

### Arc Length of $y = \ln(x)$

Express the arc length of the graph of  $y = \ln x$  between  $x = 1/10$  and  $x = 1$  as an integral. (Do not evaluate.)

#### Solution

The formula for arc length is:  $\int_{\text{start}}^{\text{finish}} ds$ .

If the arc is a portion of the graph of  $y = f(x)$  then  $ds = \sqrt{dx + dy}$  or, equivalently:

$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}.$$

In our example,  $y = \ln x$  and  $\frac{dy}{dx} = \frac{1}{x}$ , so:

$$\begin{aligned} \text{Arc length} &= \int_{1/10}^1 \sqrt{1 + \left(\frac{1}{x}\right)^2} dx \\ &= \int_{1/10}^1 \frac{\sqrt{x^2 + 1}}{x} dx. \end{aligned}$$

If we were to evaluate this integral, using numerical or other methods, we should find that its value is slightly greater than the value of  $|\ln(1/10)| = \ln(10)$ .

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