18.04 Recitation 2
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1.1. Show directly using the definition of the complex derivative that $\bar{z}$ is not complex differentiable.

Ans: Follows from the next part.

1.2. What are the values that $\lim_{z\to 0} \frac{\bar{z}}{z}$ can attain?

Ans: You can get any value on the unit circle. Use the parameterization $z = re^{i\theta}$, for which the limit becomes $\lim_{r\to 0} \frac{re^{-i\theta}}{re^{i\theta}} = e^{-2i\theta}$.

2.1. What are the real and imaginary parts of $\cos(z)$? Of $\sin(z)$?

Ans: $\cos(z) = \cos x \cosh y - i \sin x \sinh y$ and $\sin(z) = \sin(x)\cosh(y) + i\cos(x)\sinh(y)$.

2.2. What are these real and imaginary parts for $z = x + i0$? What about for $z = 0 + iy$?

Ans: Follows from above.

2.3. Is it true that $\cos(z)$ and $\sin(z)$ are bounded functions?

Ans: No. Can see this using the expressions in 2.1.

2.4. Is it true that $\cos^2 z + \sin^2 z = 1$?

Ans: Yes. Compute directly using 2.1.

3.1 Show that e^z is continuous as a function of z.

Ans: Write down the real and imaginary parts, and check that each of them is continuous.

3.2. Use this to show that $\cos(z)$ and $\sin(z)$ are continuous as functions of z.

Ans: Same as above.

3.3. Is $\bar{z}$ continuous as a function of z? Is this consistent with Problem 1?

Ans: Yes, it is continuous. Yes, it is consistent, since Problem 1 is about differentiability and there are certainly continuous functions which are not differentiable.

4.1. Express the following functions in the form $f(z) = u(x, y) + iv(x, y)$: e^z, z^2, $\cos(z)$ and $\sin(z)$ (see also Problem 2)

4.2. Compute the partial derivatives u_x, u_y, v_x, v_y for each of these functions. Do they satisfy the Cauchy-Riemann equations?

4.3. What is $f'(z)$ in each of these cases?

4.3. Repeat this for $\bar{z}$. Is this consistent with Problem 1?

Ans: See notes for these computations. The computation for $\cos(z)$ is also on the second problem set. For 4.3, the Cauchy-Riemann equations are not satisfied, which is consistent with what we saw in Problem 1.