This time, we started solving differential equations.

This is the third lecture of the term, and I have yet to solve a single differential equation in this class.

Well, that will be rectified from now until the end of the term. So, once you learn separation of variables, which is the most elementary method there is, the single, I think the single most important equation is the one that's called the first order linear equation, both because it occurs frequently in models because it's solvable, and-- I think that's enough. If you drop the course after today you will still have learned those two important methods: separation of variables, and first order linear equations. So, what does such an equation look like? Well, I'll write it in there.

There are several ways of writing it, but I think the most basic is this. I'm going to use x as the independent variable because that's what your book does.

But in the applications, it's often t, time, that is the independent variable.

And, I'll try to give you examples which show that.

So, the equation looks like this.

I'll find some function of x times y prime plus some other function of x times y is equal to yet another function of x.

Obviously, the x doesn't have the same status here that y does, so y is extremely limited in how it can appear in the equation.

But, x can be pretty much arbitrary in those places.

So, that's the equation we are talking about, and I'll put it up. This is the first version of it, and we'll call them purple. Now, why is that called the linear equation? The word linear is a very heavily used word in mathematics, science, and engineering. For the moment, the best simple answer is because it's linear in y and y prime, the variables y and y prime.

Well, y prime is not a variable.

Well, you will learn, in a certain sense, it helps to think of it as one, not right now perhaps, but think of it as linear. The most closely analogous thing would be a linear equation, a real linear equation, the kind you studied in high school, which would look like this. It would have two variables, and, I guess, constant coefficients, equal c. Now, that's a linear equation.

And that's the sense in which this is linear.

It's linear in y prime and y, which are the analogs of the variables y1 and y2. A little bit of terminology, if c is equal to zero, it's called homogeneous, the same way this equation is called homogeneous, as you know from 18.02, if the right-hand side is zero.

So, c of x I should write here, but I won't.

That's called homogeneous. Now, this is a common form for the equation, but it's not what it's called standard form. The standard form for the equation, and since this is going to be a prime course of confusion, which is probably completely correct, but a prime source of confusion is what I meant.

The standard linear form, and I'll underline linear is the first co-efficient of y prime is taken to be one.

So, you can always convert that to a standard form by simply dividing through by it. And if I do that, the equation will look like y prime plus, now, it's common to not call it b anymore, the coefficient, because it's really b over a. And, therefore, it's common to adopt, yet, a new letter for it.

And, the standard one that many people use is p.

How about the right-hand side? We needed a letter for that, too. It's c over a, but we'll call it q. So, when I talk about the standard linear form for a linear first order equation, it's absolutely that that I'm talking about.

Now, you immediately see that there is a potential for confusion here because what did I call the standard form for a first-order equation? So, I'm going to say, not this. The standard first order form, what would that be? Well, it would be y prime equals, and everything else on the left-hand side.

So, it would be y prime. And now, if I turn this into the standard first-order form, it would be negative p of x y plus q of x.

But, of course, nobody would write negative p of x. So, now, I explicitly want to say that this is a form which I will never use for this equation, although half the books of the world do.

In short, this poor little first-order equation belongs to two ethnic groups. It's both a first order equation, and therefore, its standard form should be written this way, but it's also a linear equation, and therefore its standard form should be used this way. Well, it has to decide, and I have decided for it. It is, above all, a linear equation, not just a first-order equation. And, in this course, this will always be the standard form.

Now, well, what on earth is the difference?

If you don't do it that way, the difference is entirely in the sin(p). But, if you get the sign of p wrong in the answers, it is just a disaster from that point on. A trivial little change of sign in the answer produces solutions and functions which have totally different behavior. And, you are going to be really lost in this course. So, maybe I should draw a line through it to indicate, please don't pay any attention to this whatsoever, except that we are not going to do that. Okay, well, what's so important about this equation? Well, number one, it can always be solved. That's a very, very big thing in differential equations.

But, it's also the equation which arises in a variety of models. Now, I'm just going to list a few of them. All of them I think you will need either in part one or part two of problem sets over these first couple of problem sets, or second and third maybe.

But, of them, I'm going to put at the very top of the list of what I'll call here, I'll give it two names: the temperature diffusion model, well, it would be better to call it temperature concentration by analogy, temperature concentration model.

There's the mixing model, which is hardly less important.

In other words, it's almost as important.

You have that in your problem set.

And then, there are other, slightly less important models.

There is the model of radioactive decay.

