

Example: $\int \sin^3 x dx$

The integral $\int \sin^3 x dx$ is of the form $\int \sin^n x \cos^m x dx$ with one exponent odd, and the other exponent equal to zero, so it is in the easy case. We again use the trig identity $\sin^2 x + \cos^2 x = 1$ to remove the largest power of $\sin x$ that we can from the cube:

$$\int \sin^3 x dx = \int (1 - \cos^2 x) \sin x dx$$

Substitute $u = \cos x$ and $du = -\sin x dx$ to get:

$$\begin{aligned} \int \sin^3 x dx &= \int (1 - u^2)(-du) \\ &= -u + \frac{u^3}{3} + c \\ &= -\cos x + \frac{\cos^3 x}{3} + c \end{aligned}$$

In general, any time you have an odd power in an integral of the form $\int \sin^n x \cos^m x dx$ you can integrate it using the trig identity $\sin^2 x + \cos^2 x = 1$ and a substitution.

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