DAVID JORDAN: Hello, and welcome back to recitation. In this problem, what I'd like us to do is I'd like us to sketch the graphs in three dimensions of these functions so z here is a function of x and y. On this second one, z is also a function of x and y. It just happens not to depend on y.

When you graph these, I'd suggest to consider slices, so what happens if you consider x equals 0 or if you consider z equals 0. As you graph these, let's see what you can do. So why don't you pause the video, and I'll check back with you in a moment, and I can show you how I solved these.

OK, welcome back. So why don't we start by looking at this function: z is the square root of x squared plus y squared. OK, I'll try to always draw my axes in the same way as we do in lecture. So x is pointing towards us, y to the right, and z up.

So as I suggested, I think a nice way to get started with these problems is to just try setting the variables x and y variously equal to 0, and then seeing-- instead of a surface in that case, then we'll get a curve, and we'll see what curve we get.

So for instance, if we set x equals to 0, then we just get z is the square root of y squared, so we just get that z is the absolute value of y. So what that means is that whatever this surface looks like, we know what it looks like if we slice it in the blackboard, in the plane of the blackboard. We know that it just looks like-- this is just the graph of the function absolute value of y, z equals absolute value of y.

So now, if you think about it, what I just said works just as well for x instead of for y. So if we were to graph this in the xz-plane where we set y equals to 0, then we would get-- OK, I'm going to try to draw this. So let me draw that in blue, actually. So the blue is in the xz-plane and the white is in the yz-plane.

OK, now I think what's going to be really illustrative is if we think about what happens as we fix values of z. Well, obviously, if we set z equals to 0, then there's just one solution, which is this point here. But what's going to be interesting is if we set z to be some positive value.

So, for instance, let's take z to be 2. So, for instance, we set z equals 2, then we get 2 equals the square root of x squared plus y squared. Solving this, this is the same as saying x squared plus y squared equals 4. So what that tells us is that at the height z equals 2, we're just going to have a circle of radius 2. This is just the equation for a circle of radius 2, and so at height 2, we just have a circle. And actually, as you can see, there's nothing special about 2. At every height, we're just going to have a circle, and so this is what's called a cone. OK.

Now for b, we can expect when we go over here to b that something funny is going to happen because it doesn't depend on y. So let's see if we can see how the fact that z doesn't depend on y, how this enters into our picture. So I'll just walk over here, and we'll consider z equals x squared. OK.

So again, we have our x-axis, x, y, and z axes. Now, let's consider what this looks like when we intersect with the xz-plane, so when we set y equals to 0. Well, setting y equal to 0 actually doesn't change the equation, and we get z equals x squared. So we know what that looks like. It's a parabola. And this parabola, I want you to think that it's, you know, coming out at us, so it's in the xz-plane, going in and out of the board.

But now if you think about it, what it means to say that this function doesn't depend on y, what that means is that we have the exact same picture at every value of y. So if we go out here, then we're going to have the same picture. And if we go over here, we're going to have the same picture. And, in fact, what you're going to get is you're going to get a prism. Oh, that's really hard to read. Let's see if we can-- so let me-- since that's a bit hard to read on the axes, let's draw this again. What we'll get is we're going to have a prism, which looks like a parabola, looks like a sheet that's just stretched out in the shape of a parabola. And so, this we could call a prism of a parabola.

Now let's see if we can get any more insight from these two pictures. So look what happened in this instance. So here, the function z, it obviously didn't depend on y. And we can see that by looking at the graph, because, you know, as you vary y, the picture had to be unchanged. So the fact that this was a prism in y and the fact that the function didn't depend on y are one and the same fact.

Now if we go over to the cone-- OK, so here, our function z very much depended on both x and y. But you notice that it depended on x and y only in the sense-- so z is actually equal just to the radius r, which is x squared plus y squared. So the fact that this cone-- thank you-- the square root of x squared plus y squared. So the fact that this only depends on the radius and not the relative angle of x and y is why we got what-- this is an example of a surface of revolution. So we can always expect that if the dependence

of z on the variables x and y, if you can actually just rewrite that as a dependence on r, then you'll get this nice radial symmetry, just like we had translational symmetry for the prism. And I think I'll leave it at that.

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