There's the model of a bank interest, bank account, various motion models, you know, Newton's Law type problems if you can figure out a way of getting rid of the second derivative, some motion problems.

A classic example is the motion of a rocket being fired off, etc., etc., etc. Now, today I have to pick a model. And, the one I'm going to pick is this temperature concentration model.

So, this is going to be today's model.

Tomorrow's model in the recitation, I'm asking the recitations to, among other things, make sure they do a mixing problem, A) to show you how to do it, and B) because it's on the problem sets.

That's not a good reason, but it's not a bad one.

The others are either in part one or we will take them up later in the term. This is not going to be the only lecture on the linear equation.

There will be another one next week of equal importance.

But, we can't do everything today.

So, let's talk about the temperature concentration model, except I'm going to change its name.

I'm going to change its name to the conduction diffusion model.

I'll put conduction over there, and diffusion over here, let's say, since, as you will see, the similarities, they are practically the same model. All that's changed from one to the other is the name of the ideas.

In one case, you call it temperature, and the other, you should call it concentration. But, the actual mathematics isn't identical. So, let's begin with conduction. All right, so, I need a simple physical situation that I'm modeling.

So, imagine a tank of some liquid.

Water will do as well as anything.

And, in the inside is a suspended, somehow, is a chamber. A metal cube will do, and let's suppose that its walls are partly insulated, not so much that no heat can get through.

There is no such thing as perfect insulation anyway, except maybe an absolute perfect vacuum.

Now, inside, so here on the outside is liquid. Okay, on the inside is, what I'm interested in is the temperature of this thing.

I'll call that T. Now, that's different from the temperature of the external water bath.

So, I'll call that T sub e, T for temperature measured in Celsius, let's say, for the sake of definiteness. But, this is the external temperature. So, I'll indicate it with an e.

Now, what is the model? Well, in other words, how do I set up a differential equation to model the situation?

Well, it's based on a physical law, which I think you know, you've had simple examples like this, the so-called Newton's Law of cooling, -- -- which says that the rate of change, the temperature of the heat goes from the outside to the inside by conduction only. Heat, of course, can travel in various ways, by convection, by conduction, as here, or by radiation, are the three most common. Of these, I only want one, namely transmission of heat by conduction.

And, that's the way it's probably a little better to call it the conduction model, rather than the temperature model, which might involve other ways for the heat to be traveling. So, dt, the independent variable, is not going to be x, as it was over there.

It's going to be t for time. So, maybe I should write that down. t equals time.

Capital T equals temperature in degrees Celsius.

So, you can put in the degrees Celsius if you want.

So, it's proportional to the temperature difference between these two. Now, how shall I write the difference? Write it this way because if you don't you will be in trouble.

Now, why do I write it that way?

Well, I write it that way because I want this constant to be positive, a positive constant.

In general, any constant, so, parameters which are physical, have some physical significance, one always wants to arrange the equation so that they are positive numbers, the way people normally think of these things. This is called the conductivity. The conductivity of what?

Well, I don't know, of the system of the situation, the conductivity of the wall, or the wall if the metal were just by itself. At any rate, it's a constant. It's thought of as a constant.

And, why positive, well, because if the external temperature is bigger than the internal temperature, I expect T to rise, the internal temperature to rise. That means dT / dt, its slope, should be positive. So, in other words, if Te is bigger than T, I expect this number to be positive. And, that tells you that k must be a positive constant. If I had turned it the other way, expressed the difference in the reverse order, K would then be negative, have to be negative in order that this turn out to be positive in that situation I described. And, since nobody wants negative values of k, you have to write the equation in this form rather than the other way around.

So, there's our differential equation.

It will probably have an initial condition.

So, it could be the temperature at the starting time should be some given number, T zero.

But, the condition could be given in other ways.

One can ask, what's the temperature as time goes to infinity, for example?

There are different ways of getting that initial condition.

Okay, that's the conduction model.

What would the diffusion model be?

The diffusion model, mathematically, would be, word for word, the same.

The only difference is that now, what I imagine is I'll draw the picture the same way, except now I'm going to put, label the inside not with a T but with a C, C for concentration. It's in an external water bath, let's say. So, there is an external concentration. And, what I'm talking about is some chemical, let's say salt will do as well as anything. So, C is equal to salt concentration inside, and Ce would be the salt concentration outside, outside in the water bath.

Now, I imagine some mechanism, so this is a salt solution.

