

Interpretation of the Fundamental Theorem

We'll talk about a proof of the fundamental theorem later; for now let's get a more intuitive interpretation of the theorem. We'll use the example of time and distance, rather than using area again.

So, t is a time and $x(t)$ is a position at time t . The rate of change of position with respect to time is $\frac{dx}{dt} = x'(t)$, and is also known as the speed $v(t)$. We have a function $x(t)$ and its derivative $v(t)$, so the fundamental theorem tells that:

$$\int_a^b v(t) dt = x(b) - x(a).$$

On the right hand side we have information about distance traveled from time $t = a$ to time $t = b$; this is what you would read on your odometer. The left hand side is what you would read on your speedometer during the trip. The fundamental theorem of calculus is telling us that if we know how fast we're going at every stage of a trip, we can figure out how far we traveled.

Let's go one step further with this interpretation to make a connection to Riemann sums. First imagine that you are extremely obsessive and while you're driving from time a to time b you check your speedometer every second. When you've read your speedometer in the i 'th second, you see that you're going at speed $v(t_i)$.

How far do you go in that second? You go this speed times the time interval Δt ; in this case $\Delta t = 1$ second. The distance traveled in the i th second is:

$$v(t_i)\Delta t.$$

Over the entire trip, you travel the sum of all these distances:

$$\sum_{i=1}^n v(t_i)\Delta t,$$

where n is some ridiculous number of seconds. The value of that sum is very close to the distance traveled recorded on your odometer because your speed doesn't change much over the course of a second. In other words,

$$\sum_{i=1}^n v(t_i)\Delta t \approx \int_a^b v(t) dt.$$

The Riemann sum, on the left, is approximately how far you traveled. The integral, on the right, is exactly how far you traveled.

MIT OpenCourseWare
<http://ocw.mit.edu>

18.01SC Single Variable Calculus
Fall 2010

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.