18.02SC Multivariable Calculus, Fall 2010
Transcript – Recitation 11, Distance of a Point to a Plane

JOEL LEWIS: Hi. Welcome back to recitation. In lecture, you've been learning about planes and their equations and various different geometric problems relating to them. So I have an example of such a problem here.

So I've got a point, which happens to be the origin-- which I'm going to call P-- 0, 0, 0. And I've got a plane which has the equation 2x plus y minus 2z is equal to 4. And what I'd like you to do is compute the distance from that point to that plane.

So just to remind you, so there are lots of points on a plane, of course. And our point in question has a distance to each of them. When we talk about the distance between a point and a plane, what we mean is the shortest distance. So the perpendicular distance. So if we have the plane and we have the point, so we want to drop a perpendicular from the point to the plane, and then we're asking for that length of that segment. So that's the distance between the point and the plane.

So why don't you pause the video, take a little while to figure this out, come back, and we can figure it out together.

So hopefully you had some luck working out this problem. Let's think about it a little bit. We have a point and we have a plane, and we want to figure out what the perpendicular distance from the point to the plane is.

So one thing that's going to be important then is definitely knowing what direction that vector is. Right? We have the plane and we want to find a perpendicular segment to it. And so in order to do that, it's useful to know what direction is that segment pointing in. So luckily, we're given the plane in this simple equation form.

So the normal to the plane-- and when you're given an equation of a plane in this form-- the normal vector is just given by the coefficients of x, y, and z. So in our case, the plane is 2x plus y minus 2z equals 4, so the normal vector to this plane is the vector 2, 1, minus 2. So this is the direction in which we need to go from our point P in order to get to the plane by the shortest distance.

So now what we need is we need to know the component of-- or sorry, rather-- we need to know the actual distance we have to travel in that direction. So one way to do this is if we go back to our little picture here. We don't know what this point is. We don't know when we start from P and head in the direction perpendicular to the plane, we don't know what point we're going to land on the plane at. But what we could do is, if we knew some other point on the plane-- somewhere-- we could look at the vector connecting P to that other point, and then we could project it onto the normal direction.

So we could take the component, so let's call this other point Q. So if we choose any point Q in the plane that we're looking at, we could take the vector PQ and we can project it onto this normal vector. And if we take the component of this vector, project it onto that normal vector-- if we take the component of this vector in the direction of the normal vector-- what that will give us is exactly the length of this segment. Yeah? That projection will be exactly the perpendicular segment we're looking for. And its length, the component-- or the absolute value of the component, perhaps-- will be exactly that distance. So good.

So then we just have to compute, well we need to find a point Q and we need to compute a component. So we need any point on the plane. So, actually I'm going to walk back over here. And to find a point on the plane, we can just do this by looking at the equation.

So one way to go about this, for example, is that you pick a variable that appears in the equation. So x appears in the equation. And now you could just set all the other variables equal to 0. And that will give you something you can solve for x.

So in particular, you know, there's a point on this plane with y equals z equals 0, and that point has 2x equals 4. So we can take, for example, Q to be the point 2, 0, 0. So this is a point on the plane.

So this is our point on the plane, and so we have, what we want to do is we want to project-- so PQ, the vector from P to PQ, we get by subtracting the coordinates of P from those of Q. Q minus P. So this is the vector 2, 0, 0. And we want the component of PQ in the direction N.

So the distance in question is the-- and really, when I say component in this case, I mean the positive component I want. Because the distance has to be positive. So if I get a negative component, I really want its absolute value here. So the distance is the positive component in the direction N.

So what's that equal to? Well, it's just equal to the absolute value of-- so we know the component of PQ in the direction N is what we get when we take-- PQ and we dot it with N divided by the length of N, and then to make sure it's positive at the end, I want to throw in these absolute values signs. So OK. So this is, and this is now, you know, we have our vector PQ and we have our vector N, so it should be straightforward to compute this final expression.

So we know that N is equal to 2, 1, minus 2. So the length of N-- which is in the denominator here-- is equal to the square root of 2 squared plus 1 squared plus minus 2 squared. And in the numerator, we have the absolute value of PQ dot N. So PQ dot N is going to be 2 times 2, plus 0 times 1, plus 0 times minus 2. OK. And so if we-- and this is just a fraction bar here. And so we simplify that a little bit. So up top, we just have 4-- plus 0 plus 0 is 4. And on the bottom, we have the square root of 2 squared plus 1 squared plus 2 squared. That's going to be the square root of 9, which is 3. So this is just equal to 4/3.

So there we go. The distance in question is 4/3. The way we got it is we realized that that distance is just the component of any segment-- any vector-- connecting our point P to the plane in the direction of the normal. So you choose any vector PQ. So, you know, you just have to come up with a point Q on the plane, which you can do by inspection from the equation. So that gives you a vector that gets you from P to some point on the plane, and then you choose the component in the normal direction. And so once you do that, you get this distance: your answer.

So I'll end there.