That's a salt solution. And, I imagine some mechanism by which the salt can diffuse, it's a diffusion model now, diffuse from here into the air or possibly out the other way.

And that's usually done by vaguely referring to the outside as a semi-permeable membrane, semi-permeable, so that the salt will have a little hard time getting through but permeable, so that it won't be blocked completely. So, there's a membrane.

You write the semi-permeable membrane outside, outside the inside. Well, I give up.

You know, membrane somewhere. Sorry, membrane wall.

How's that? Now, what's the equation?

Well, the equation is the same, except it's called the diffusion equation. I don't think Newton got his name on this. The diffusion equation says that the rate at which the salt diffuses across the membrane, which is the same up to a constant as the rate at which the concentration inside changes, is some constant, usually called k still, okay.

Do I contradict? Okay, let's keep calling it k1.

Now it's different, times Ce minus C.

And, for the same reason as before, if the external concentration is bigger than the internal concentration, we expect salt to flow in. That will make C rise.

It will make this positive, and therefore, we want k to be positive, just k1 to be positive for the same reason it had to be positive before.

So, in each case, the model that I'm talking about is the differential equation.

So, maybe I should, let's put that, make that clear. Or, I would say that this first order differential equation models this physical situation, and the same thing is true on the other side over here.

This is the diffusion equation, and this is the conduction equation. Now, if you are in any doubt about the power of differential equations, the point is, when I talk about this thing, I don't have to say which of these I'm following. I'll use neutral variables like Y and X to solve these equations.

But, with a single stroke, I will be handling those situations together. And, that's the power of the method. Now, you obviously must be wondering, look, these look very, very special. He said he was going to talk about the first, general first-order equation.

But, these look rather special to me.

Well, not too special. How should we write it?

Suppose I write, let's take the temperature equation just to have something definite.

Notice that it's in a form corresponding to Newton's Law.

But it is not in the standard linear form.

Let's put it in standard linear form, so at least you could see that it's a linear equation. So, if I put it in standard form, it's going to look like DTDTD little t plus KT is equal to K times TE. Now, compare that with the general, the way the general equation is supposed to look, the yellow box over there, the standard linear form.

How are they going to compare? Well, this is a pretty general function. This is general.

This is a general function of T because I can make the external temperature. I could suppose it behaves in anyway I like, steadily rising, decaying exponentially, maybe oscillating back and forth for some reason. The only way in which it's not general is that this K is a constant.

So, I will ask you to be generous.

Let's imagine the conductivity is changing over time.

So, this is usually constant, but there's no law which says it has to be. How could a conductivity change over time? Well, we could suppose that this wall was made of slowly congealing Jell-O, for instance. It starts out as liquid, and then it gets solid. And, Jell-O doesn't transmit heat, I believe, quite as well as liquid does, as a liquid would. Is Jell-O a solid or liquid?

I don't know. Let's forget about that.

So, with this understanding, so let's say not necessarily here, but not necessarily, I can think of this, therefore, by allowing K to vary with time.

And the external temperature to vary with time.

I can think of it as a general, linear equation.

So, these models are not special.

They are fairly general. Well, I did promise you I would solve an equation, and that this lecture, I still have not solved any equations.

OK, time to stop temporizing and solve.

So, I'm going to, in order not to play favorites with these two models, I'll go back to, and to get you used to thinking of the variables all the time, that is, you know, be eclectic switching from one variable to another according to which particular lecture you happened to be sitting in. So, let's take our equation in the form, Y prime plus P of XY, the general form using the old variables equals Q of X. Solve me.

Well, there are different ways of describing the solution process. No matter how you do it, it amounts to the same amount of work and there is always a trick involved at each one of them since you can't suppress a trick by doing the problem some other way.

The way I'm going to do it, I think, is the best.

That's why I'm giving it to you.

It's the easiest to remember. It leads to the least work, but I have colleagues who would fight with me about that point.

So, since they are not here to fight with me I am free to do whatever I like. One of the main reasons for doing it the way I'm going to do is because I want you to get what our word into your consciousness, two words, integrating factor. I'm going to solve this equation by finding and integrating factor of the form U of X. What's an integrating factor?

Well, I'll show you not by writing an elaborate definition on the board, but showing you what its function is. It's a certain function, U of X, I don't know what it is, but here's what I wanted to do. I want to multiply, I'm going to drop the X's a just so that the thing looks less complicated. So, what I want to do is multiply this equation through by U of X.

