18.02SC Multivariable Calculus, Fall 2010 Transcript – Recitation 6, Volume of a Parallelepiped

DAVID JORDAN: Hello, and welcome back to recitation. I'd like to work on a problem with you, which is to compute the volume of a parallelepiped using 3 by 3 determinants.

So here we've got the parallelepiped drawn. It has, one of its vertices is at the origin-- 0, 0, 0-- and the other three edges are given to us with these coordinates here. So why don't you take some time to work out this problem, pause the video, and check back with me and I'll show you how I solved it.

OK, welcome back. Let's get started. So the first thing that we need to do is we need to remember that computing volumes of parallelepipeds is the same thing as computing 3 by 3 determinants. So the volume is just equal to the determinant, which is built out of the vectors, the row vectors determining the edges.

So we have-- well, that's almost true. This determinant will be either a positive or negative number and we always want to take the positive number. So I'm going to write plus or minus here, and we'll have to remember at the end that we want a positive number.

So we can compute this determinant using Laplace expansion as we did in the last video. So for Laplace expansion, we take, can do Laplace expansion on the first row. And that will again be nice, because this 0 here will make our computation simpler.

And so, remember what we do is we take the first entry in the row, and then we need to multiply by the minor that we get by covering up that row and that column. So we have this little 2 by 2 determinant, which looks like minus 1.

And now we need to subtract the next entry in the row times its minor, which is now this matrix 1, 1, 0, 1. So taking that determinant, we get 1 minus 0. So altogether, we get 1.

So what that tells us is that our determinant is minus 4. But then that tells us that our volume is plus 4. And I'll leave it at that.

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