area of this region that they bound between those two points. So why don't you take a couple of minutes, work through that, come back and we can work through it together.

All right, welcome back. So, from this picture it's pretty easy to see what the region of integration is. I mean what the bounds on x will be. As we said, x has to go the left, I mean, you could-- sorry, let me start over. You could do any two consecutive intersection points you want, right? The area is the same in any case because of the symmetry of these two functions. So, OK. But the first two that are easiest for me to find are this pi over 4 and this 5*pi over 4. So I'm going to do those two. If you wanted, you could have done it with a different pair of consecutive points. But, once we've agreed that sort of these first two are the ones I'm going to do, then it's easy. I know that they're pi over 4 and 5*pi over 4. So that's the interval over which I'm going to be integrating.

And then, what I want to do is just view this region as cut into a lot of little rectangles. And I want to integrate the height of those rectangles in order to get the area of the whole region. So in this case, the upper curve is y equals sine x and the lower curve is y equals cosine of x. So the height of a little-- you know if I draw in a little rectangle here-- the height of that rectangle is going to be sine x minus cosine x. Its width is dx. And then I add them all up by integrating.

So the area is just the integral from pi over 4 to 5*pi over 4 of sine x minus cosine x dx. Top minus bottom to get the height. If you did it backwards; if you wrote cosine minus sine, what you would get is exactly the negative of the area. So it's, all right, from pi over 4 to 5, pi over 4.

So now, we just have to integrate this. So integral of sine, the function whose derivative is sine is minus cosine x. And the function whose derivative is cosine is sine. So it's minus sine x between pi over 4, 5*pi over 4. And OK, so now, we just have to plug in the values. So this is

equal to minus cosine 5*pi over 4 minus sine 5*pi over 4 minus minus cosine pi over 4 minus sine pi over 4. And I'm sure you can work out for yourself that this is equal to 2 times the square root of 2 if I haven't botched anything terribly. So I'll end there.