## Volumes and determinants

1. a) Find the volume of the parallelepiped with edges given by the origin vectors $\langle 1,2,4\rangle$, $\langle 2,0,0\rangle, \quad\langle 1,5,2\rangle$

Answer: The figure below shows the box.
The volume is $|\operatorname{det}(\mathbf{A}, \mathbf{B}, \mathbf{C})|=\left|\operatorname{det}\left(\begin{array}{lll}1 & 2 & 4 \\ 2 & 0 & 0 \\ 1 & 5 & 2\end{array}\right)\right|=|-2 \cdot(-16)|=32$.

2. We know $\left|\begin{array}{lll}1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9\end{array}\right|=0$.

What does this say about the origin vectors $\langle 1,2,3\rangle,\langle 4,5,6\rangle$ and $\langle 7,8,9\rangle$ ?
Answer: Call the three vectors $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$. Since $\operatorname{det}(\mathbf{A}, \mathbf{B}, \mathbf{C})=0$ the volume of the parallelepiped with these vectors as edges is 0 . This means all three origin vectors lie in a plane.
To see this consider the figure in problem 1. It shows the opposite case, when the vectors are not in a plane the resulting parallelepiped is really three dimensional and has non-zero volume.

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