

Numerical Integration

Compare the trapezoidal rule to the left Riemann sum. The area of each trapezoid is calculated using twice as much information (left *and* right endpoints) as the area of each rectangle. This leads one to expect that applying the trapezoidal rule with $n = 6$ should produce a result comparable to the one obtained from a Riemann sum with $n = 12$.

- a) Open the Riemann Sums mathlet. Set the function to $x^3 - 2x$ and select the trapezoidal rule. Make sure $n = 6$. Record the mathlet's estimate of the integral.
- b) Select "Evaluation point" to change to the Riemann sum approximation. Move the slider to 0.5, setting the evaluation point to be the midpoint of the interval. Set n equal to 12 and record the mathlet's estimate of the integral.
- c) Calculate $\int_{-1}^2 x^3 - 2x dx$ by hand. Was the accuracy of the Riemann sum with $n = 12$ comparable to that of the trapezoidal rule with $n = 6$? Why or why not?

Solution

- a) Open the Riemann Sums mathlet. Set the function to $x^3 - 2x$ and select the trapezoidal rule. Make sure $n = 6$. Record the mathlet's estimate of the integral.

Estimate = 0.93750

- b) Select "Evaluation point" to change to the Riemann sum approximation. Move the slider to 0.5, setting the evaluation point to be the midpoint of the interval. Set n equal to 12 and record the mathlet's estimate of the integral.

Estimate = 0.72656

- c) Calculate $\int_{-1}^2 x^3 - 2x dx$ by hand. Was the accuracy of the Riemann sum with $n = 12$ comparable to that of the trapezoidal rule with $n = 6$? Why or why not?

$$\begin{aligned}\int_{-1}^2 x^3 - 2x dx &= \left[\frac{1}{4}x^4 - x^2 \right]_{-1}^2 \\ &= \left(\frac{1}{4} \cdot 16 - 4 \right) - \left(\frac{1}{4} - 1 \right) \\ &= \frac{3}{4} = 0.75\end{aligned}$$

In this case, the Riemann sum gives a much more accurate estimate than the trapezoidal rule.

Although computing the area of each individual trapezoid requires more information than computing the area of a rectangle, the trapezoidal rule as a whole uses only one more piece of information than the left Riemann sum does. Compare the two formulas:

$$\begin{aligned}\text{trapezoidal rule estimate} &= \Delta x \left(\frac{y_0}{2} + y_1 + y_2 + \dots + y_{n-1} + \frac{y_n}{2} \right), \\ \text{left Riemann sum estimate} &= \Delta x (y_0 + y_1 + y_2 + \dots + y_{n-1}).\end{aligned}$$

For a large enough number n of subdivisions, the results obtained using the trapezoidal rule are very similar to those obtained using Riemann sums; verify this using the mathlet. As Professor Jerison mentioned in lecture, the estimate given by the trapezoidal rule is exactly equal to the average of the left Riemann sum and the right Riemann sum.

In contrast, doubling the number of subdivisions does approximately double the amount of information used in an approximation and so significantly improves the estimate's likelihood of being close to the exact answer.

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