CHRISTINE BREINER: Welcome back to recitation. In this video, what I want to work on is using what we know about matrix multiplication and finding inverses of matrices to solve a system of equations. So we've set up the system already as if it's already in matrix form.

And what I'd like us to do is, for this particular A-- this 3-by-3 matrix A-- find a vector x, so that A*x equals b. Where b is equal to these two things. So you're going to do two problems.

You're going to do when b equals 1, 2, negative 3. And you're going to do when b is equal to [0, 0, 0]. So you want to find vector x so that A*x equals this value here.

And what I'd like you to do is I'd like you to use the strategy that you saw in the lecture, which is find A inverse, and then take A inverse b. So we really want to practice understanding how to find the inverse of a matrix. So why don't you work on this, pause the video. When you feel comfortable, confident, that you have the right answer, then bring the video back up, and you can compare your work with mine.

OK, welcome back. Well, hopefully you were able to make some headway and you feel confident in your answers for 1 and 2. I am going to find the inverse of the matrix A first, and then solve the problem. And because there's a lot of computation, I may make a mistake. So I'm going to have to check every once in a while that I'm doing OK. So hopefully, it's-- too bad you can't tell me if I've made a mistake, but hopefully my studio audience will help me out.

So the first thing I need to do is I need to find the determinant of A. So I'm going to do that first, and then I'm going to find the cofactor matrix and go from there. So if I want to find the determinant of A-- I guess I'll just use the first row here, because it's pretty easy.

So the determinant of A is going to be 3 times the determinant of this matrix, this 2-by-2 matrix. So it's going to be 3 times-- and then I get a 2 times negative 1, which is negative 2, and then minus 0, so I get a 3 times negative 2.

Oops. And I was about to write plus, but I should write minus. I take minus 1 times-- because this is my minus, I take negative of this thing times the matrix that is these two components in the first column and these two components in the second column. Right? We take away the column and the row that the 1 is contained in and we look at what remains, the 2-by-2 matrix that remains. And we find the determinant of that.

So we get negative 1 times negative 1, which gives me a 1. And then negative 1 times 0 gives me a 0. So I just have the negative 1 from the row 1, column 2 spot, and then the determinant of the matrix that remains is 1, OK-- of the minor matrix that remains.

And then the last one I should put a plus, but notice that it is a minus already, so I'm going to put just minus 1 times what remains. What's this minor? This one is this 2-by-2 matrix I'm looking at, right? So I need to take the determinant of this 2-by-2 matrix and multiply it by that negative 1 to get the third component here I have to add in.

Negative 1 times negative 1 is 1. And then I subtract negative 1 times 2. So this is where I have to be careful. It's 1 minus negative 2. So I'm going to get a 3. OK. 1 here minus a negative 2-- so 1 plus 2-- I'm going to get a 3. OK.

And so negative 6 minus 1 minus 3-- looks like I get a negative 10. That's good, because I think that's what I'm supposed to get. OK.

Now what I want to do is I want to find the matrix of minors for A. And then I'm going to find-- so I'm going to find the matrix of minors first, and then I'm going to switch the signs appropriately so I get the cofactors correct. OK? So some of them I already have. But, the whole matrix of minors, I'm going to just go through and do it again, to be very careful.

So the first one I delete. For the first row and column spot, I delete row 1 and column 1, and I look at the determinant of that matrix. That's 2 times negative 1 is negative 2, minus 0, so I get a negative 2 there.

For the first row, second column I come back, and I'm now again looking-- I'm deleting this column and row, and so I'm looking at the determinant of this matrix. So I get negative 1 times negative 1 is 1, minus 0, so I get a 1.

Again, I'm going to change all the signs later. So I'm going to do that in the second step.

Now I'm in row 1, column 3. So I'm going to delete row 1, column 3 and look at the determinant of that matrix. I get negative 1 times negative 1 is 1, minus the negative 2, so there's my 3.

Those I already knew, but I didn't want to just plop them in from here. But notice that is what you get here. Negative 2, 1, and 3. That's exactly where they come from, right? We got them by the same method. OK, and so now I want to find the minors for the rest of it.

So let's look at-- when I delete row 2, column 1, I'm left with 1, negative 1 here. Negative 1, negative 1 here. Right? So 1 times negative 1 is negative 1. And then negative and negative is positive. So it's negative 1 minus negative 1, so I get negative 2.

That one was a lot of signs, so you might want to check. Maybe I should check. OK, maybe I should check.

I'm deleting this column and this row, so I get 1 times negative 1. That's a negative 1, right? Negative 1 minus-- negative 1 times negative 1 is 1-- and so there's negative 1 minus 1. That looks good. Negative 2. Right? Negative, negative, negative. Yeah. OK.

