Concept questions

Concept question 1. Normal distributions
$X$ has normal distribution, standard deviation $\sigma$.

(a) $P(-\sigma < X - \mu < \sigma)$ is approximately
   (i) 0.025  (ii) 0.16  (iii) 0.68  (iv) 0.84  (v) 0.95

(b) $P(X > \mu + 2\sigma)$ is approximately
   (i) 0.025  (ii) 0.16  (iii) 0.68  (iv) 0.84  (v) 0.95

Board questions

Problem 1. Standardization
Suppose $X$ is a random variable with mean $\mu$ and standard deviation $\sigma$. Let $Z$ be the standardization of $X$.

(a) Give the formula for $Z$ in terms of $X$, $\mu$ and $\sigma$.

(b) Use the algebraic properties of mean and variance to show $Z$ has mean 0 and standard deviation 1.

Problem 2. CLT
(a) Carefully write the statement of the central limit theorem.

(b) To head the newly formed US Dept. of Statistics, suppose that 50% of the population supports the team of Alessandre, Gabriel, Sarah and So Hee, 25% support Jen and 25% support Jerry.

A poll asks 400 random people who they support. What is the probability that at least 55% of those polled prefer the team?

(c) What is the probability that less than 20% of those polled prefer Jen?

Problem 3. Sampling from the standard normal distribution
How would you approximate a single random sample from a standard normal distribution using 9 rolls of a ten-sided die?

Note: $\mu = 5.5$ and $\sigma^2 = 8.25$ for a single roll of a 10-sided die.

Hint: CLT is about averages.
Extra problems

Bonus problem
An accountant rounds to the nearest dollar. We’ll assume the error in rounding is uniform on [-0.5, 0.5]. Estimate the probability that the total error in 300 entries is more than $5.