

Introduction to Series

Remember that we're dealing with infinity; we'll be studying *infinite* series. The most important and useful series is the geometric series. We'll start with a concrete example:

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \cdots$$

We can visualize this infinite sum by marking its partial sums on the number line, as shown in Figure 1. The figure shows the results of adding 1, then $\frac{1}{2}$, then $\frac{1}{4}$, then $\frac{1}{8}$. Note that the value of each partial sum is midway between the value of the previous sum and 2. We say that the series converges to 2.

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \cdots = 2.$$

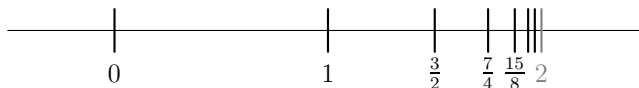


Figure 1: Adding a term gives a number half way between the previous value and 2.

Note that we never get to 2, we just get halfway there infinitely often. This is known as Zeno's paradox — if a rabbit gives a tortoise a head start in a race, the rabbit can never pass the tortoise because it must cross half the distance between itself and the tortoise infinitely many times. One resolution of this paradox requires understanding time as a continuum, which Zeno failed to do.

In general, a *geometric series* is a series of the form:

$$1 + a + a^2 + a^3 + \cdots = \frac{1}{1 - a} \quad (|a| < 1).$$

We'll see how this formula is derived later.

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