That's why it's called a factor because you're going to multiply everything through by it. So, it's going to look like UY prime plus PUY equals QU, and now, so far, it's just a factor. What makes it an integrating factor is that this, after I do that, I want this to turn out to be the derivative of something with respect to X. You see the motivation for that. If this turns out to be the derivative of something, because I've chosen U so cleverly, then I will be able to solve the equation immediately just by integrating this with respect to X, and integrating that with respect to X.

You just, then, integrate both sides with respect to X, and the equation is solved.

Now, the only question is, what should I choose for U?

Well, if you think of the product formula, there might be many things to try here.

But there's only one reasonable thing to try.

Try to pick U so that it's the derivative of U times Y.

See how reasonable that is? If I use the product rule on this, the first term is U times Y prime.

The second term would be U prime times Y.

Well, I've got the Y there. So, this will work.

It works if, what's the condition that you must satisfy in order for that to be true?

Well, it must be that after it to the differentiation, U prime turns out to be P times U.

So, is it clear? This is something we want to be equal to, and the thing I will try to do it is by choosing U in such a way that this equality will take place.

And then I will be able to solve the equation.

And so, here's what my U prime must satisfy.

Hey, we can solve that. But please don't forget that P is P of X. It's a function of X.

So, if you separate variables, I'm going to do this.

So, what is it, DU over U equals P of X times DX. If I integrate that, so, separate variables, integrate, and you're going to get DU over U integrates to the be the log of U, and the other side integrates to be the integral of P of X DX.

Now, you can put an arbitrary constant there, or you can think of it as already implied by the indefinite integral. Well, that doesn't tell us, yet, what U is. What should U be?

Notice, I don't have to find every possible U, which works. All I'm looking for is one.

All I want is a single view which satisfies that equation.

Well, U equals the integral, E to the integral of PDX.

That's not too beautiful looking, but by differential equations, things can get so complicated that in a week or two, you will think of this as an extremely simple formula.

So, there is a formula for our integrating factor.

We found it. We will always be able to write an integrating factor. Don't worry about the arbitrary constant because you only need one such U.

So: no arbitrary constant since only one U needed.

And, that's the solution, the way we solve the linear equation. OK, let's take over, and actually do it. I think it would be better to summarize it as a clear-cut method.

So, let's do that. So, what's our method?

It's the method for solving Y prime plus PY equals Q.

Well, the first place, make sure it's in standard linear form. If it isn't, you must put it in that form. Notice, the formula for the integrating factor, the formula for the integrating factor involves P, the integral of PDX.

So, you'd better get the right P.

Otherwise, you are sunk. OK, so put it in standard linear form. That way, you will have the right P. Notice that if you wrote it in that form, and all you remembered was E to the integral PDX, the P would have the wrong sign.

If you're going to write, that P should have a negative sign there. So, do it this way, and no other way. Otherwise, you will get confused and get wrong signs. And, as I say, that will produce wrong answers, and not just slightly wrong answers, but disastrously wrong answers from the point of view of the modeling if you really want answers to physical problems. So, here's a standard linear form. Then, find the integrating factor. So, calculate E to the integral, PDX, the integrating factor, and that multiply both, I'm putting this as both, underlined that as many times as you have room in your notes.

Multiply both sides by this integrating factor by E to the integral PDX. And then, integrate.

OK, let's take a simple example.

Suppose we started with the equation XY prime minus Y equals, I had X2, X3, something like that, X3, I think, yeah, X2.

OK, what's the first thing to do?

Put it in standard form. So, step zero will be to write it as Y prime minus one over X times Y equals X2.

Let's do the work first, and then I'll talk about mistakes. Well, we now calculate the integrating factor. So, I would do it in steps.

You can integrate negative one over X, right?

That integrates to minus log X. So, the integrating factor is E to the integral of this, DX.

So, it's E to the negative log X.

Now, in real life, that's not the way to leave that. What is E to the negative log X? Well, think of it as E to the log X to the minus one. Or, in other words, it is E to the log X is X. So, it's one over X.

So, the integrating factor is one over X.

OK, multiply both sides by the integrating factor.

Both sides of what? Both sides of this:

the equation written in standard form, and both sides. So, it's going to be one over XY prime minus one over X2 Y is equal to X2 times one over X, which is simply X. Now, if you have done the work correctly, you should be able, now, to integrate the left-hand side directly. So, I'm going to write it this way. I always recommend that you put it as extra step, well, put it as an extra step the reason for using that integrating factor, in other words, that the left-hand side is supposed to be, now, one over X times Y prime.

