Hi.

Welcome back to recitation.

In class, you've been learning about convergence tests for infinite series.

I have here three examples of series that I happen to like.

So the first one is the sum from n equals 0 to infinity of 5n plus 2 divided by n cubed plus 1.

The second one is the sum from n equals 1 to infinity of the quantity 1 plus the square root of 5 over 2, all that to the nth power.

And the third one is the sum from n equals 1 to infinity of the natural log of n divided by n squared.

So what I'd like you to do for each of these three series is to figure out whether or not they converge.

So whether they converge, or whether they diverge.

So why don't you pause the video, take some time to work that out, come back, and we can work on it together.

Welcome back.

Hopefully you had some luck working on these series.

So let's talk about how we do them.

So I'll start with the first one here.

So this first one, the function that we're, that we're summing is this 5n plus 2 divided by n cubed plus 3.

And so you want to know, does that sum of that expression, of n goes from 1 to infinity, does that converge?

Does it reach some finite value?

Or does it diverge?

Does it either, you know, oscillate, or go to infinity, or something like that?

So for one thing you can always do, is something like a comparison test. And so what you want to do there, is you want to extract the information from the expression for the summand.

You want to figure out, you know, about how big this is.

What are the important parts of it?

So in this case, you have a ratio.

And so when you have a ratio, what you want to do, is you want to look at what's the magnitude of the top, and what's the magnitude of the bottom.

So roughly speaking, the magnitude of the top is of the order of n.

Right?

It's 5n.

The 5 is just a constant multiple.

That's not going to make a whole big difference.

And the plus 2 is just much, much, much smaller than the n when n gets very, very large.

So the, we can say that the order of magnitude of the top here is of the order of n, and similarly, the order of magnitude of the bottom is of the order of n cubed.

Right?

The plus 1 is much, much, much smaller than the n cubed, so it's not, it's almost insignificant.

So roughly speaking, we should expect this to be of the same behavior as if we just had n over n cubed, which is 1 over n squared.

And we know that 1 over n squared, the series 1 over n squared from n equals 1 to infinity, converges.

So that's sort of just a sort of a way of thinking about this.

And we can formalize it using the limit comparison test.

So the idea here for part A is to use the limit comparison test. So what we want to compute-- so I'm going to put a 5 in.

So I'm going to make it 5 over n squared, because we had this 5 up here.

So we have an is equal to 5n plus 2 over n cubed plus 1.

And I want to limit compare it with-- I'm going to compare my series with the series sum from n equals 1 to infinity of 5 over n squared.

Now-- I'm talking about this 5, right?

This 5 doesn't matter.

If this series-- if 1 over n squared, that sum converges then 5 over n squared, that sum converges to exactly 5 times as much.

And if, you know, if I wrote a divergent thing here, and then multiplied it by 5, the result multiplied by 5 would still diverge.

So this constant multiple isn't going to matter.

And I'm just choosing, I'm just putting the 5 in there because of this 5 we saw over here.

So OK.

So let's select work it out, then.

So the limit comparison test says-- so we look at the limit as n goes to infinity of the ratio of the two things that we're interested in.

So in this case, this is 5n and plus 2 over n cubed plus 1, divided by 5 over n squared.

And you can-- OK.

So this is a ratio of two ratios, so we can rewrite it by multiplying upstairs, and we get that this is equal to the limit as n goes to infinity of 5n cubed plus 2 n squared over 5n cubed plus 5.

And so we've seen before, when we were dealing with limits, that when you have a ratio of two polynomials, as the variable goes to infinity, the limit is what you get just by comparing the leading terms. So in this case, we just have to look at 5n cubed over 5n cubed, and that's indeed 1.

OK, so this is 1.

So by the limit comparison test, our series, the sum of this an, converges if and only if this series converges.

And we already said that we know sum 1 over n squared converges.

So by the limit comparison test, since sum 1 over n squared converges, our series converges.

Great.

OK.

So that's the first one.

Let's look at the second one.

So the second one here is the sum from n equals 1 to infinity of 1 plus the square root of 5 over 2.

That whole thing to the nth power.

Now, if you look at this, the thing you should recognize this as, is just a particular geometric series.

Right?

This is, if you were to replace 1 plus the square root of 5 over 2 with x, this is just x plus x squared plus x cubed plus dot dot dot dot dot.

It's just a geometric series with constant ratio-- well, x .

1 plus the square root of 5 over 2.

So we know exactly when geometric series converge.

They converge exactly when the constant ratio is between minus 1 and 1.

So bigger than minus 1 and less than 1.

So this series converges, then, if and only if 1 plus the square root of 5 over 2 is between minus 1 and 1.

So then we just have to think about, is this number between minus 1 and 1.

And OK.

So this is not that hard to do.

Square root of 5 is bigger than 2, so 1 plus the square root of 5 is bigger than 3, so 1 plus the square root of 5 over 2 is bigger than 3/2.

