18.04 Problem Set 2, Spring 2018

Calendar
M Feb. 12: Reading: Topic 2 sections 1-5
W Feb. 14: Reading: Topic 2 sections 6-9
R Feb. 15: Recitation
F Feb. 16: Reading: Review of 18.02

Coming next

Feb. 20-23: Analytic functions; Cauchy's theorem

Problem 1. (20: 10,10 points)
(a) Show that $\cos(z)$ is an analytic for all z, i.e. it's an entire function. Compute its derivative and show it equals $-\sin(z)$.

(b) Give the region where $\cot(z)$ is analytic. Compute its derivative.

Problem 2. (20: 10,10 points)
(a) Let $P(z) = (z - r_1)(z - r_2) \dots (z - r_n)$. Show that $\dfrac{P'(z)}{P(z)} = \displaystyle\sum_{j=1}^{n} \frac{1}{z - r_j}$

Suggestion: try $n = 2$ and $n = 3$ first.

(b) Compute and simplify $\dfrac{d}{dz}\left(\dfrac{az+b}{cz+d}\right)$.
What happens when $ad - bc = 0$ and why?

Problem 3. (10 points)
Why does $\log(e^z)$ not always equal z?

Hint: This is true for any branch of log. Start with the principal branch.

Problem 4. (20: 10,10 points)
(a) Let $f(z)$ be analytic in a D a disk centered at the origin. Show that $F_1(z) = \overline{f(\overline{z})}$ is analytic in D.

(b) Let $f(z)$ be as in part (a). Show that $F_2(z) = f(\overline{z})$ is not analytic unless f is constant.

Hint for both parts: Use the Cauchy-Riemann equations.

Problem 5. (10 points)
Let $f(z) = |z|^2$. Show the $\dfrac{df}{dz}$ exists at $z = 0$, but nowhere else.

Problem 6. (10 points)
Using the principal branch of log give a region where $\sqrt{z^2 - 1}$ is analytic.