Problem 1

a) *(10)* Evaluate the integral $\int_1^2 \int_x^{x^2} 12x \; dy dx$.

b) *(15)* Sketch the region of integration, and express the integral in the order $dxdy$.
Use two parts; do not evaluate.

Problem 2

(15) Find the polar moment of inertia I_0 for the half-disk $x^2 + y^2 < a^2$, $x > 0$, with density $\delta(x, y) = x^2$.

Problem 3

Let $\overrightarrow{\mathbf{F}} = axy\,\hat{\imath} + (e^y + 2x^2)\,\hat{\jmath}$.

a) *(5)* Find a so that $\overrightarrow{\mathbf{F}}$ is conservative.

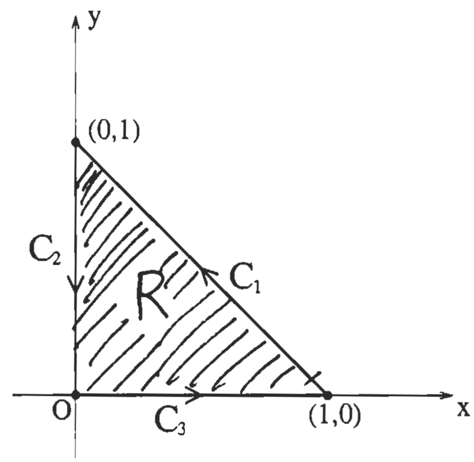
b) *(5)* For the value of a you found in part (a), find a potential function for $\overrightarrow{\mathbf{F}}$.

c) *(5)* For the same value of a as in parts (a) and (b), find the work done by $\overrightarrow{\mathbf{F}}$ along the path $x = t$, $y = \cos t$, $0 \le t \le \pi$.

Problem 4

Suppose that $\vec{\mathbf{F}} = (2xy + y)\,\hat{\imath} + x^2\,\hat{\jmath}$ and $C = C_1 + C_2 + C_3$ is the loop around the triangle as pictured.

a) *(10)* Compute the line integral $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ using Green's theorem.



b) *(15)* Compute the same line integral directly from the definition without using part (a) or Green's theorem.

Problem 5

a) *(10)* Compute the Jacobian factor $\quad du\,dv = \left|\frac{\partial(u,v)}{\partial(x,y)}\right| dx\,dy$

for the change of variable $u = x^2/y$, $v = xy$.

b) *(10)* Use this change of variable to find the area of the region in the xy-plane given by $1 \le x^2/y \le 2$, $0 \le xy \le 1$.