

18.04 Recitation 6
Vishesh Jain

1. Let $A = \{z : |z| \leq 2\}$, and let $u(x, y)$ be a harmonic function on A . Let $B = \{z : |z| = 2\}$. Express the following in terms of u and B :
 - 1.1. The maximum value of u on A .
 - 1.2. The minimum value of u on A .
 - 1.3. The value $u(0, 0)$.

2. Let $\Phi(z) = \phi(z) + i\psi(z)$ be an analytic function mapping a region B to another region A . Let $u(x, y)$ be a harmonic function on A .
 - 2.1. Under the assumption that A is simply connected, show that $u(\phi(x, y), \psi(x, y))$ is a harmonic function on B .
 - 2.2. Can we drop the assumption that A is simply connected?

3. Consider the complex potential for the double source: $\Phi(z) = \log(z - 1) + \log(z + 1) = \log(z^2 - 1)$.
 - 3.1. Find the flow F .
 - 3.2. Show that on the y -axis, the flow is along the axis.
 - 3.3. What are the stagnation points for this flow?
 - 3.4. See the notes for Topic 6 to see the stream lines for this potential and some further discussion.

MIT OpenCourseWare
<https://ocw.mit.edu>

18.04 Complex Variables with Applications
Spring 2018

For information about citing these materials or our Terms of Use, visit: <https://ocw.mit.edu/terms>.