

# Class 11 in-class problems, 18.05, Spring 2022

## Concept questions

### Concept question 1. Learning from experience

(a) Which treatment would you choose?

1. Treatment 1: cured 100% of patients in a trial.
2. Treatment 2: cured 95% of patients in a trial.
3. Treatment 3: cured 90% of patients in a trial.

(b) Which treatment would you choose?

1. Treatment 1: cured 3 out of 3 patients in a trial.
2. Treatment 2: cured 19 out of 20 patients treated in a trial.
3. Standard treatment: cured 90000 out of 100000 patients in clinical practice.

## Board questions

### Problem 1. Learning from data

- A certain disease has a prevalence of 0.005.
- A screening test has 2% false positives and 1% false negatives.

Suppose a random patient is screened and has a positive test.

- (a) Represent this information with a tree and use Bayes' theorem to compute the probabilities the patient does and doesn't have the disease.
- (b) Identify the data, hypotheses, likelihoods, prior probabilities and posterior probabilities.
- (c) Make a full [likelihood table](#) containing all hypotheses and possible test data.
- (d) Redo the computation using a [Bayesian update table](#). Match the terms in your table to the terms in your previous calculation.

### Problem 2. Dice

I have five dice: 4-sided, 6-sided, 8-sided, 12-sided, 20-sided.

I pick one at random, roll it and report that the roll was a 13.

Goal: Find the probabilities the die is 4, 6, 8, 12 or 20 sided.

- (a) Identify the hypotheses.
- (b) Make a likelihood table with columns for the data 'rolled a 13', 'rolled a 5' and 'rolled a 9'.
- (c) Make a Bayesian update table and compute the posterior probabilities that the chosen die is each of the five dice.
- (d) Same question if I had reported a 5.
- (e) Same question if I had reported a 9.



### Problem 3. Iterated updates

Suppose I rolled a 9 and then a 5.

(a) Do the Bayesian update in two steps:

Step 1: First update for the 9.

Step 2: Then update the update for the 5.

(b) Do the Bayesian update in one step.

That is, the data is  $\mathcal{D} = \text{'9 followed by 5'}$

### Problem 4. Probabilistic prediction (Probably won't get here till next time)

With the same setup as before let:

$\mathcal{D}_1 = \text{result of first roll,}$   $\mathcal{D}_2 = \text{result of second roll}$

(a) Find  $P(\mathcal{D}_1 = 5)$ .

(b) Find  $P(\mathcal{D}_2 = 4 | \mathcal{D}_1 = 5)$ .

### Extra problems

#### Extra 1. Bayesian updating: terminology, trees, tables

I have a bag with one 4-sided die and 999 6-sided dice. I pick one at random and roll it.

Suppose I get a 3.

Goal: find the probabilities the chosen die was 4-sided or 6-sided.

(a) Identify the hypotheses.

(b) Use Bayes' theorem to compute the posterior probabilities. Organize the computation using trees.

(c) Connect all the Bayesian updating terminology with the parts of the computation.

(d) Redo the computation using a Bayesian updating table.

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18.05 Introduction to Probability and Statistics

Spring 2022

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