Part II Problems

Problem 1: [Laplace transform] **(a)** Suppose that $F(s)$ is the Laplace transform of $f(t)$, and let $a > 0$. Find a formula for the Laplace transform of $g(t) = f(at)$ in terms of $F(s)$, by using the integral definition and making a change of variable. Verify your formula by using formulas and rules to compute both $\mathcal{L}(f(t))$ and $\mathcal{L}(f(at))$ with $f(t) = t^n$.

(b) Use your calculus skills: Show that if $h(t) = f(t) * g(t)$ then $H(s) = F(s)G(s)$. Do this by writing $F(s) = \int_0^\infty f(x)e^{-sx}\,dx$ and $G(s) = \int_0^\infty g(y)e^{-sy}\,dy$; expressing the product as a double integral; and changing coordinates using $x = t - \tau$, $y = \tau$.

(c) Use the integral definition to find the Laplace transform of the function $f(t)$ with $f(t) = 1$ for $0 < t < 1$ and $f(t) = 0$ for $t > 1$. What is the region of convergence of the integral?

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