

Laplace Table

Properties and Rules

We assume that $f(t) = 0$ for $t < 0$.

<u>Function</u>	<u>Transform</u>	
$f(t)$	$F(s) = \int_0^{\infty} f(t)e^{-st} dt$	(Definition)
$a f(t) + b g(t)$	$a F(s) + b G(s)$	(Linearity)
$e^{at} f(t)$	$F(s - a)$	(s -shift)
$f'(t)$	$sF(s) - f(0)$	
$f''(t)$	$s^2 F(s) - s f(0) - f'(0)$	
$f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - \dots - f^{(n-1)}(0)$	
$t f(t)$	$-F'(s)$	
$t^n f(t)$	$(-1)^n F^{(n)}(s)$	
$f(t - a)$	$e^{-as} F(s)$	(t -translation or t -shift)
$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$	(integration rule)
$\frac{f(t)}{t}$	$\int_s^{\infty} F(\sigma) d\sigma$	

Function Table

<u>Function</u>	<u>Transform</u>	<u>Region of convergence</u>
1	$1/s$	$\text{Re}(s) > 0$
e^{at}	$1/(s - a)$	$\text{Re}(s) > \text{Re}(a)$
t	$1/s^2$	$\text{Re}(s) > 0$
t^n	$n!/s^{n+1}$	$\text{Re}(s) > 0$
$\cos(\omega t)$	$s/(s^2 + \omega^2)$	$\text{Re}(s) > 0$
$\sin(\omega t)$	$\omega/(s^2 + \omega^2)$	$\text{Re}(s) > 0$
$e^{at} \cos(\omega t)$	$(s - a)/((s - a)^2 + \omega^2)$	$\text{Re}(s) > \text{Re}(a)$
$e^{at} \sin(\omega t)$	$\omega/((s - a)^2 + \omega^2)$	$\text{Re}(s) > \text{Re}(a)$
$\delta(t)$	1	all s
$\delta(t - a)$	e^{-as}	all s
$\cosh(kt) = \frac{e^{kt} + e^{-kt}}{2}$	$s/(s^2 - k^2)$	$\text{Re}(s) > k$
$\sinh(kt) = \frac{e^{kt} - e^{-kt}}{2}$	$k/(s^2 - k^2)$	$\text{Re}(s) > k$
$\frac{1}{2\omega^3}(\sin(\omega t) - \omega t \cos(\omega t))$	$\frac{1}{(s^2 + \omega^2)^2}$	$\text{Re}(s) > 0$

$\frac{t}{2\omega} \sin(\omega t)$	$\frac{s}{(s^2 + \omega^2)^2}$	$\operatorname{Re}(s) > 0$
$\frac{1}{2\omega}(\sin(\omega t) + \omega t \cos(\omega t))$	$\frac{s^2}{(s^2 + \omega^2)^2}$	$\operatorname{Re}(s) > 0$
$t^n e^{at}$	$n!/(s - a)^{n+1}$	$\operatorname{Re}(s) > \operatorname{Re}(a)$
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$	$\operatorname{Re}(s) > 0$
t^a	$\frac{\Gamma(a + 1)}{s^{a+1}}$	$\operatorname{Re}(s) > 0$

MIT OpenCourseWare
<https://ocw.mit.edu>

18.04 Complex Variables with Applications
Spring 2018

For information about citing these materials or our Terms of Use, visit: <https://ocw.mit.edu/terms>.