I always put it that because there's always a chance you made a mistake or forgot something.

Look at it, mentally differentiated using the product rule just to check that, in fact, it turns out to be the same as the left-hand side.

So, what do we get? One over X times Y prime plus Y times the derivative of one over X, which indeed is negative one over X2. And now, finally, that's 3A, continue, do the integration.

So, you're going to get, let's see if we can do it all on one board, one over X times Y is equal to X plus a constant, X, sorry, X2 over two plus a constant. And, the final step will be, therefore, now I want to isolate Y by itself.

So, Y will be equal to multiply through by X.

X3 over two plus C times X. And, that's the solution.

OK, let's do one a little slightly more complicated.

Let's try this one. Now, my equation is going to be one, I'll still keep two, Y and X, as the variables.

I'll use T and F for a minute or two.

One plus cosine X, so, I'm not going to give you this one in standard form either.

It's a trick question. Y prime minus sine X times Y is equal to anything reasonable, I guess.

I think X, 2X, make it more exciting.

OK, now, I think I should warn you where the mistakes are just so that you can make all of them.

So, this is mistake number one. You don't put it in standard form. Mistake number two: generally people can do step one fine.

Mistake number two is, this is my most common mistake, so I'm very sensitive to it. But that doesn't mean if you make it, you'll get any sympathy from me.

I don't give sympathy to myself.

You are so intense, so happy at having found the integrating factor, you forget to multiply Q by the integrating factor also. You just handle the left-hand side of the equation, if you forget about the right-hand side. So, the emphasis on the both here is the right-hand, please include the Q.

Please include the right-hand side.

Any other mistakes? Well, nothing that I can think of. Well, maybe only, anyway, we are not going to make any mistakes the rest of this lecture. So, what do we do?

We write this in standard form. So, it's going to look like Y prime minus sine X, sine X divided by one plus cosine X times Y equals, my heart sinks because I know I'm supposed to integrate something like this.

And, boy, that's going to give me problems.

Well, not yet. With the integrating factor?

The integrating factor is, well, we want to calculate the integral of negative sine X over one plus cosine.

That's the integral of PDX. And, after that, we have to exponentiate it. Well, can you do this?

Yeah, but if you stare at it a little while, you can see that the top is the derivative of the bottom.

That is great. That means it integrates to be the log of one plus cosine X. Is that right, one over one plus cosine X times the derivative of this, which is negative cosine X. Therefore, the integrating factor is E to that. In other words, it is one plus cosine X. Therefore, so this was step zero. Step one, we found the integrating factor. And now, step two, we multiply through the integrating factor.

And what do we get? We multiply through the standard for equation by the integrating factor, if you do that, what you get is, well, Y prime gets the coefficient one plus cosine X, Y prime minus sign X equals 2X. Oh, dear.

Well, I hope somebody would giggle at this point.

What's giggle-able about it? Well, that all this was totally wasted work. It's called spinning your wheels. No, it's not spinning your wheels. It's doing what you're supposed to do, and finding out that you wasted the entire time doing what you were supposed to do. Well, in other words, that net effect of this is to end up with the same equation we started with. But, what is the point?

The point of having done all this was because now the left-hand side is exactly the derivative of something, and the left-hand side should be the derivative of what?

Well, it should be the derivative of one plus cosine X times Y, all prime. Now, you can check that that's in fact the case. It's one plus cosine X, Y prime, plus minus sine X, the derivative of this side times Y. So, if you had thought, in looking at the equation, to say to yourself, this is a derivative of that, maybe I'll just check right away to see if it's the derivative of one plus cosine X sine. You would have saved that work.

Well, you don't have to be brilliant or clever, or anything like that. You can follow your nose, and it's just, I want to give you a positive experience in solving linear equations, not too negative.

Anyway, so we got to this point.

So, now this is 2X, and now we are ready to solve the equation, which is the solution now will be one plus cosine X times Y is equal to X2 plus a constant, and so Y is equal to X2 divided by X2 plus a constant divided by one plus cosine X. Suppose I have given you an initial condition, which I didn't.

But, suppose the initial condition said that Y of zero were one, for instance. Then, the solution would be, so, this is an if, I'm throwing in at the end just to make it a little bit more of a problem, how would I put, then I could evaluate the constant by using the initial condition. What would it be?