And then I'm looking at row 2, column 2. So now I'm deleting this row and this column. All right. And so I have these sort of diagonals here. That's what I'm interested in, right?

So I get 3 times negative 1. That's negative 3. And then minus 1, because I have negative 1 times negative 1 is positive 1. So negative 3 minus 1. So I should get negative 4. Right? And then I'm over here.

So I need to delete this column and this row. So I get 3 times negative 1 is negative 3. Minus the negative 1, that's plus 1. So negative 3 plus 1 is negative 2.

And before I go on, I'm going to check those first 2 rows. Because if I made a mistake now, it's only going to get worse. What did I have? Yes. OK. So far so good. Whew. All right.

Next, final row. OK, final row is, I'm going to delete this column and row here, and I'm looking at this matrix. 1 times 0 is 0. 2 times negative 1 is negative 1, but I subtract that. So it's 0 minus negative 2, so it's 2.

And then row 3, column 2. So row 3, I delete row 3 and column 2. 3 times 0 is 0. 0 minus-- negative 1 times negative 1 is 1-- so 0 minus 1, that's negative 1.

And then the last spot, I'm deleting this row and this column. So 3 times 2 is 6, minus negative 1. I get 7. All right, let's check that row. 2, negative 1, 7. OK.

I have not done the cofactor matrix yet, because now I need to change the appropriate signs. OK, so if this is the matrix of minors, then if I want to change it to the cofactor matrix, what do I have to do? I'm going to scratch this out and write the cofactor matrix so that we can just change the signs appropriately. I'm going to do it all right here. And how does it work?

Well, remember I'm going to go plus, minus, plus; minus, plus, minus; plus, minus, plus. I have to do this grid that starts with plus and alternates minus.

So this sign stays the same, this sign switches, this sign stays the same. That's the plus, minus, plus.

This one is going to be minus, plus, minus. So the minus switches that. Plus keeps that the same. Minus switches that.

And then I was at minus, plus, minus. So I'm going to have plus, minus, plus. And so these two stay the same, and this one switches.

So a lot of things that were negative became positive. And I had to change-- maybe I threw in one negative, maybe not. But, so all the signs I kept, this one stayed the same, this one stayed the same, this one stayed the same, these two stayed the same, and then these four switched, because it's the plus, minus, plus sort of grid that I have to put on top of this.

OK, so that's the cofactor matrix. We're getting closer. OK, now we need the transpose of this, right? So if I look at the transpose-- actually, know what I'm going to do? Because I'm also just going to have to take the transpose and then multiply it by 1 over the determinant, I'm going to do that all at once. OK. Because we can do that all at once, and then we don't have to worry about it.

So A inverse I know is going to be negative 1/10, because the determinant was minus 10. So it's 1 over the determinant times the transpose of this matrix. So the transpose of this matrix-- remember what I'm going to do is essentially you fix the diagonal and you're going to flip. That's really what, in the square matrix, that's how you can think about it. But every column is going to become a row.

So I'm going to write this as my first row. This first column is going to become my first row. So it's going to be negative 2, 2, 2 as my first row.

And then the next column is going to be negative 1, negative 4, 1. I mean next row. I will take a column and change it to a row. The next row is going to be negative 1, negative 4, 1.

And then the last one. I take this column and I change it to a row. It's going to be 3, 2, 7.

OK. And because again, I want to make sure-- this one is really messy-- I want to make sure I have something similar for that, or exactly that. OK. I think I'm still doing all right.

Now, let's get to solving the problem. Because so far, we just were finding the inverse matrix. So I'm going to leave it in this form, instead of dividing by 10 in every spot, because that will be annoying. So let's think about how do I want to solve the system that I had. I had A*x equals b.

And actually, I mean, my strategy is to find the inverse matrix. I didn't talk to you about why we know the inverse matrix actually exists. But ultimately, you haven't even seen this yet in the lecture videos, really. Except that you know that the determinant of A being non-zero gives you an inverse matrix. That's all you know, I think, at this point.

You have the determinant of A. It's non-zero, so you can find an inverse matrix. Makes sense based on the formulation you have, because if the determinant is 0, then this quantity 1 over the determinant of A, you run into quite a bit of trouble. So that's just as a little sidebar, we know the inverse matrix exists for A.

So what we do-- this is again the strategy-- you multiply A inverse A times x on the left side.

Ooh. Is equal to-- sorry-- that should be the lowercase b. Should be a vector there.

It is equal to A inverse b on the right-hand side. And you notice, it's very important, in the matrix multiplication video we saw that it's very important the order in which you multiply matrices. And since I'm putting A inverse on the far left of this side of the equality, I have to put it on the far left of the right-hand side of the equality. Right? And in fact, you would run into trouble if you tried to switch the order of these. OK? We wouldn't be able to multiply them. All right?