So 3/2 is bigger than 1.

So this common ratio in this series is bigger than 1.

So the terms of this series are blowing ip.

You know, when you, when n gets bigger and bigger, you're adding larger and larger numbers here.

This is blowing up.

It's a divergent geometric series.

So this series does not converge.

So b, I'll just write that here.

B diverges, because it's geometric with common ratio bigger than 1.

So that's the reason that part B diverges.

Finally, we're left with question C.

So I'm going to come over and write it over here again, so we can see it.

So part C asks the sum for n equals 1 to infinity of log n over n squared.

So this one's a little trickier, and it requires a little bit more thought.

The thing to-- do let's start just by-- it can't be solved just by a straightforward application of the limit comparison test that we've learned.

So we need to think a little bit more about what ways could we solve it.

So one thing to remember here is, is we should think about, what are the magnitudes of these things?

So we know that sum 1 over n squared converges.

But log n is a function that grows.

So this individual term is bigger than 1 over n squared.

So we can't just compare it to 1 over n squared.

But log n grows very, very slowly.

How slowly?

Well, it grows more slowly than any power of x.

Right?

So, or-- sorry-- any power of n in this case, because the variable is n.

So if you remember, we know, we've shown, that the limit of ln x over x to the p as x goes to infinity is equal to 0 for any positive p.

So ln, log n, ln n, is going to infinity, but it's going to infinity much slower than any power of n.

In this case.

Or x, down here.

So OK.

So what does that mean?

Well, one thing you could do, is you could say, oh, OK.

So ln n over n, that's getting really small.

And then what we're left with is 1 over n.

So you can say, OK.

So this is much smaller than 1 over n.

The problem is that the sum 1 over n diverges.

Yeah?

So that doesn't help us, really, right?

So we've shown this is bigger than the sum 1 over n squared, which converges, and it's smaller than the sum 1 over n, which diverges.

But that still doesn't tell us, you know?

It could be something that, you know, there's a lot of room bigger than a particular convergent series, and smaller than a particular divergent series.

And in particular, there are both convergent and divergent series in between.

So we still need, we need either something that converges that our thing is less than, or we need something that diverges that our thing is bigger than.

Right?

If we can bound our series above by something convergent, then our series converges.

Because [UNINTELLIGIBLE] has positive terms. This is important.

Or if we can bound it below by something that diverges, then we would know it diverges.

And so far, we haven't been able to do that.

But maybe we can think of a-- so I said we could write this, a second ago, I said we could write ln n over n squared is equal to ln n and over n times 1 over n.

So that was what we just said a minute ago, at which we showed is eventually less than 1 over n.

So this is true, but it wasn't useful, because the sum 1 over n diverges.

But maybe we can, we can do something even a little more tricky.

Because here we saw that x, we could use it in the base, we could use any power of the variable, any positive power.

So here, we tried it with the power n to the 1.

We tried to split this 2 as 1 plus 1, and we kept one of the n's to knock out the log n, and we kept the other n over here.

But we don't need a whole power of n to knock out the log n.

Any positive power of n would do.

So in particular, we could split this using, say, just a small power of n, even smaller than the first power here, and leave more of it over here.

So we also know, we also have that ln n over n squared is equal to, for example, ln n over-- well, you know, for example, n to the 1/2 times 1 over n to the 3/2.

And now we know that ln n over n to the 1/2 goes to 0 as n gets large.

So this is thing is getting smaller and smaller and smaller.

So as this gets smaller, we have that this has to be less than 1 over n to the 3/2.

So this thing that we're adding up here is smaller than 1 over n to the 3/2.

Well, what's the significance of that?

Well, we know 1 over n to the 3/2, that series converges.

Yeah?

We know the sum 1 over n to the 3/2 from n equals 1 to infinity converges, because 3/2 is bigger than 1.

OK?

So what does that mean?

Well, we have our series, and we've shown that the terms of our series are eventually bounded below 1 over n to the 3/2.

And we know that the sum of 1 over n to the 3/2 converges, so our series is bounded above by a convergent series.

So whenever you have a series bounded above, a series of non-negative terms bounded above, by a convergent series, that means your series also has to converge.

OK, so this converges.

So it converges by a comparison to this other series.

Using this cute trick that we can replace a log with any small positive power of n that happens to be convenient.

And of course, if you wanted to, maybe you could have made this 1/2 a 1/10 and that would have been fine, or you could've even made it a 9/10, and then you would here be left with n to the 11/10, and that would still be OK.

Because that would be 11/10, and 11/10, then it would still be bigger than 1.

All right.

So this is a nice use of a, of a comparison test. We didn't use exactly the limit comparison test as it was described in lecture, but it's a very closely related process that we went through to show that this second series-- sorry, this third series-- also converges.

So I'll end there.