This would be, on the left-hand side, one, on the right-hand side would be C over two.

So, I would get one equals C over two.

Is that correct? Cosine of zero is one, so that's two down below. Therefore, C is equal to two, and that would then complete the solution.

We would be X2 plus two over one plus cosine X.

Now, you can do this in general, of course, and get a general formula. And, we will have occasion to use that next week. But for now, why don't we concentrate on the most interesting case, namely that of the most linear equation, with constant coefficient, that is, so let's look at the linear equation with constant coefficient, because that's the one that most closely models the conduction and diffusion equations. So, what I'm interested in, is since this is the, of them all, probably it's the most important case is the one where P is a constant because of its application to that.

And, many of the other, the bank account, for example, all of those will use a constant coefficient. So, how is the thing going to look? Well, I will use the cooling.

Let's use the temperature model, for example.

The temperature model, the equation will be DTDT plus KT is equal to. Now, notice on the right-hand side, this is a common error. You don't put TE.

You have to put KTE because that's what the equation says.

If you think units, you won't have any trouble.

Units have to be compatible on both sides of a differential equation. And therefore, whatever the units were for capital KT, I'd have to have the same units on the right-hand side, which indicates I cannot have KT on the left of the differential equation, and just T on the right, and expect the units to be compatible. That's not possible.

So, that's a good way of remembering that if you're modeling temperature or concentration, you have to have the K on both sides.

OK, let's do, now, a lot of this we are going to do in our head now because this is really too easy.

What's the integrating factor? Well, the integrating factor is going to be the integral of K, the coefficient now is just K.

P is a constant, K, and if I integrate KDT, I get KT, and I exponentiate that.

So, the integrating factor is E to the KT.

I multiply through both sides, multiply by E to the KT, and what's the resulting equation?

Well, it's going to be , I'll write it in the compact form. It's going to be E to the KT times T, all prime. The differentiation is now, of course, with respect to the time.

And, that's equal to KTE, whatever that is, times E to the KT. This is a function of T, of course, the function of little time, sorry, little T time. OK, and now, finally, we are going to integrate.

What's the answer? Well, it is E to the, so, are we going to get E to the KT times T is, sorry, K little t, K times time times the temperature is equal to the integral of KTE.

I'll put the fact that it's a function of T inside just to remind you, E to the KT, and now I'll put the arbitrary constant. Let's put in the arbitrary constant explicitly. So, what will T be?

OK, T will look like this, finally.

It will be E to the negative KT.

That's on the outside. Then, you will integrate.

Of course, the difficulty of doing this integral depends entirely upon how this external temperature varies.

But anyways, it's going to be K times that function, which I haven't specified, E to the KT plus C times E to the negative KT. Now, some people, many, in fact, that almost always, in the engineering literature, almost never write indefinite integrals because an indefinite integral is indefinite.

In other words, this covers not just one function, but a whole multitude of functions which differ from each other by an arbitrary constant.

So, in a formula like this, there's a certain vagueness, and it's further compounded by the fact that I don't know whether the arbitrary constant is here.

I seem to have put it explicitly on the outside the way you're used to doing from calculus.

Many people, therefore, prefer, and I think you should learn this, to do what is done in the very first section of the notes called definite integral solutions. If there's an initial condition saying that the internal temperature at time zero is some given value, what they like to do is make this thing definite by integrating here from zero to T, and making this a dummy variable. You see, what that does is it gives you a particular function, whereas, I'm sorry I didn't put in the DT one minus two. What it does is that when time is zero, all this automatically disappears, and the arbitrary constant will then be, it's T.

So, in other words, C times this, which is one, is that equal to [T?].

In other words, if I make this zero, that I can write C as equal to this arbitrary starting value.

Now, when you do this, the essential thing, and we're going to come back to this next week, but right away, because K is positive, I want to emphasize that so much at the beginning of the period, I want to conclude by showing you what its significance is. This part disappears because K is positive. The conductivity is positive.

This part disappears as T goes to zero.

This goes to zero as T goes to infinity.

So, this is a solution that remains.

This, therefore, is called the steady state solution, the thing which the temperature behaves like, as T goes to infinity. This is called the transient.

because it disappears as T goes to infinity.

It depends on the initial condition, but it disappears, which shows you, then, in the long run for this type of problem the initial condition makes no difference.

The function behaves always the same way as T goes to infinity.