So A inverse A, we know is just the identity matrix. So you get the identity matrix times x is equal to A inverse b. So you can find x by finding A inverse times b. Right? And so now we have A inverse. Let's see if we can solve the problem.

One point I want to make is that now that you have A inverse-- I've tried to ask you to solve the problem for two different b's-- you don't have to go and find A inverse again, right? You're done finding A inverse. You just now have to do the multiplication.

So now for number 1, we had b was equal to-- I'm going to write it here, so I don't have to keep looking over-- 1, 2, negative 3. So A inverse b is going to be equal to-- well I should get another vector, so I should just have three components here. And I'm probably going to have to write out what I get, because it might be long.

But let's see-- actually, you know what I'm going to do to make it easier? Because there's a lot of junk going on here. So what I'm going to do to make it easier is put the negative 1/10 in front to start. Because that negative 1/10 is going to come along with every term, so I'm just going to put the negative 1/10 in front and deal with it at the end. OK?

So now I'm just going to multiply b-- which is this 1, 2, negative 3-- by this big matrix here without the negative 1/10 in front. OK? So let's look at that.

We're just going to have first row times the column, and that's going to give me the first position. So negative 2 times 1 is negative 2. I'm going to write them all down. Plus 2 times 2 is 4. Plus 2 times negative 3 is negative 6. So that's the first position. We'll simplify in a moment.

So the next one, I get negative 1 times 1. That's negative 1. Then I get negative 4 times 2. That's negative 8. So minus 8. And then I get 1 times negative 3, so minus 3. So we've got two of the rows done. We just have to simplify them in a moment.

And now we just do this third component. So it's the third row of A inverse without that scalar in front, times the only column of b to give me the last position. Right? So 3 times 1 is 3, plus 2 times 2 is 4, so I get 3 plus 4, and then 7 times negative 3 is minus 21. OK.

So what do I get when I write it all out? I get negative 1/10. And then-- so negative 8 plus 4, that looks like a minus 4. Right? 8, 9, 10, 11, 12. That looks like a negative 12. It's a lot of adding for me. I make a lot of adding mistakes, so we should be careful. This looks like negative 14. OK.

So this is a matrix that, it's just a vector, right? All the negative signs will drop out. I'll get some fractions. But if it is the correct answer-- which I'm really hoping it is, because I just did this whole problem and I hope it's the correct answer-- if it's the correct answer, then what should it do?

When I take the original A that I had and I multiply it by this, I should get b. I should get 1, 2, negative 3. So you can check your work very easily to see if it works. You can take A times this, and see if you get b. Right? And then you'll know if this is the x we were looking for. OK?

And then let's look at number two. I just said that b equals [0, 0, 0]. And the point I want to make there is that since this has an inverse, A inverse-- since A has an inverse, A inverse b is going to be-- in this case-- A inverse times [0, 0, 0], which is going to give you [0, 0, 0].

So the only solution we have in this case-- because A inverse, if I look and I try and multiply every row by this column, right, I'm going to get 0 in the first spot, 0 in the second spot, and 0 in the third spot-- so the solution I get-- the x I'm looking for so that Ax equals [0, 0, 0]-- is [0, 0, 0]. And what I just want to mention to you, is that that is true because A is invertible. If A were not invertible, you could get other solutions. Other things might work. And that's also true, actually, in this case as well, but it's a little harder to see that it's-- that could be potentially a weird thing.

To solve A*x equals [0, 0, 0], it's sort of like, naturally we see [0, 0, 0] is a solution. Right away you can see that, and that's one that we get. The point I want to make is because A is invertible, that's the only solution. And if A were not invertible, you could get other solutions to that. So that's something that we haven't seen yet-- we haven't dealt with yet-- but that is something that can happen. So I just want to point out that there could be an oddity if A were not invertible. But since A is invertible, we get just one solution for both of these things. OK. So I'm going to go back and just remind you of a few things of how we found the inverse matrix, and then I will stop. So we were given a matrix A. And to go through the steps of finding the inverse matrix, what did we do? The first thing we did was we found the determinant.

Then we found the matrix of minors. And then I just took that matrix of minors, put the plus-minus grid on top of it so that I got the cofactor matrix. Right? And then once I had the cofactor matrix, you just have to transpose it.

So I came over here. I transposed that, and I put 1 over the determinant of A in front. So the scalar is 1 over the determinant of A, times the transpose of the cofactor matrix. And that's what gives me A inverse.

So there are a fair number of steps, but you can do them very systematically, and then you have the inverse matrix that you're looking for. And then you can solve for x, when you're looking for A*x equals b, and you know b and you know A. And you do this same process we just outlined here again, and that gives it to you.

OK, I think I'll stop there.