DAVID JORDAN: Hello, and welcome back to recitation. The problem I'd like to work with you today is the intersection of two parametrized lines.

So we have two lines here. L1, given with the parametrization in terms of the variable t. And L2, also given with the parametrization in terms of the variable t. So the first question that I want us to answer is do these lines intersect? And if so, then we want to find out where do they intersect.

So why don't you pause the video and work on this. And we can check back in a moment and we'll see how I solved it.

OK, welcome back. Let's get started. So we have these two lines in space. Before we start doing any computations, I find it useful to draw a picture. So let's see what's going on.

OK. So we have these two lines. We can just find some common points on the lines.

So, well, if we put in t equals 0 here, then it looks like we get the point 2 comma 1. OK. And now if we plug in, let's say-- t is minus 1 here-- then we get, if we plug in t equals minus 1 we get x is 3 and y is 0. So there's our line L1.

And now let's see, L2. If we plug in t equals 0, we get 2-- 1, 2, 3-- 4. And if we plug in, let's say, t equals minus 1 again, then we get 1 and 2. OK? So there is L2.

And indeed, it does look like they intersect. We could have probably guessed that they intersect by looking back over here at the formulas for L1 and L2, because we see that the sort of direction that this is moving in, we can take derivatives in t. And we see that L2 is, this line is moving in the direction 1 comma 2. And L1 is moving in the direction minus 1 comma 1. And so those directions are not parallel.

And so we know that the only way the two lines could fail to intersect is if they're parallel. So actually, even without drawing this, we could have guessed that these lines do intersect. So now we know that these lines intersect.

And in fact, it even looks, you know, it kind of looks like they intersect-- from our sketch-- at the point-- it looks like-- 1 comma 2. It seems to be the point of intersection. But, you know, we got a little bit lucky with our sketch here. So let's see if that's actually true. Let's see if we can verify this in the general way that we discussed in lecture.

So, now there is one place where we have to be careful. We have two lines here, and we parametrized both of these with the variable t. But we need to keep in mind that t is what's called a dummy variable. It doesn't have any geometric meaning to the problem.

And in particular, what I want to caution you about is if we just start solving these two equations algebraically as they're given to us with the variable t, the problem that we could run into is that, you kow, we're sort of moving-- as we vary t-- we're moving along this line, and as we vary t again, we're moving along this line. And you see we're moving at the same time. And so that's really solving a different problem. That's not asking about when did these lines intersect, but that would be asking about when do two particles on these lines collide, which is a harder problem.

So instead, what we need to do is we need to give a change of variables for the line L2. So what I want to do is I'm going to write L2. I'm just going to write the same equations, but I'm going to introduce a new variable, u.

So x is 2 plus u, and y is 4 plus 2u. OK. So now once we've done that, to find the point of intersection, well, the point of intersection is going to precisely be a point on L2 where the x-coordinate and the y-coordinate agree with another point on L1 with the same x- and y-coordinate. So that is, we have the, what we want to do is we want to set the x-coordinate for L2.

We want to set this equal to the x-coordinate for L1, which is 2 minus t. So this was for L1. And similarly here, we want to set the y-coordinate for L2 equal to the y-coordinate for L1.

So now if you think about it, if we can solve this system of equations, then what we've done is we've simultaneously found a point which is on L1 and on L2. And that's our goal. So that will be a point of intersection. OK, so now we just have this system of two linear equations and two variables, and we just need to solve it.

Now we could do, you know, in general, with an equation like this, we might try to add or subtract the equations. But this one is so simple, that I see that the top equation is just the same thing as t equaling to minus u. That's what the top equation says if we cancel the 2's.

And so if we plug that into the next equation, then we get 4 plus 2u equals 1 minus u. And so then we can solve this, and we get, so it looks like 3 equals minus 3u, which tells us that u equals minus 1, and then that tells us that t equals plus 1. OK?

So we found our parameters t and u. And we're not quite done yet. What we need to do is we need to go back to our parametrization. So let me go back over to our original parametrization here, and we have L1 was 2 minus t and 1 plus t. And over here, we found that t equals 1 was the value that we're after.

So that tells us that x is 1-- 2 minus 1-- and y is 2. Excuse me. I wrote that in a funny way. x is 1 and y is 2.

Now, just as a reality check. We also found that if we solved for L2, we wanted the variable u to be equal to minus 1. So we had 2 plus u, and 4 plus 2u.

And so let's see what happens when we plug in u equals minus 1 here. We again get x equals 2 plus minus 1 is 1. And y equals 4 plus minus 2 is 2. So we just double check that this is a point of intersection of both lines. And I'll leave it at that.

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