DAVID JORDAN: Hello, and welcome back to recitation.

So the problem I'd like to work with you today is this one here. It's just to compute this two-variable integral, and the integrand that we're going to be computing is e to the u over u.

And what you might notice right away is that this inner integral, it's an integral over u of e to the u over u, and this is not an integral that we have a nice formula for from one-variable calculus. So I'm going to suggest that, as you try to solve this, you think about how can you use the fact that this is a multivariable integral, maybe swapping the order of integration, et cetera, to solve this. So why don't you go ahead and work this problem on your own. Check back with me in a few minutes, and I'll see how I did it.

OK, welcome back. So as I suggested, I think what we should do is we should see what happens if we switch the order of integration. I don't know how to do this inside integral, and so maybe if we switch the order of integration, then something's going to work out.

So in order to get started doing that, we need to draw the region of integration, so why don't we do that over here. So I'll just walk over here. So we've got-- our variables are t and u, so I've drawn the t- and u-axes here. And now, let's look at the region of integration. So t is running from 0 to 1/4. So we'll just draw 1/4 about there.

And now the range for u, the bottom range is the square root of t, so I'm going to draw the curve u is the square root of t. It just looks like a parabola on its side. And then the top bound is at u equals 1/2. And notice that when t is 1/4, that means that u is 1/2, because u is just the square root of t. And so what we're really interested in is this region here, the region between u is the square root of t and between u is 1/2, so this is our region.

So let's rewrite the integral by swapping the order of integration, so I'll do that here. So now on the outside, we want to put the range of u first. So the range of u, we can see on the graph here, u ranges from 0 to 1/2, so that's going to be easy. And now t, so t is always starting right here at t equals 0, and

it's always ending at this curve, which is t equals u squared. So we have these little integrals here. And so our ranges for t is going to be t is running from 0 to u squared. Then we have the same integrand e to the u over u, and now we have dt du.

All right. Now we see that this was a nice thing to do, because look: The first integral that we need to take is an integral in t, but our integrand doesn't involve the variable t, so this is going to be a very easy integral to take. So I just take that integrand and I just multiply it by the constant t, so we just have e to the u over u times t, and then it's a definite integral which ranges from u squared to 0 du. And so this is just going to be-- 1/2 here-- this is really just going to be ue to the u du, all right?

So let me write that up over here again. So we're at integral from u equals 0 to 1/2 of ue to the u du. And now we want to remember the method of integration by parts from one-variable calculus. So integration by parts, you'll remember, will tell us that the integral of ue to the u is going to be ue to the u minus e to the u. So that's just applying integration by parts. And then this is a definite integral, so we have a range 1/2 to 0.

Well, now, we can just plug this in, so we get 1/2e to the 1/2 minus e to the 1/2 minus the quantity, so we just get 0 minus e to the 0. And so altogether, we have-- let's see, we have a negative e to the 1/2 and then we have a plus 1. And negative e to the 1/2 over 2, because we had 1/2 and a minus a whole, and then plus 1. And that's our solution.

MIT OpenCourseWare
http://ocw.mit.edu

18.02SC Multivariable Calculus
Fall 2010